# Correlation-Enhanced Heat-Bath Algorithmic Cooling

**Nayeli A. Rodríguez-Briones**

R. Laflamme, E. Martín-Martínez, A. Kempf

25 June, 2019

# *Outline*

1) Introduction to algorithmic cooling

2) Improvement due to correlated relaxation processes between qubit-environment

3) Improvement due to correlations present in the initial state

# Introduction

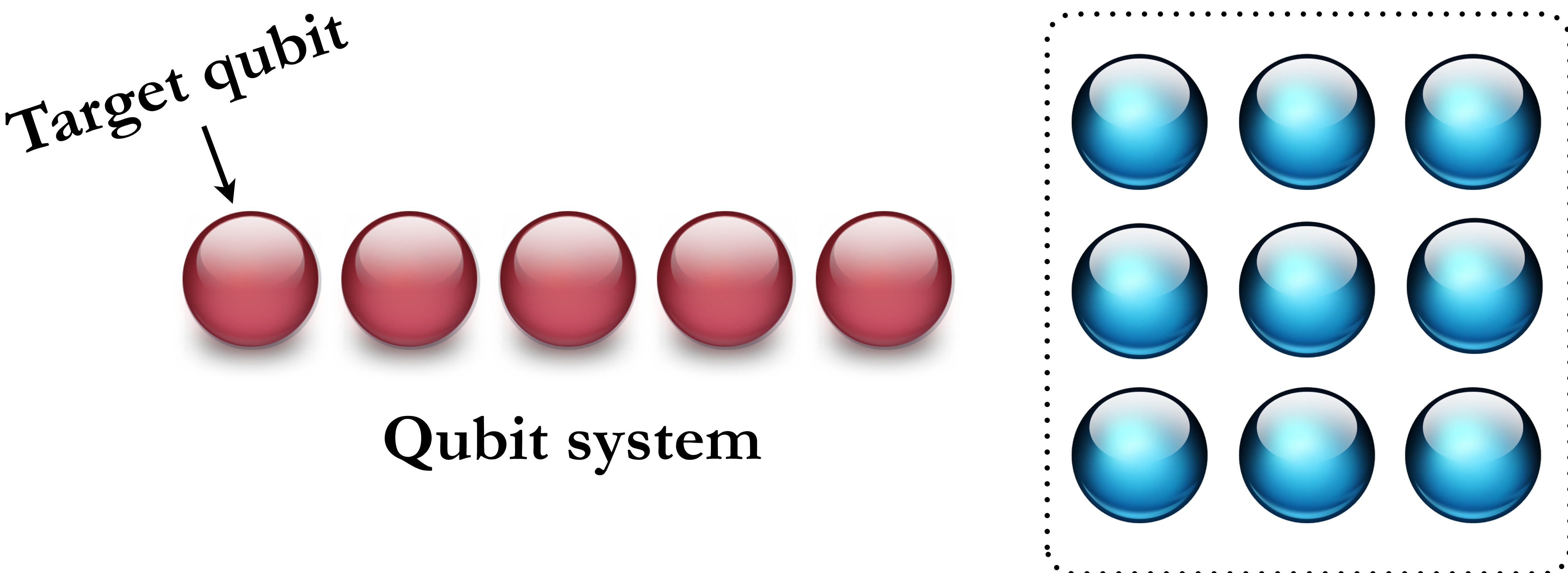
## Cooling physical systems

**Processing of information** ⟶ **Entropy manipulation**

Tools of quantum information processing give us new techniques for cooling, in non conventional ways.

# Algorithmic Cooling

How can we cool some qubits using unitary operations and contact with a bath?

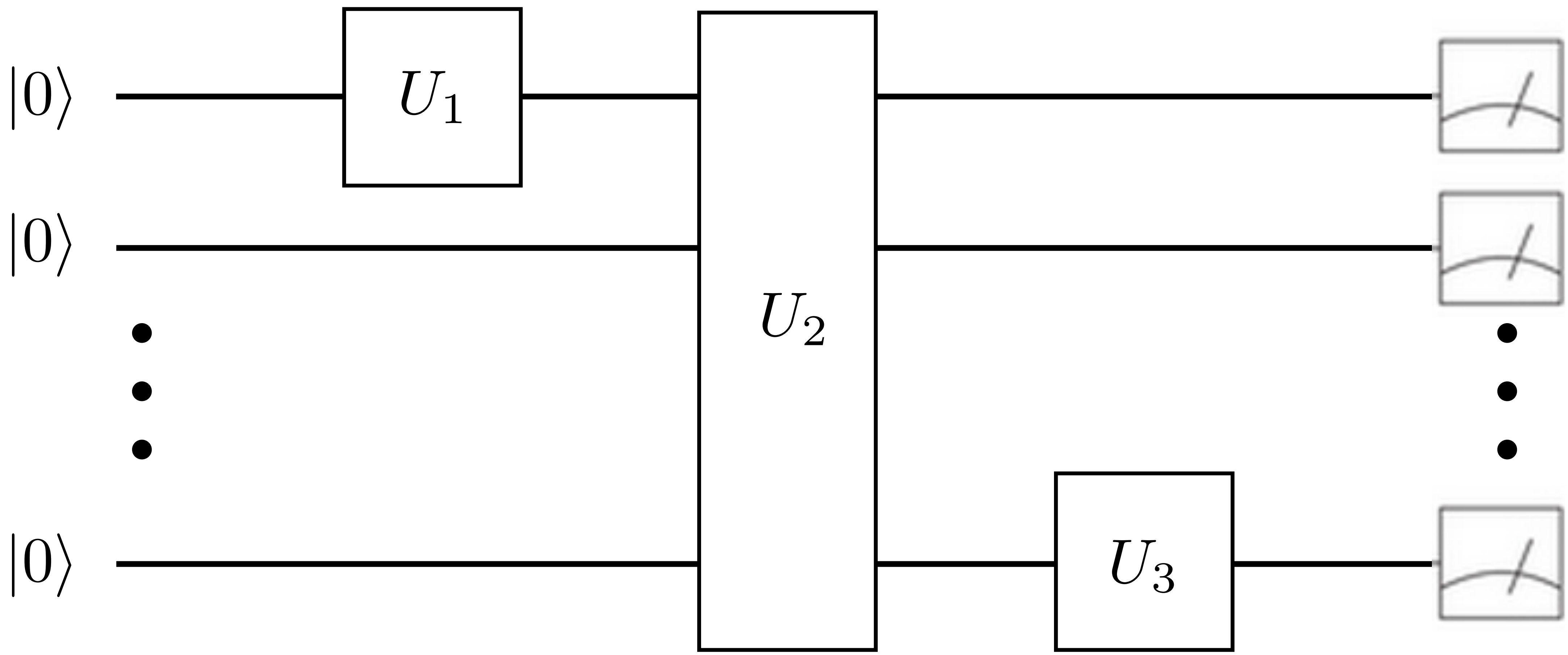



Heat-bath

(and projective measurements not allowed. *)

# Motivation

Preparation of highly pure quantum states



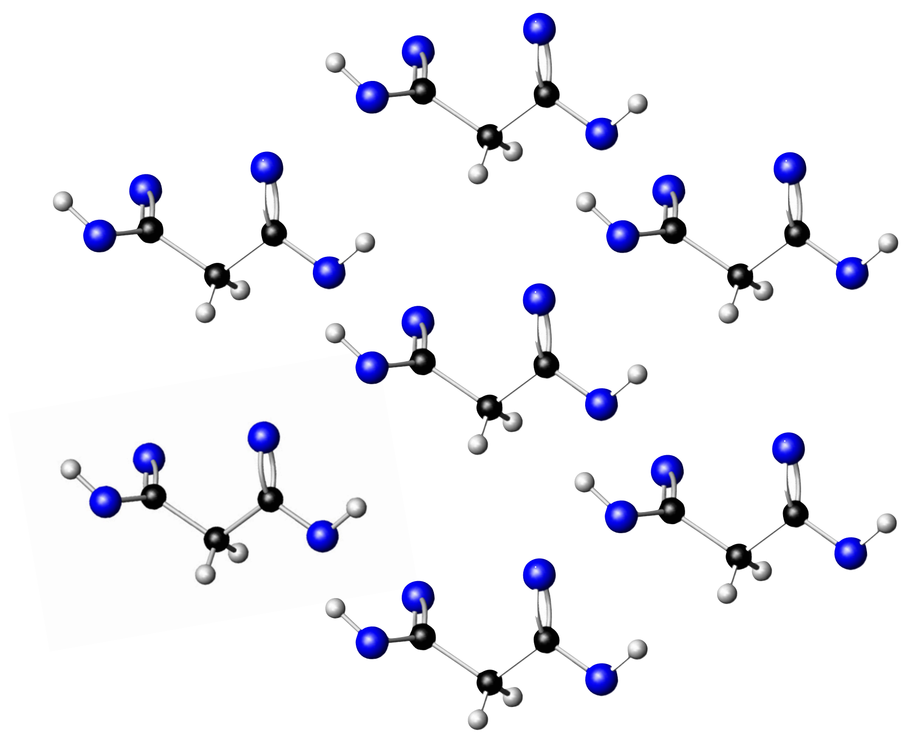

A quantum computer needs a constant supply of highly pure qubits:

