Entanglement Preserving Local Thermalization

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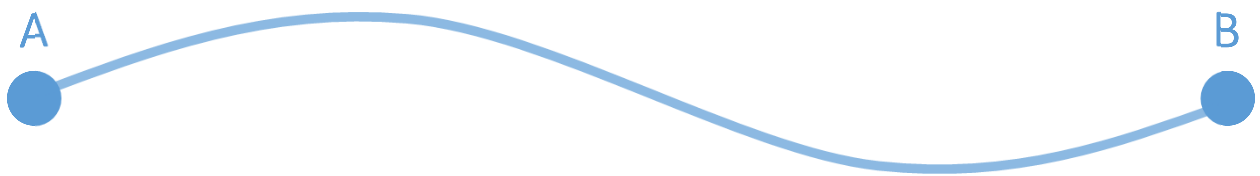
Chung-Yun Hsieh | Matteo Lostaglio | Antonio Acín

Motivation

Entanglement Preserving Local Thermalization

N. Brunner *et al.*, Rev. Mod. Phys. **86**, 419 (2014); R. Horodecki *et al.*, Rev. Mod. Phys. **81**, 865 (2007).

Entanglement Preserving Local Thermalization



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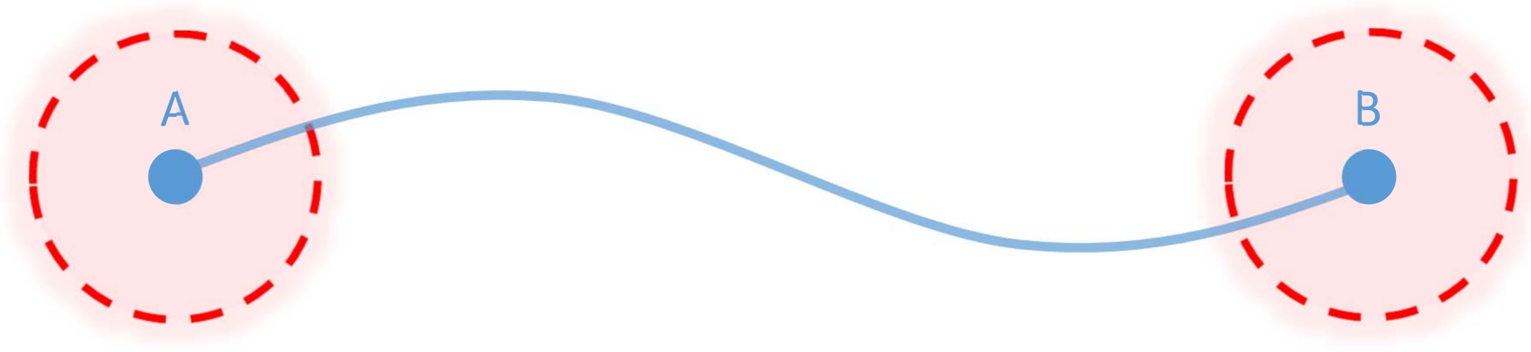