- As initial state (ex. |0000>) for quantum algorithms

-As ancillas for quantum error correction

# State preparation without projective measurements is a big challenge in Quantum Computing

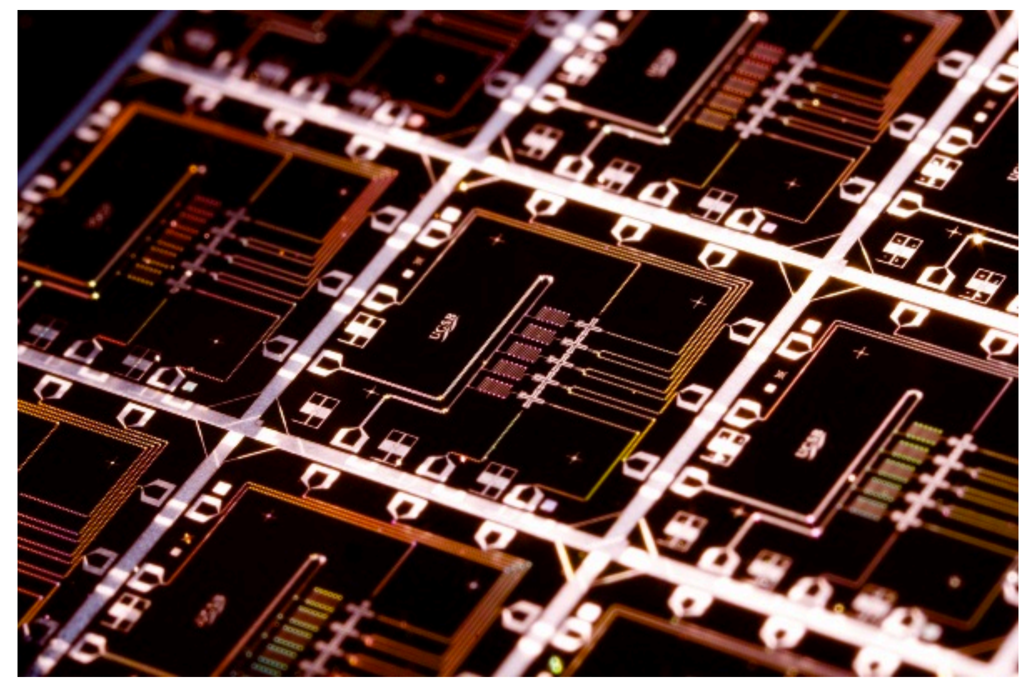
**Ensemble implementations**



Ex. NMR, ESR

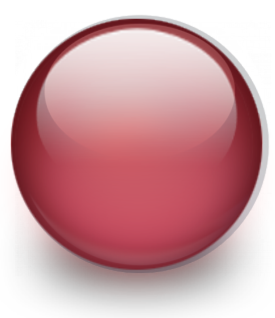
Highly mixed states

**In the lab**



**Non-perfect** projective measurements

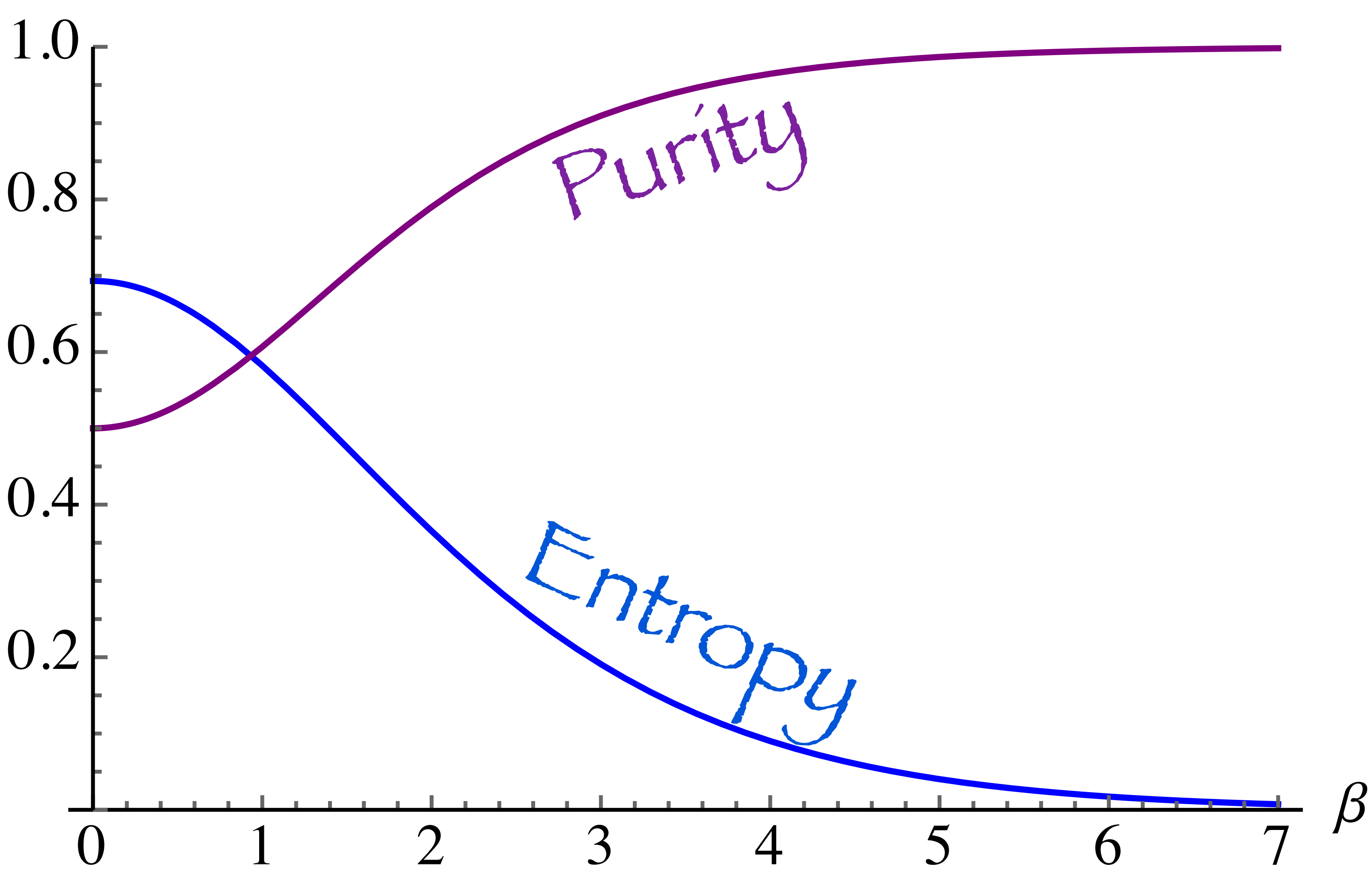
# For the target qubit:



$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Entropy $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$

Purity $\mathcal{P} = \mathrm{Tr}\left(\rho^2\right)$




Local thermal state

$$\rho = \frac{e^{-\beta H}}{Z}, \quad H = \frac{1}{2}\omega\sigma_z$$

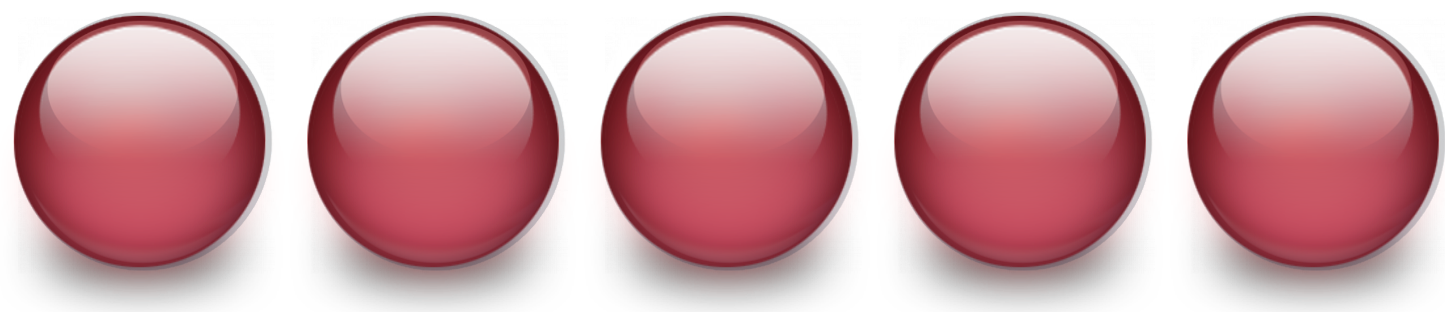
$$\mathcal{P} = \frac{1}{2}(1 + \tanh^2[\omega\beta/2])$$

$$T = \beta^{-1}$$

# Chapter one

# Introduction to algorithmic cooling

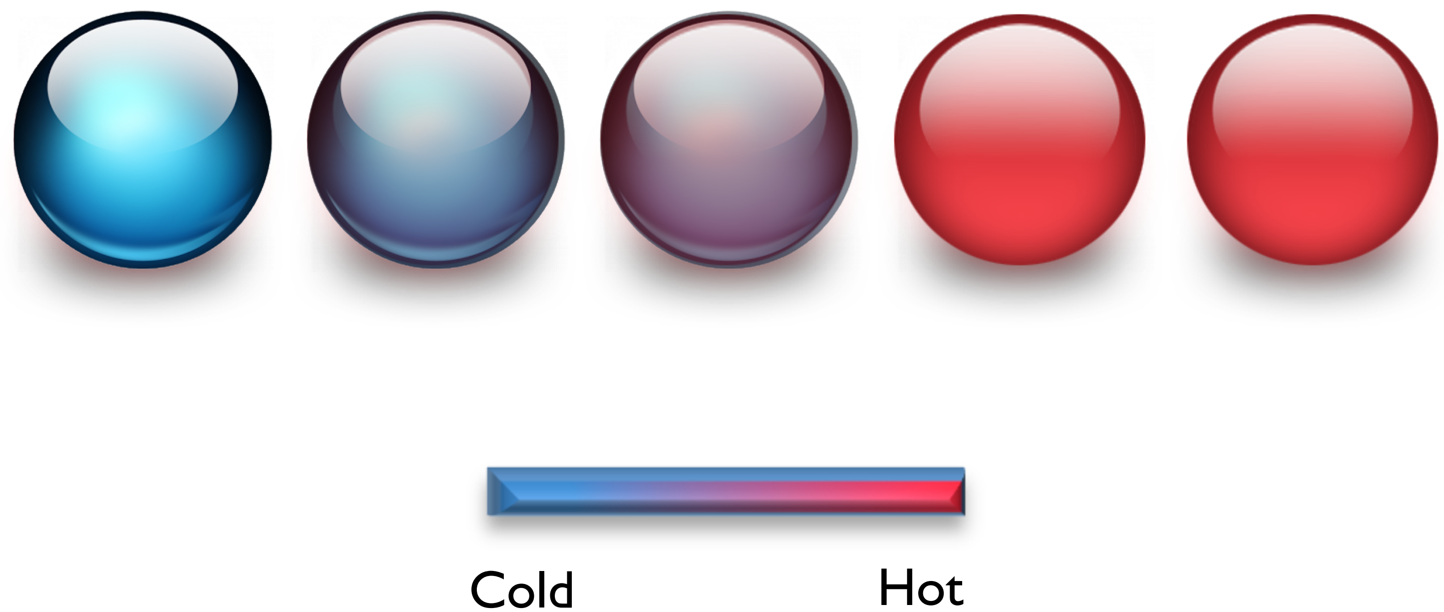
# Algorithmic Cooling (AC)



"A quantum mechanical heat engine"

* Schulman and Vazirani, 1990

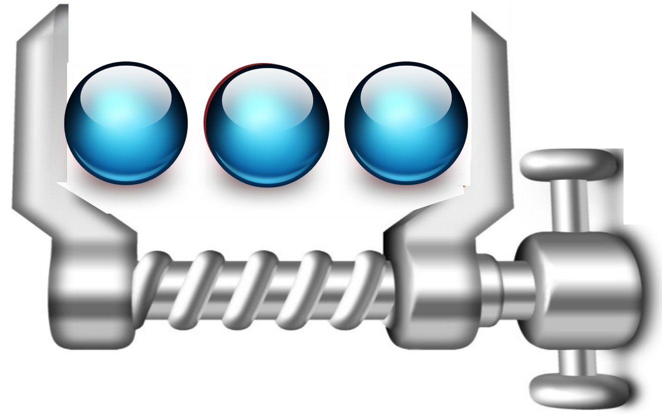
# Algorithmic Cooling (AC)




The idea is to re-distribute the entropy within a group of qubits using a unitary operation $U$.

$$\rho_{1\mathrm{st}} = \mathrm{Tr}_{\overline{1\mathrm{st}}}\left(U\rho_{\mathrm{total}}U^{\dagger}\right) \qquad S\left(\rho_{1\mathrm{ST}}\right) \to 0$$

# Simple example

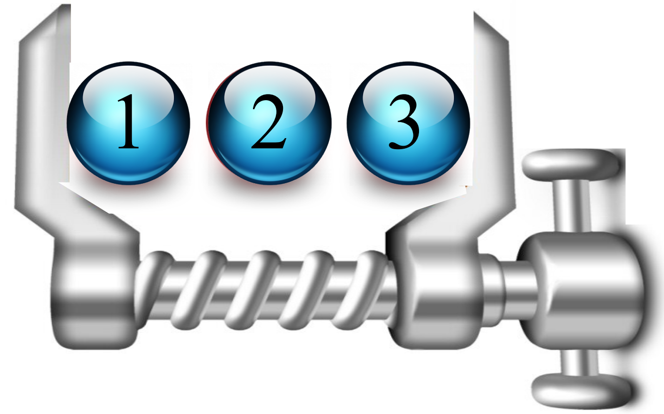


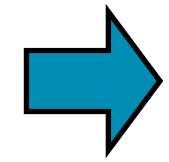
$$\rho_{\epsilon_0} = \frac{1}{2}\begin{pmatrix} 1+\epsilon_0 & 0 \\ 0 & 1-\epsilon_0 \end{pmatrix}$$

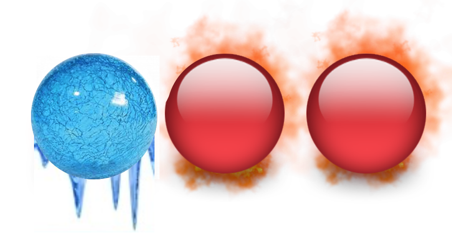
$$\mathrm{diag}\left(\rho_\epsilon^{\otimes 3}\right) = \frac{1}{2^3}\begin{bmatrix} (1+\epsilon_0)^3 \\ (1+\epsilon_0)^2(1-\epsilon_0) \\ (1+\epsilon_0)^2(1-\epsilon_0) \\ (1+\epsilon_0)(1-\epsilon_0)^2 \\ (1+\epsilon_0)^2(1-\epsilon_0) \\ (1+\epsilon_0)(1-\epsilon_0)^2 \\ (1+\epsilon_0)(1-\epsilon_0)^2 \\ (1-\epsilon_0)^3 \end{bmatrix}$$

000
001
010
011
100
101
110
111

# Simple example

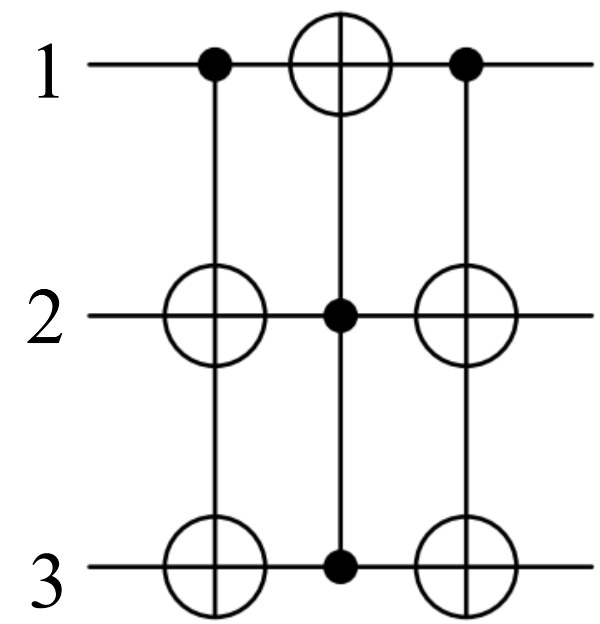




$$U = |011\rangle\langle 100| + |100\rangle\langle 011|$$

$U \equiv$




Example, when $\beta = \dfrac{1}{k_B T_0} \ll 1$:

Target qubit effective temperature

$$T_0 \rightarrow \frac{2}{3} T_0$$

Second and third qubit effective temperatures

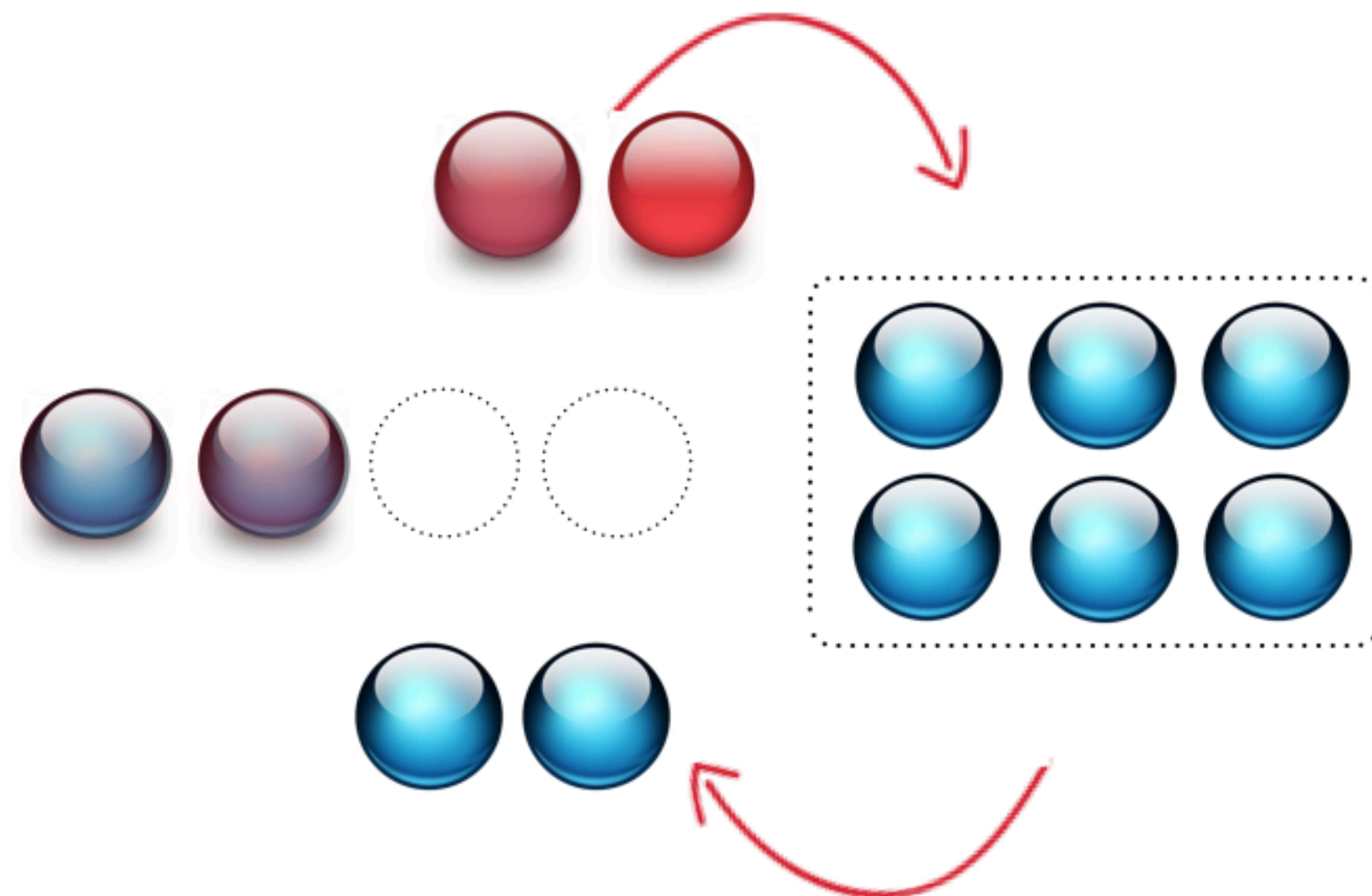
$$T_0 \rightarrow 2T_0$$

# Limits of entropy compression using unitary dynamics for a close system

- Shannon Entropy bound
  (Limited by unitary dynamics)
- Conservation of eigenvalues of the total system
- Still uses a lot of resources

  Ex. $\epsilon_b \sim 10^{-5} \longrightarrow 10^{12}$ qubits

# Heat-Bath Algorithm Cooling



**Refreshing step**

$$\rho' \rightarrow \rho'' = \mathrm{Tr}_m\left(\rho\right) \otimes \rho_{\epsilon_b}^{\otimes m}$$

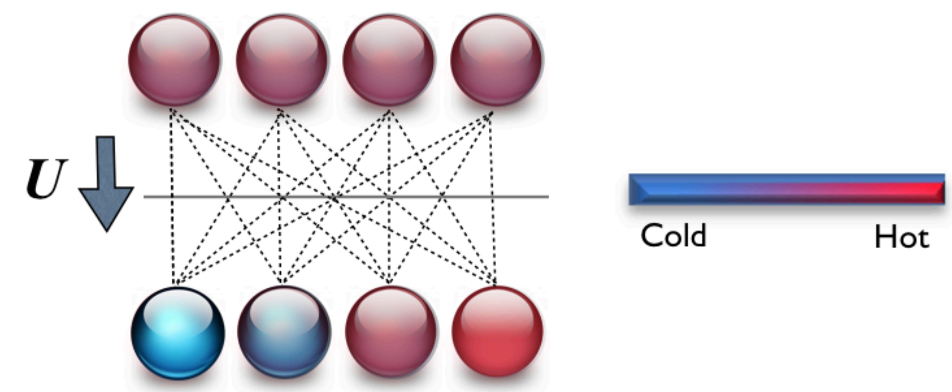
P. O. Boykin, et al. Proceedings of the National Academy of Sciences 99, 3388 (2002).

# The Partner Pairing Algorithm (PPA-HBAC)

PPA-HBAC is the iteration of the two steps:

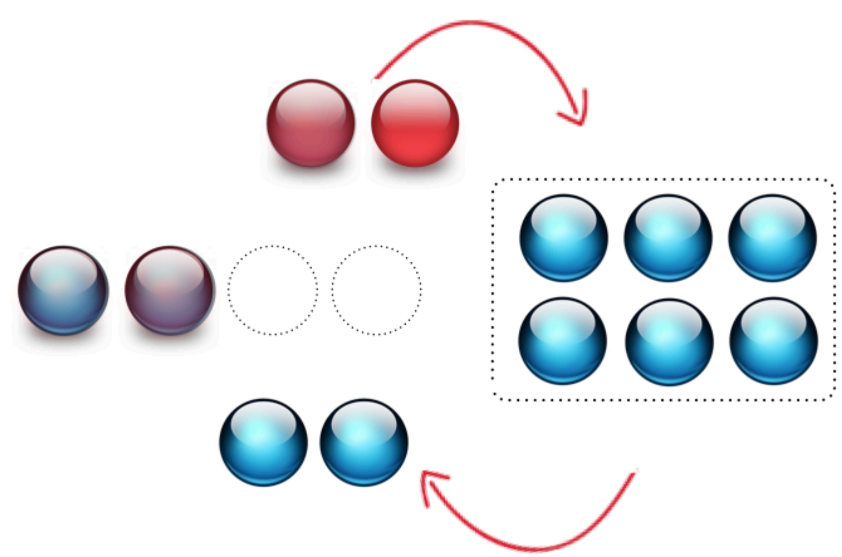
① **Entropy Compression Step.**

$$\rho \to \rho' = U \rho U^\dagger$$




$U \equiv$ **Descending SORT of the diagonal elements.**

② **Refresh Step.**



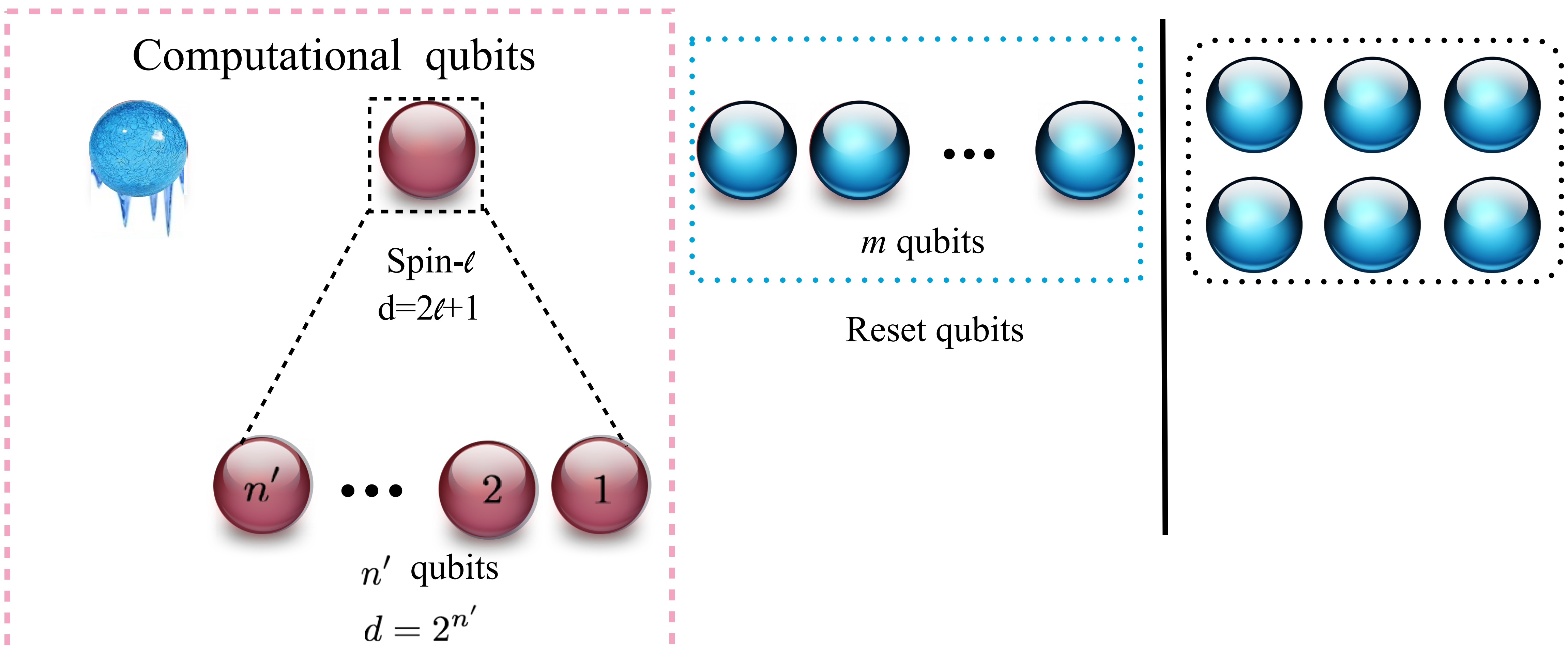
$$\rho' \to \rho'' = \mathrm{Tr}_m(\rho) \otimes \rho_{\epsilon_b}^{\otimes m}$$

L. J. Schulman, T. Mor, and Y. Weinstein, Phys. Rev. Lett. 94, 120501 (2005).

# Cooling Limit

$$\Phi\left(\rho\right) := \mathrm{Tr}_m\left(U\left(\rho\right)\rho\, U^{\dagger}\left(\rho\right)\right)\otimes\rho_b^{\otimes m}$$

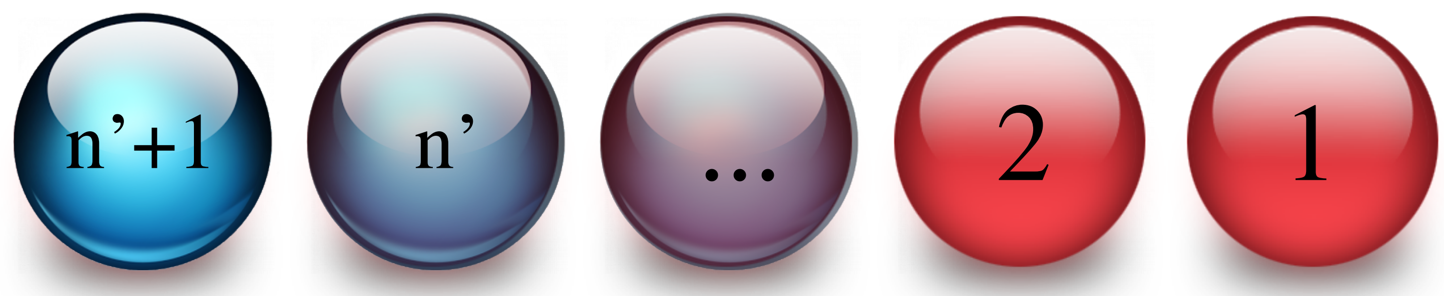
In the limit: $\rho = \Phi\left(\rho\right)$




Rodríguez-Briones and Laflamme: PRL. 116,170501, 2016

## Steady State

$$\rho_T^{\infty} = \bigotimes_{j=1}^{n'+1} \rho_{j^{th}}^{\infty} = \bigotimes_{j=1}^{n'+1} \frac{1}{2}\begin{pmatrix} 1+\epsilon_j^{\infty} & 0 \\ 0 & 1-\epsilon_j^{\infty} \end{pmatrix}$$

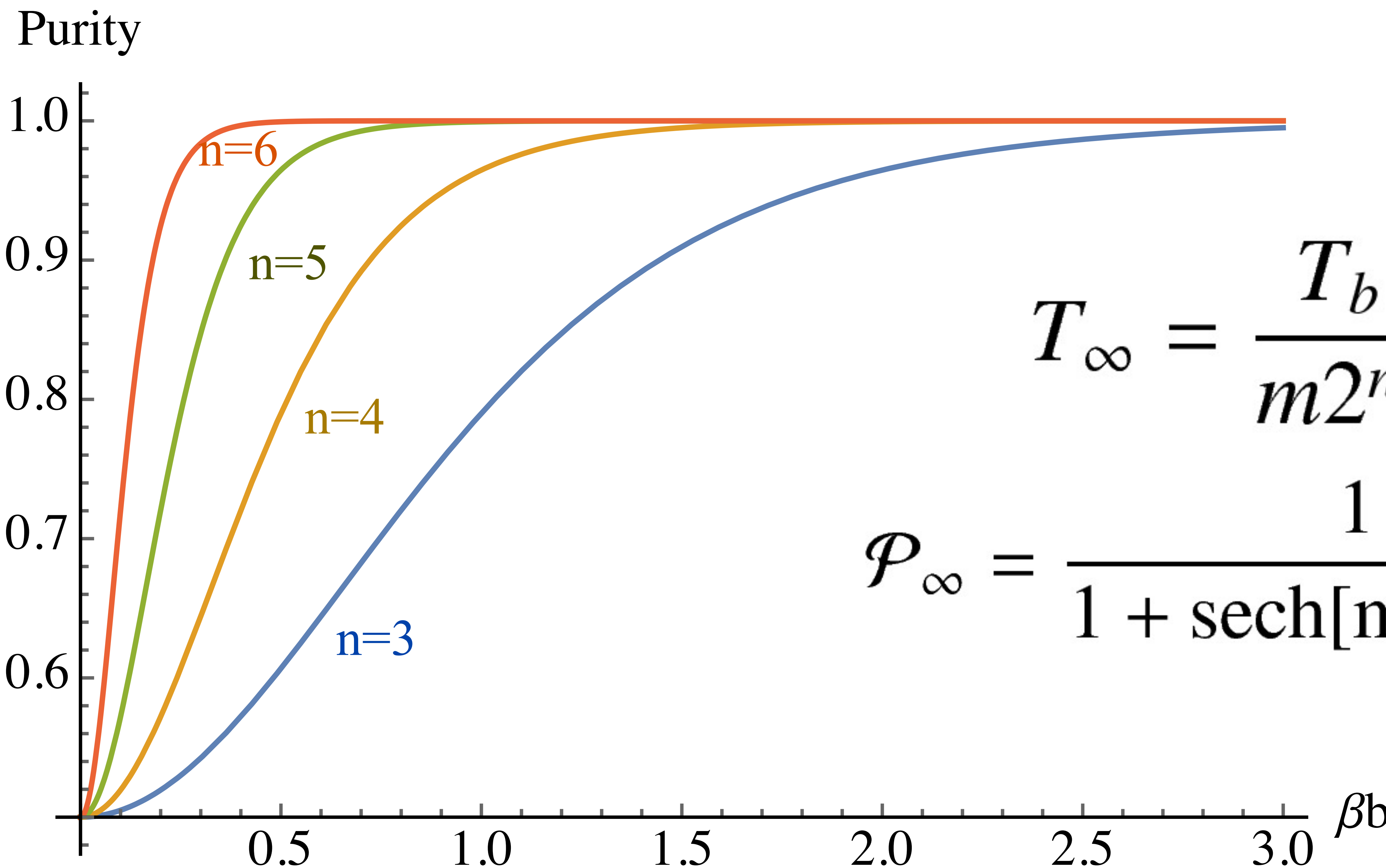




$$\epsilon_j^{\infty} = \frac{(1+\epsilon_b)^{m2^{j-1}} - (1-\epsilon_b)^{m2^{j-1}}}{(1+\epsilon_b)^{m2^{j-1}} + (1-\epsilon_b)^{m2^{j-1}}}$$

Rodríguez-Briones and Laflamme: PRL. 116,170501, 2016
S. Raesi and M. Mosca, PRL 114, 100404, 2016

# Cooling limits of PPA-HBAC



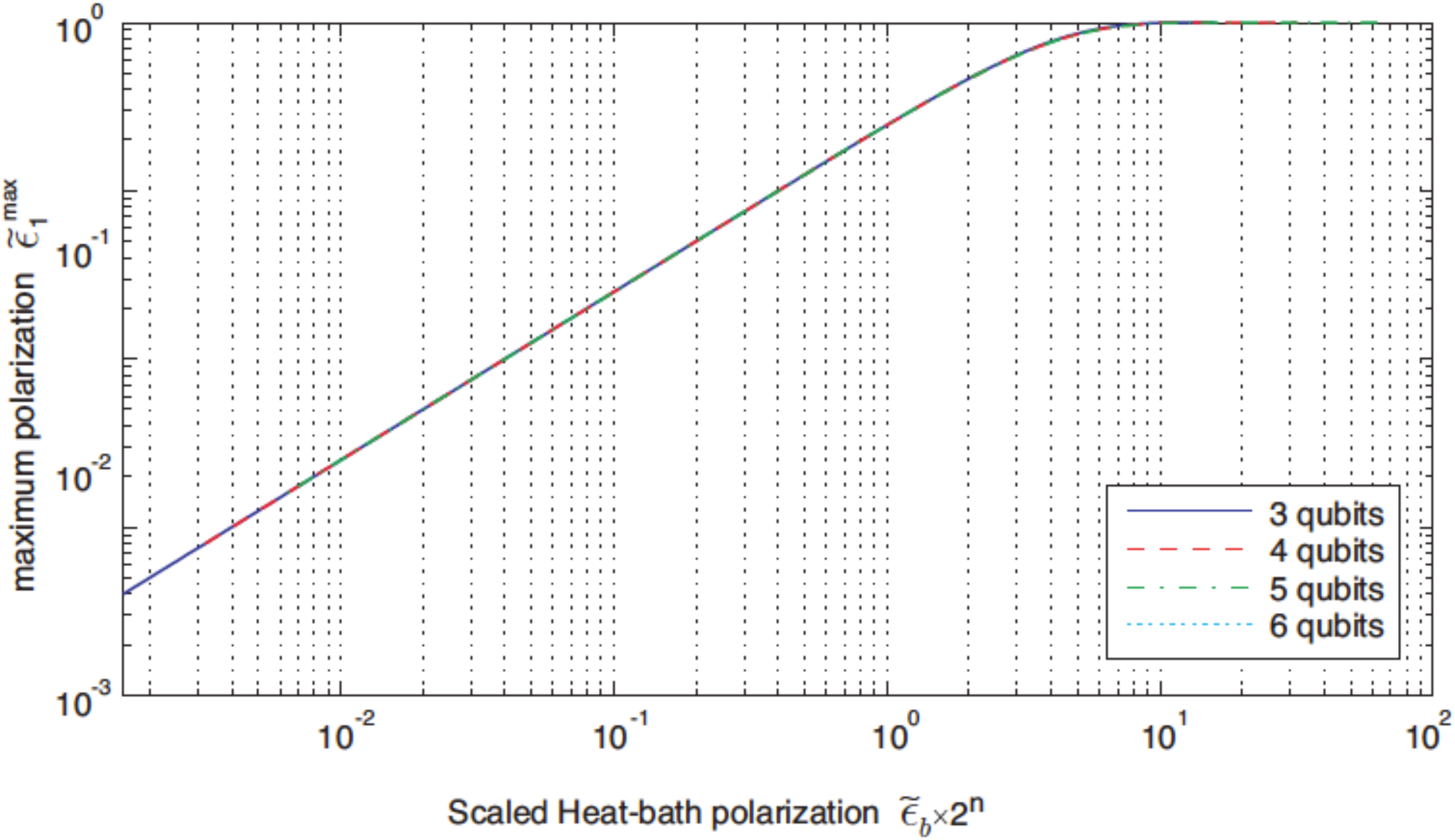

$$T_\infty = \frac{T_b}{m2^{n'}}$$

$$\mathcal{P}_\infty = \frac{1}{1 + \mathrm{sech}[\mathrm{m}2^{\mathrm{n}'}\omega\beta_{\mathrm{b}}]}$$

Rodríguez-Briones and Laflamme: PRL. 116,170501, 2016

(n´= computational qubits: n-1-m)

# Threshold effect




$\epsilon << \dfrac{1}{2^n}$ Can cool to only $2^{n-2}\epsilon$

$\epsilon >> \dfrac{1}{2^n}$ Can purify to order unity.

Moussa thesis 2005; and Schulman et. al. "Physical limitations of Heat- Bath Algorithmic cooling" 2005

Is this the fundamental cooling limit?

Some people claim it is....

S. Raesi and M. Mosca, PRL 114, 100404 (2016)

L. Schulman, T. Mor, and Y. Weinstein, PRL 94, 120501 (2005)

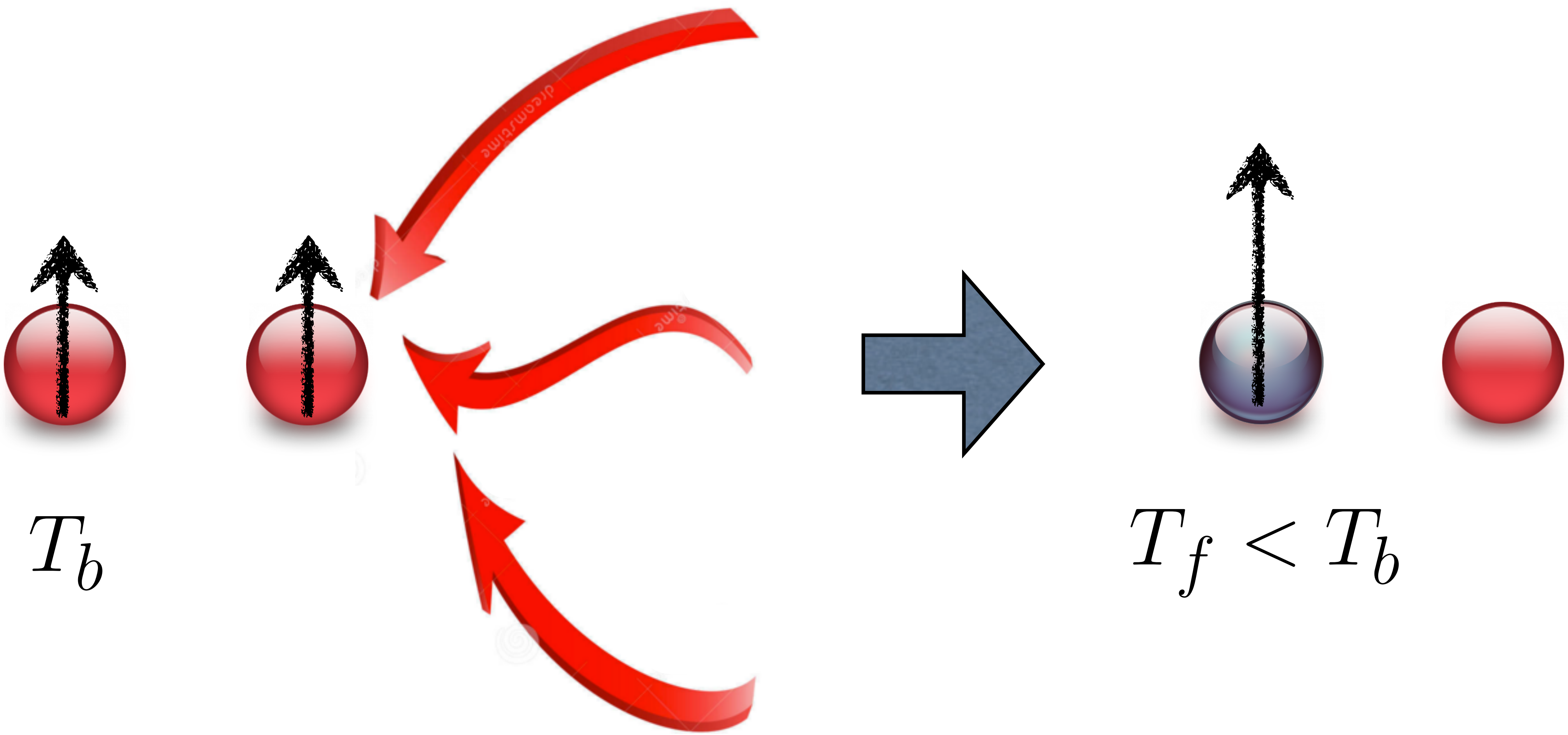
However, we found new techniques to circumvent this cooling limit by taking advantage of correlations

N. Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, **NJP 19 (11), 113047**
N. Rodríguez-Briones, E.Martín Martínez, A. Kempf, R. Laflamme **PRL 119 (5), 050502, 2017**

# Chapter two

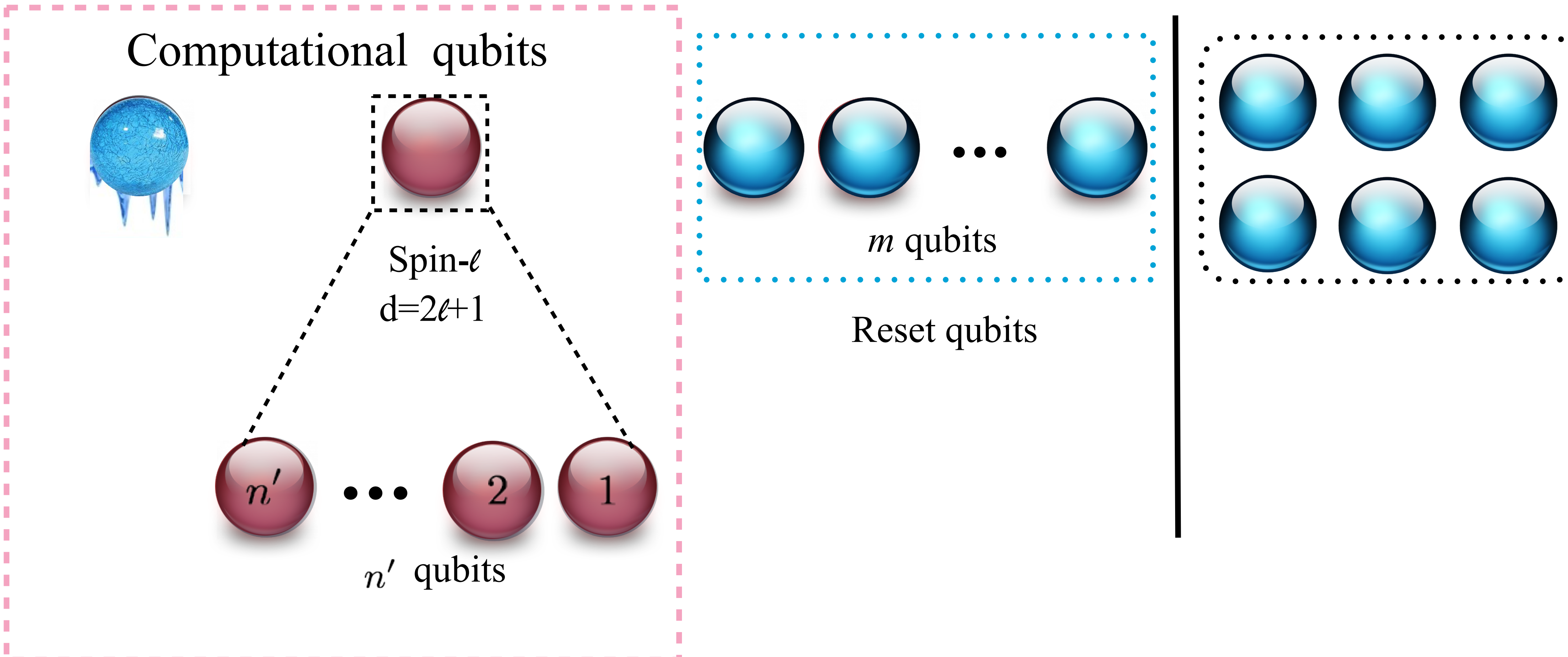
# Correlated relaxation

# Nuclear Overhauser Effect (NOE), 1953




A. Overhauser, Phys. Rev. 89, 689 (1953).

# Cooling limits of PPA-HBAC




$$T_\infty = \frac{T_b}{m2^{n'}}$$

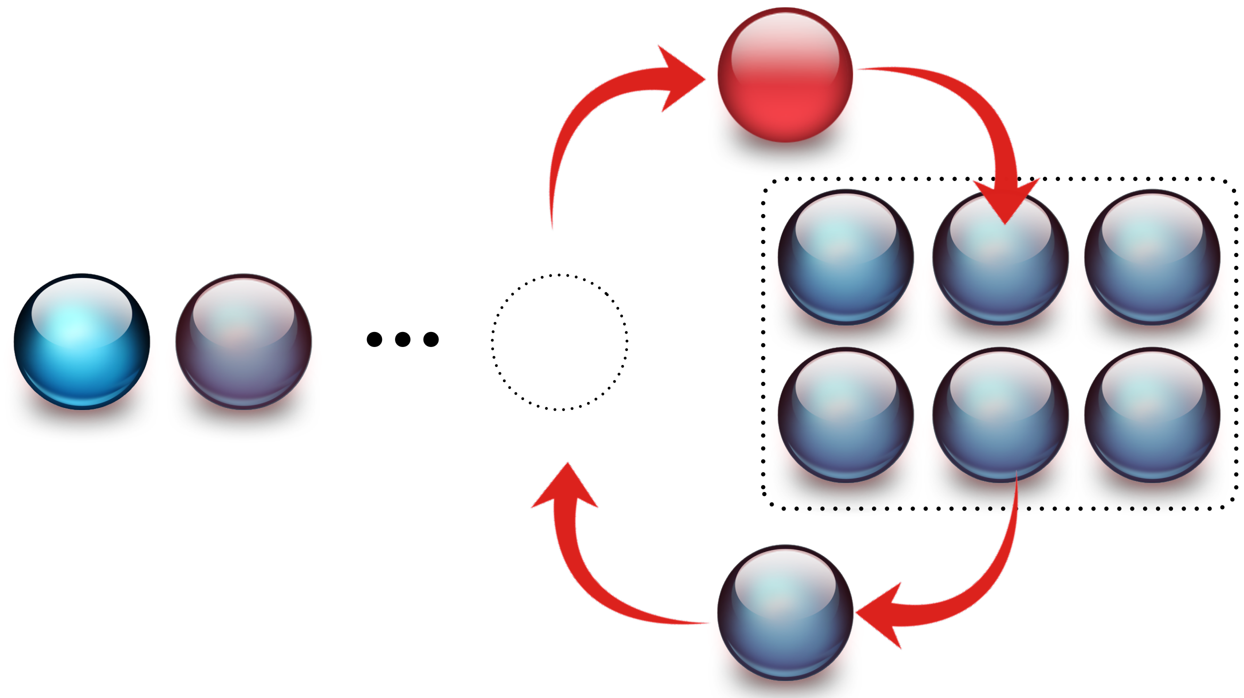
For 2-qubit system:
PPA-HBAC: $T_\infty = T_b$

# Thermalization of qubits

## PPA-HBAC

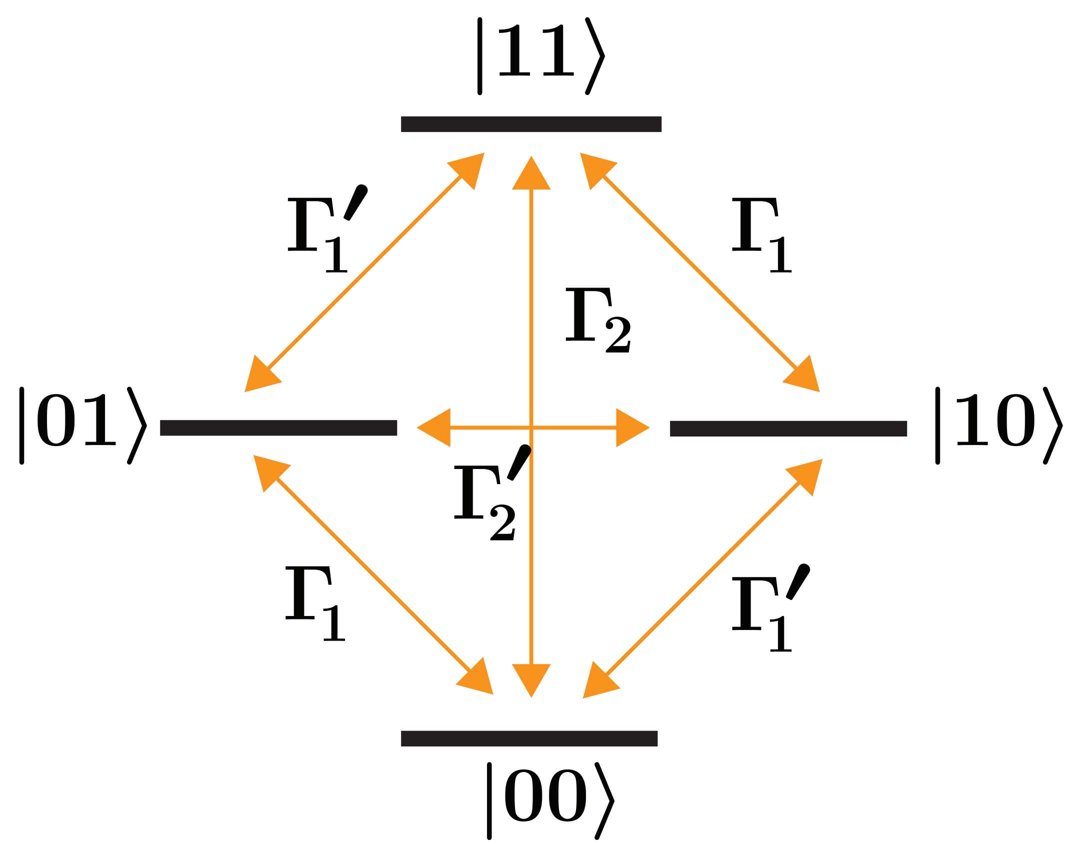
$$\rho_{total} \rightarrow \mathrm{Tr}_m(\rho_{total}) \otimes \rho_b^{\otimes m}$$

$m \equiv$ number of refreshed qubits

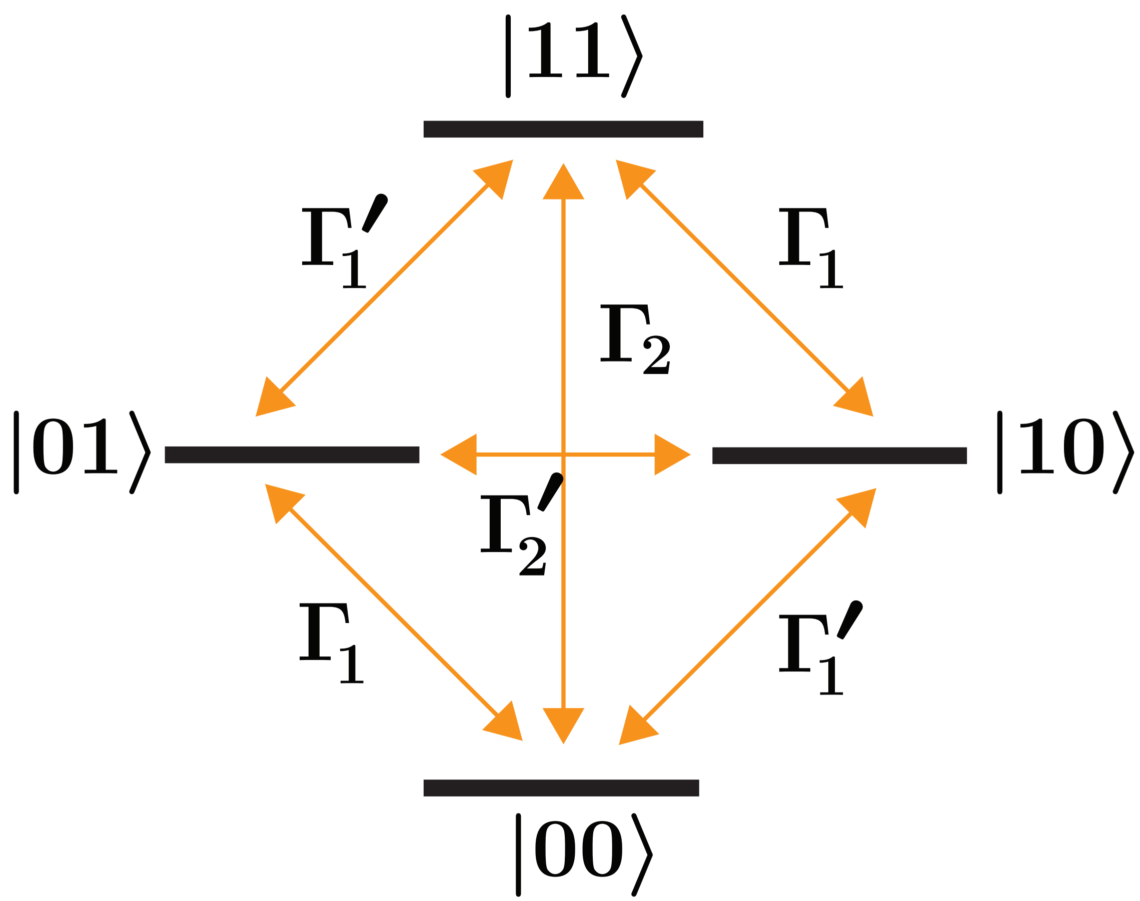



## NOE

Crossed-relaxation



# Crossed-relaxation




$$\Gamma_2 \neq 0$$

&

$$\Gamma_1 = \Gamma_1' = \Gamma_2' = 0$$

$$\mathrm{diag}\,(\rho) = \begin{bmatrix} A_{00} \\ A_{01} \\ A_{10} \\ A_{11} \end{bmatrix} \xrightarrow{\Gamma_2} \begin{bmatrix} (A_{00} + A_{11})\, P_2 \\ A_{01} \\ A_{10} \\ (A_{00} + A_{11})\,(1 - P_2) \end{bmatrix}$$

$$P_2 = \frac{e^{2\xi}}{2\cosh(2\xi)}$$

$$\xi = \tanh^{-1}(\epsilon_b)$$

N. Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, NJP 19 (11), 113047

# Wrong implicit assumption in all previous work:

- The reset step with the bath is optimal when swapping qubits to pump entropy out of the system.

# Reset of states $\Gamma_2$

Relaxation/equilibration

$|00\rangle \longleftrightarrow |11\rangle$

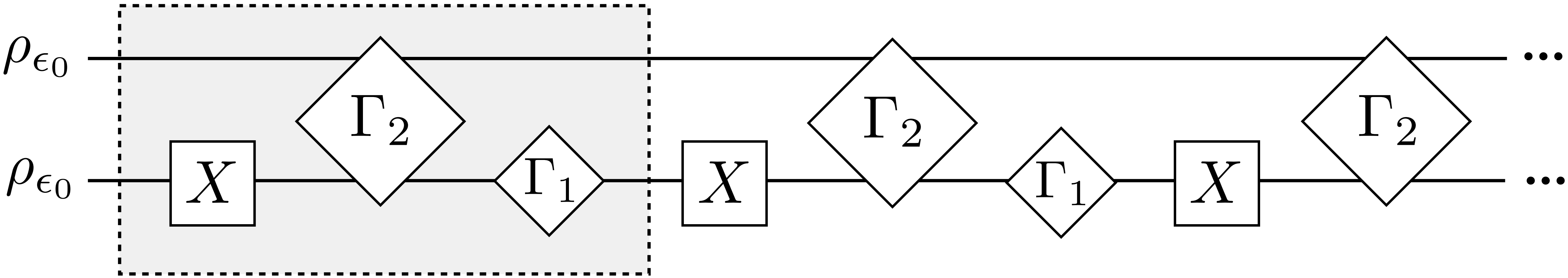
**Kraus Operators**

$$A_1^{(n=2)} = \sqrt{p_2}|00\rangle\langle 00|$$
$$A_2^{(n=2)} = \sqrt{p_2}|00\rangle\langle 11|$$
$$A_3^{(n=2)} = \sqrt{1-p_2}|11\rangle\langle 11|$$
$$A_4^{(n=2)} = \sqrt{1-p_2}|11\rangle\langle 00|$$
$$A_5^{(n=2)} = |01\rangle\langle 01|$$
$$A_6^{(n=2)} = |10\rangle\langle 10|$$

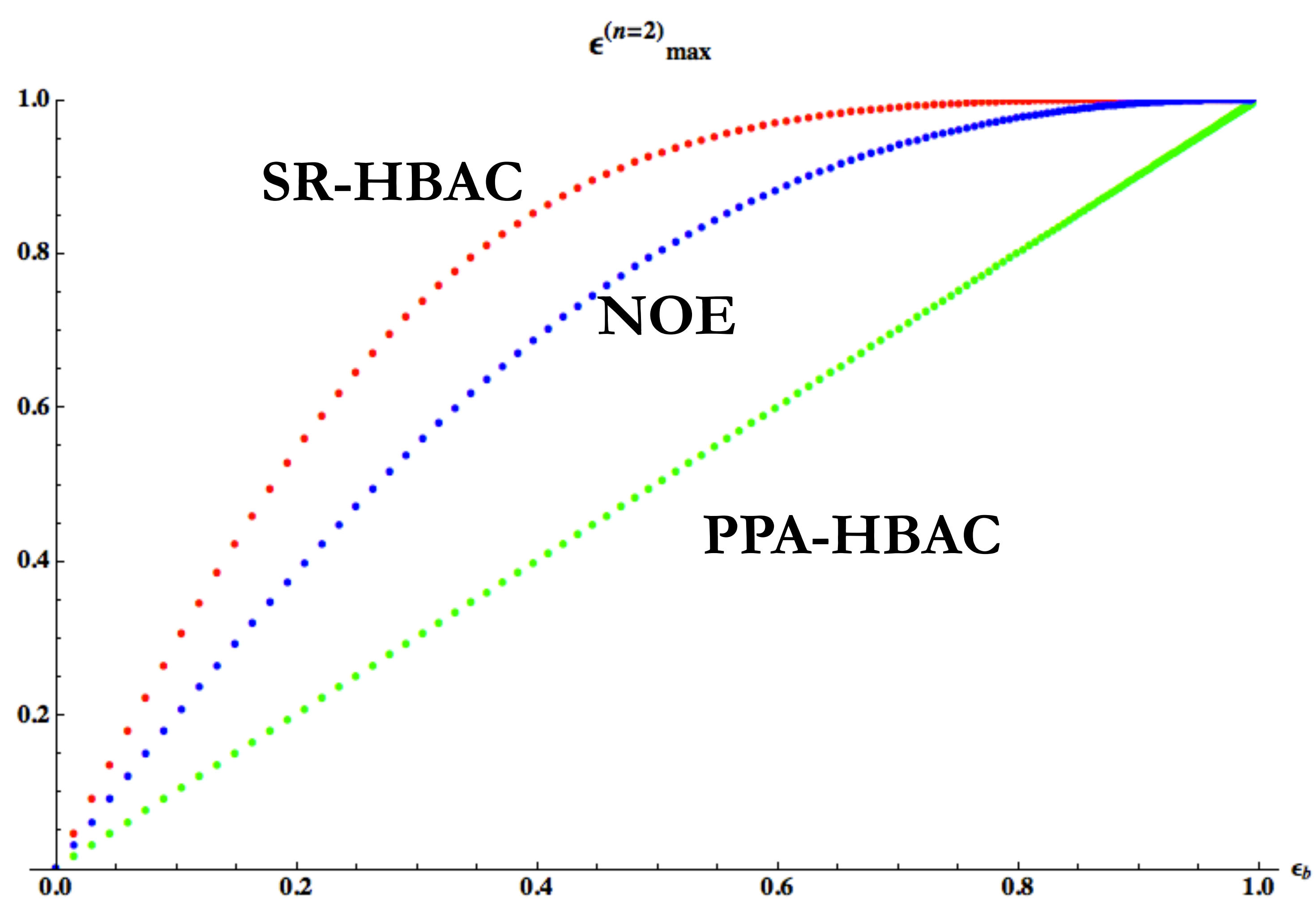

$$P_2 = \frac{e^{2\xi}}{2\cosh(2\xi)}$$
$$\xi = \tanh^{-1}(\epsilon_b)$$

N. Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, NJP 19 (11), 113047

# SR-HBAC algorithm, for n=2




**Round**$_{(n=2)}$



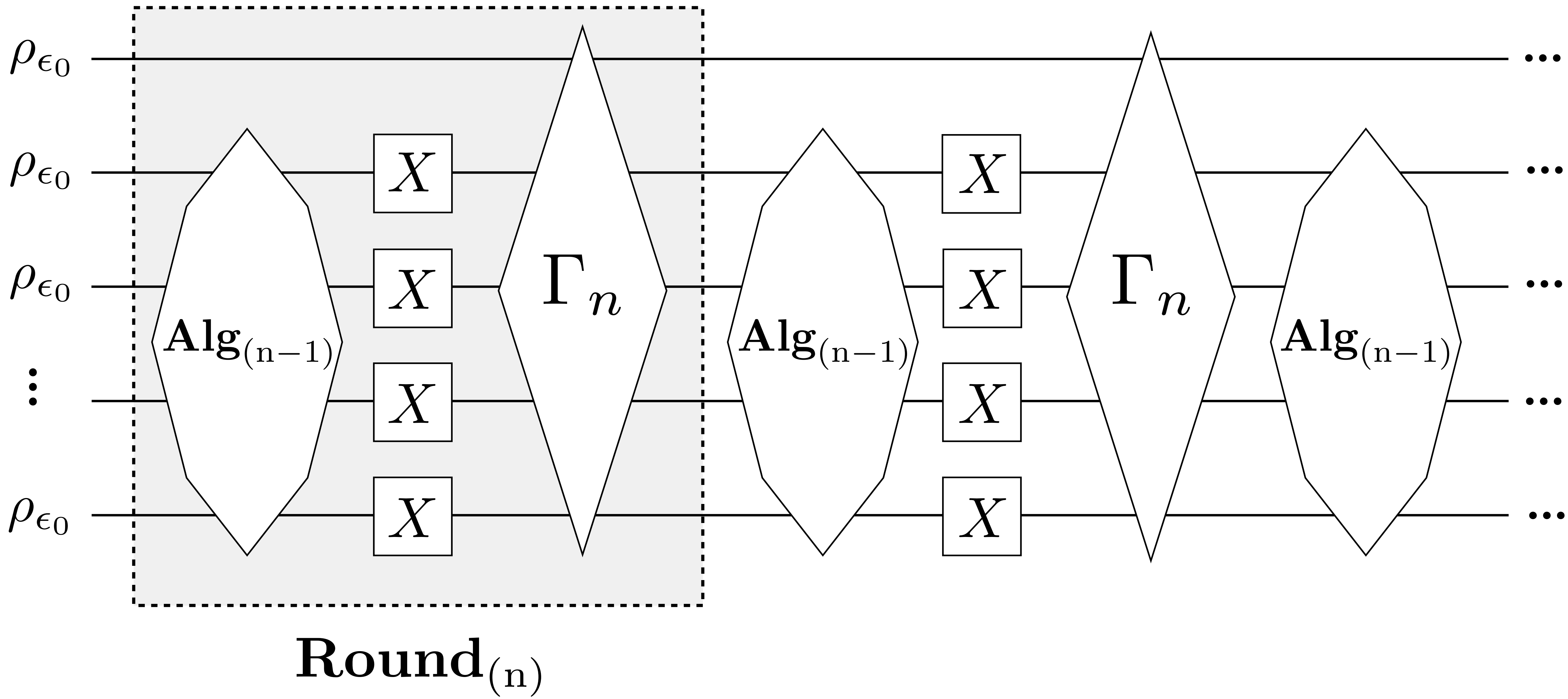

$$\epsilon_{max}^{(n=2)} = \tanh(3\xi)$$

$$\xi = \tanh^{-1}(\epsilon_b)$$

$$\epsilon_k^{(n=2)} = \left(3 - \frac{1}{2^{k-1}}\right)\epsilon_b$$

N. Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, NJP 19 (11), 113047

# SR-HBAC algorithm for n qubits




$$\epsilon_{max}^{(n)} = \tanh\left[\left(2^{\mathrm{n}} - 1\right)\xi\right]$$

$$\xi = \tanh^{-1}\left(\epsilon_b\right)$$

N. Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, NJP 19 (11), 113047

# Final temperatures

**PPA-HBAC** $$T_\infty = \frac{T_b}{md} \cdot \frac{\Delta E_t}{\Delta E_r}$$

using string of qubits with only one reset qubit: $$T_\infty = \frac{T_b}{2^{n-2}} \cdot \frac{\Delta E_t}{\Delta E_r}$$

**SR-HBAC** $$T_\infty = \frac{T_b}{2^n - 1} \cdot \frac{\Delta E_t}{\Delta E_r}$$

# Chapter three

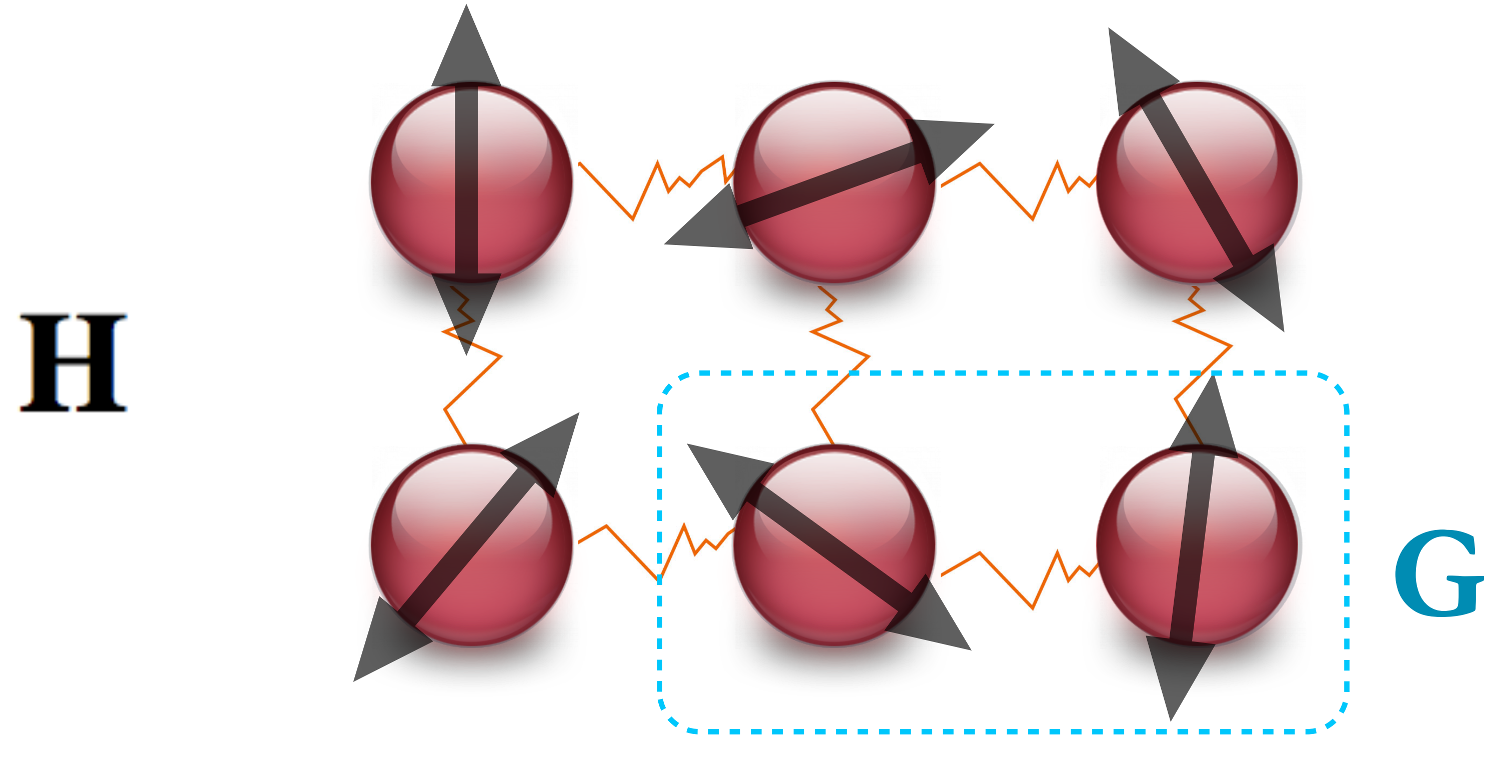
# Correlations in the initial state

PPA-HBAC assumes that the initial state of the system is in a product state.

**Idea:** Remove this assumption, and use initial correlations for cooling

# Motivation:

Purifying qubits from low temperature **interacting** quantum systems. I.e., cooling local quantum systems subject to many-body interactions




N. Rodríguez-Briones, E.Martín Martínez, A. Kempf, R. Laflamme **PRL 119 (5), 050502, 2017**

# Bad news for local cooling of interacting systems?

• Cool down the system:
Ground state is entangled $\longrightarrow$ Subsystems are mixed

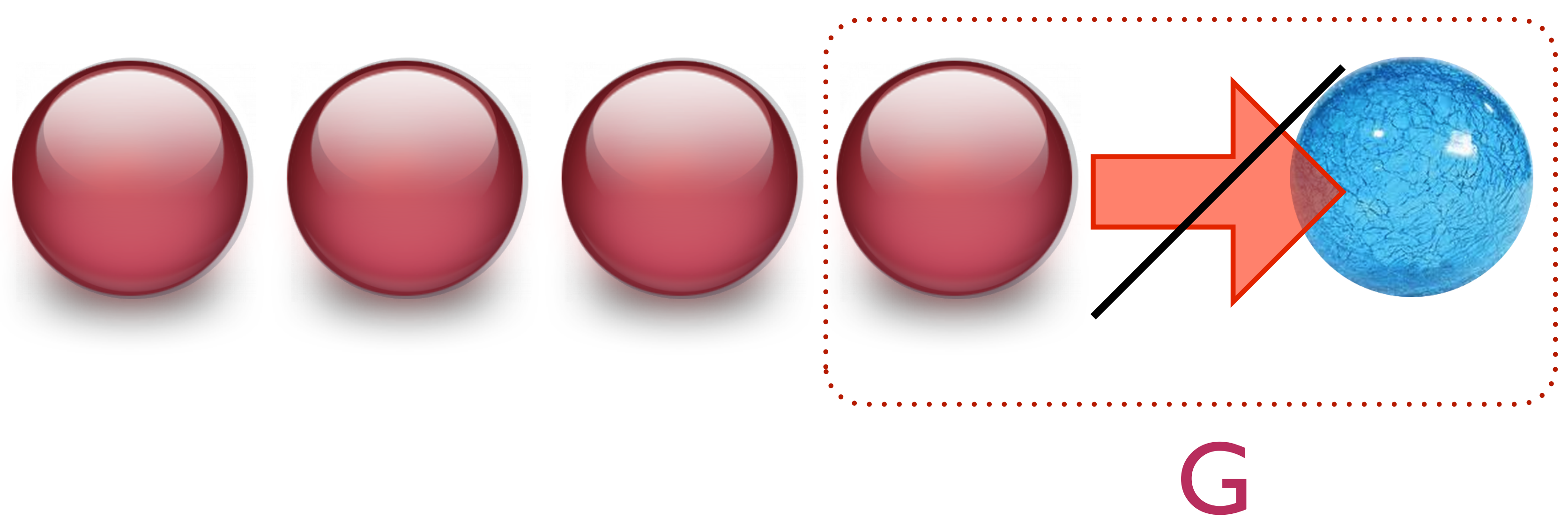
• Break entanglement through interaction with environment:
System in a Gibbs state $\longrightarrow$ Subsystems are mixed

• Apply local operations on the subsystem:
Strong local passivity $\longrightarrow$ Subsystem's energy increases

How do we cool down a part of an interacting system?

# Strong local passive states

A system state is defined to be SL passive if no general quantum operation G applied locally to a system can extract positive energy from the system.



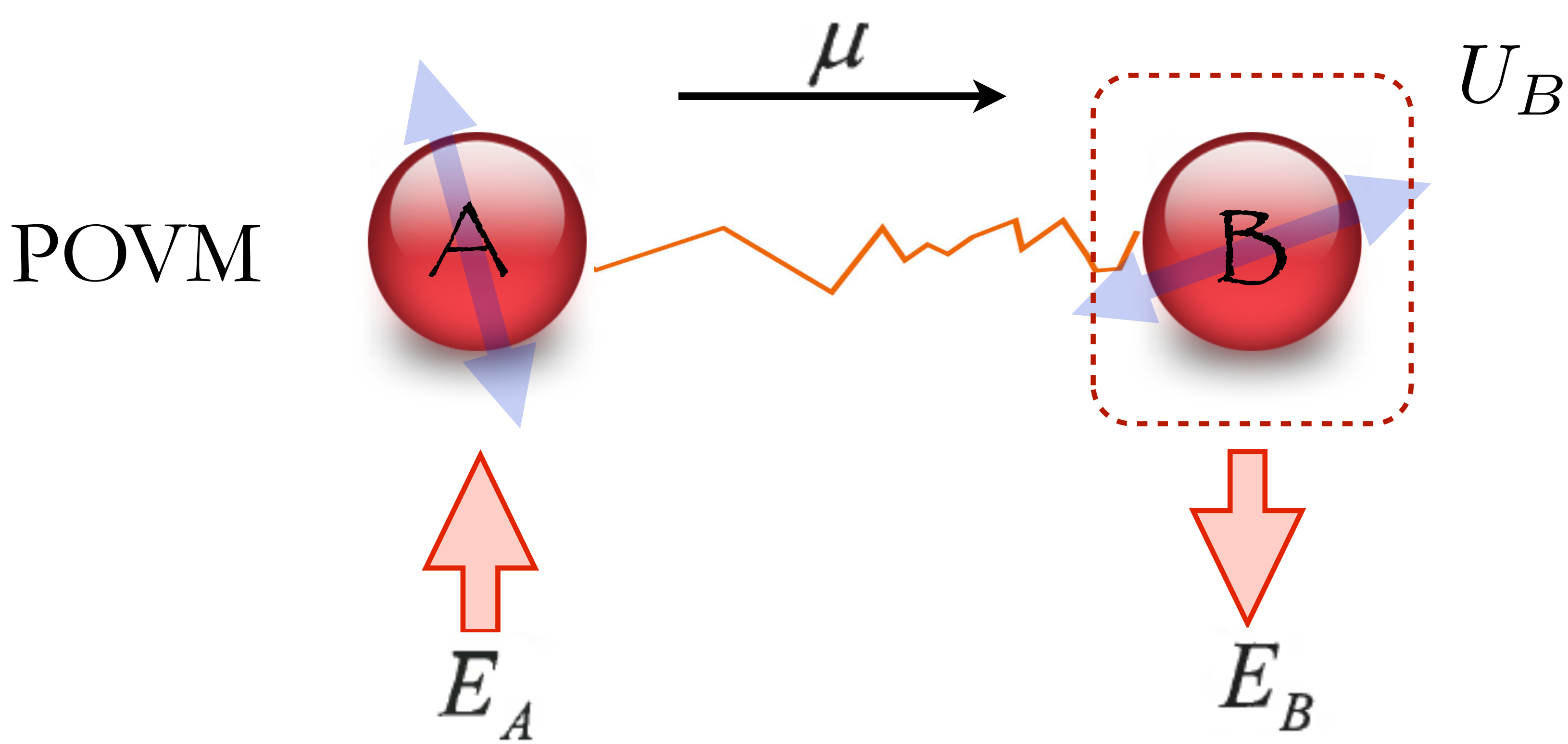

$$\Delta E\left(\rho\right) = \mathrm{Tr}\left[\mathrm{H}\left(\mathrm{I}\otimes \mathrm{G}\right)\rho\right] - \mathrm{Tr}\left[\mathrm{H}\rho\right]$$

$$\forall G,\ \Delta E \geq 0$$

# Taking advantage of pre-existing correlations

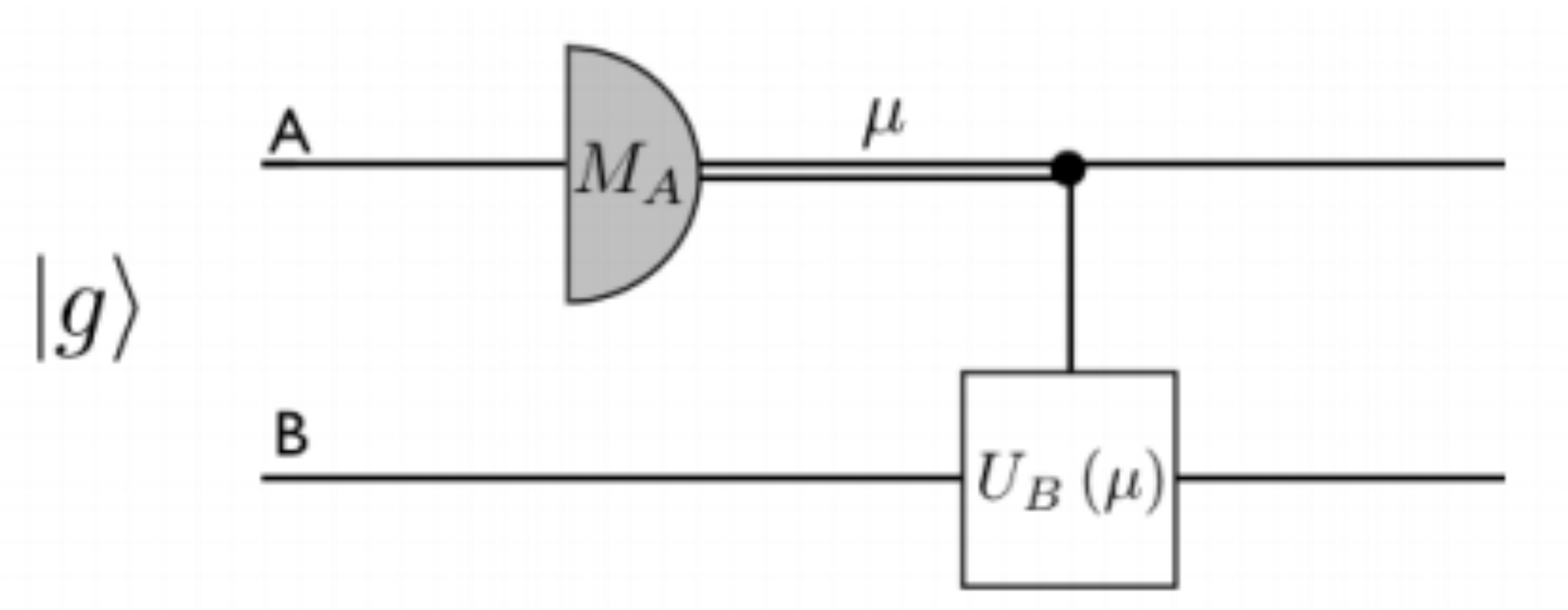
## *How?*

Using an idea from quantum field theory, a method called quantum energy teleportation to consume correlations while extracting work locally from the target qubit.




M. Hotta, Physics Letters A 374, 3416 (2010)

# QET Circuit

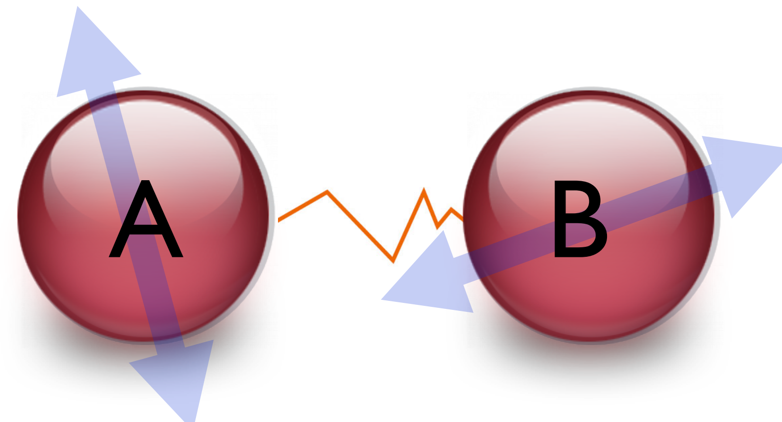



POVM case with measurement operators



M. Hotta, Physics Letters A 374, 3416 (2010)

# Two interacting qubits




$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{V} \geq 0$$

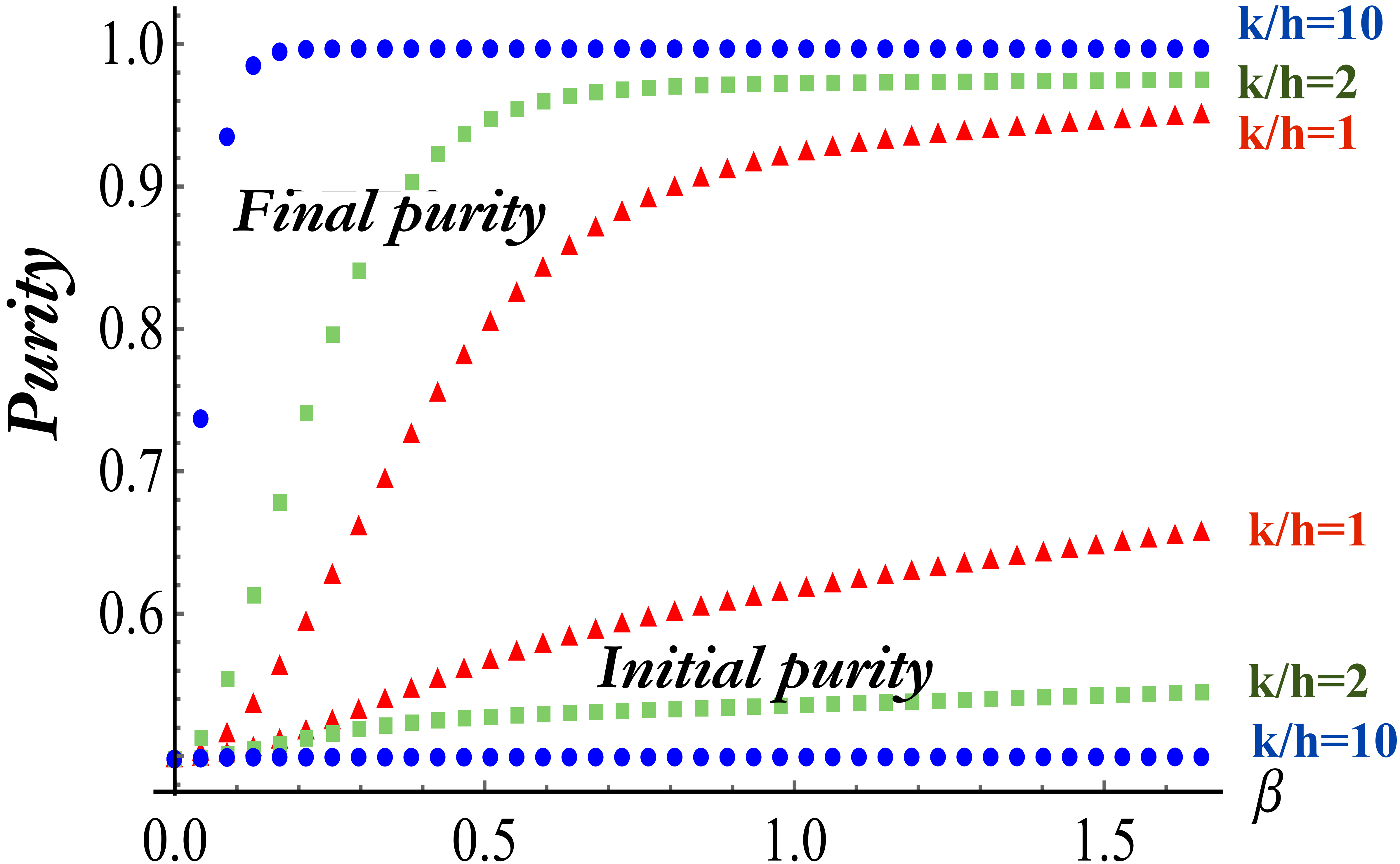
$$\hat{H}_A = h\hat{\sigma}^z{}_A + \frac{h^2}{\sqrt{h^2 + k^2}},$$

$$\hat{H}_B = h\hat{\sigma}^z{}_B + \frac{h^2}{\sqrt{h^2 + k^2}},$$

$$\hat{V} = 2k\hat{\sigma}^x{}_A\hat{\sigma}^x{}_B + \frac{2k^2}{\sqrt{h^2 + k^2}}.$$

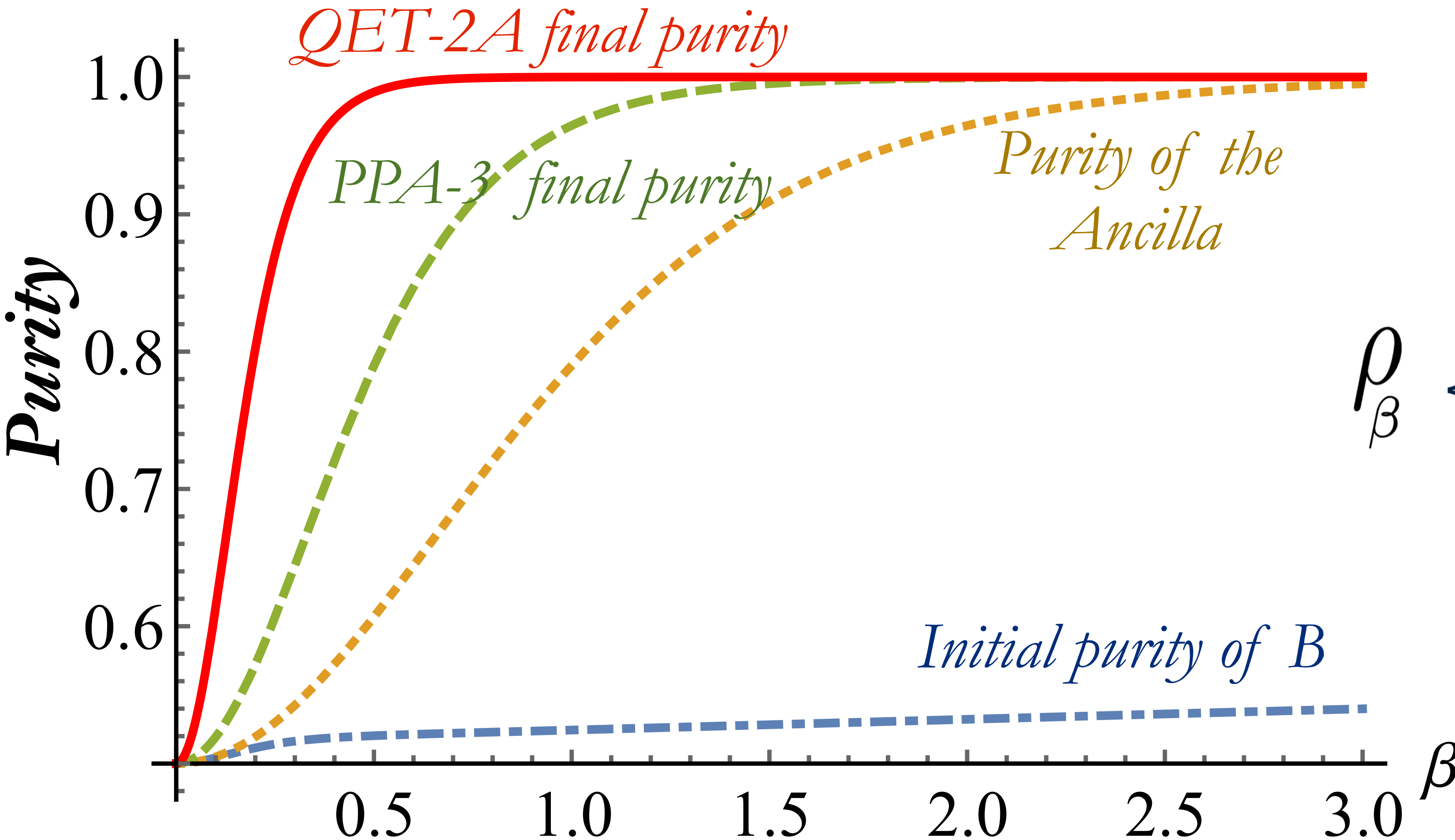
$$(h > 0, k > 0)$$

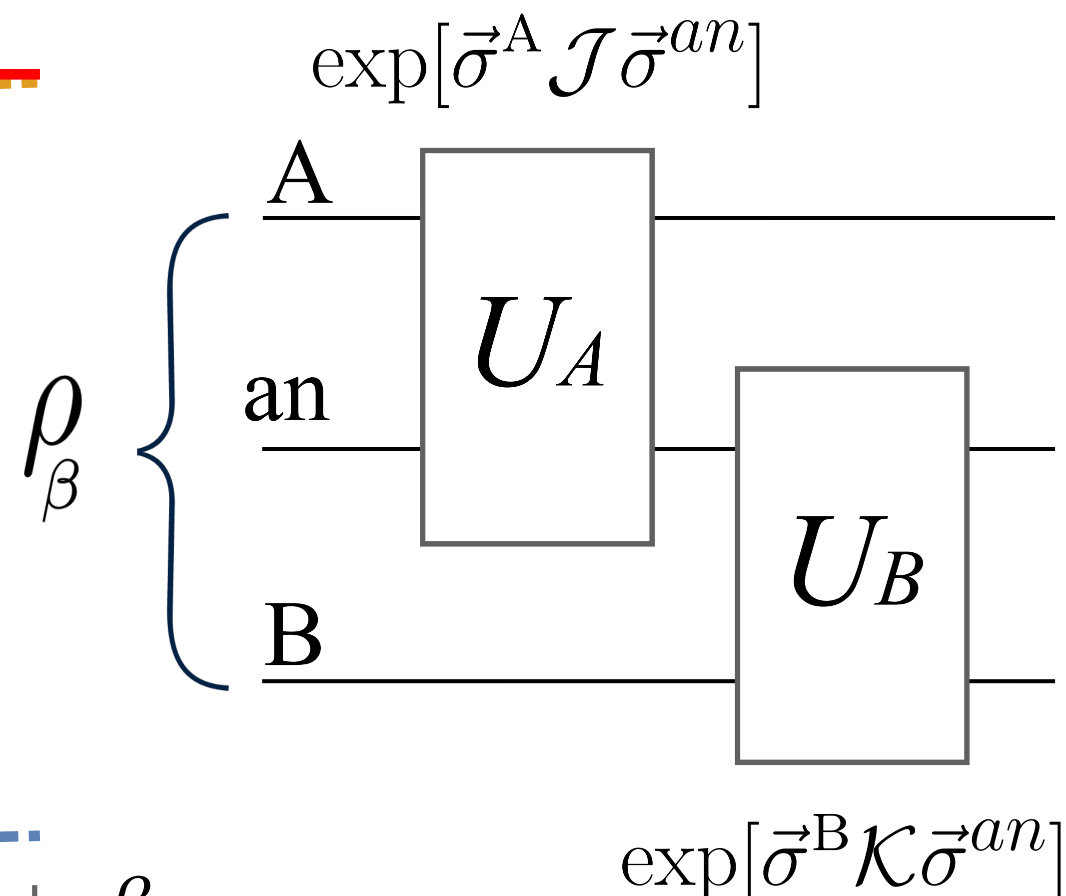
# Results for Minimal QET




N. Rodríguez-Briones, E.Martín Martínez, A. Kempf, R. Laflamme **PRL 119 (5), 050502, 2017**

# Fully unitary picture




N. Rodríguez-Briones, E.Martín Martínez, A. Kempf, R. Laflamme **PRL 119 (5), 050502, 2017**

# Conclusions:

**1)** We give analytical results for the achievable cooling limits in the conventional HBAC.

**We circumvented previous limits by using correlations:**

**2)** Taking advantage of correlations that can be created during the re-thermalization step with the heat-bath.

**3)** Using correlations present in the initial state induced by the internal interactions of the system.

[1] NA Rodríguez-Briones, R Laflamme. PRL 116 (17), 170501
[2] DK Park, NA Rodriguez-Briones, G Feng, R Rahimi, J Baugh, R Laflamme. Electron Spin Resonance (ESR) Based Quantum Computing, 227-255
[3] NA Rodriguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme. NJP 19 (11), 113047
[4] NA Rodríguez-Briones, E Martín-Martínez, A Kempf, R Laflamme. PRL 119 (5), 050502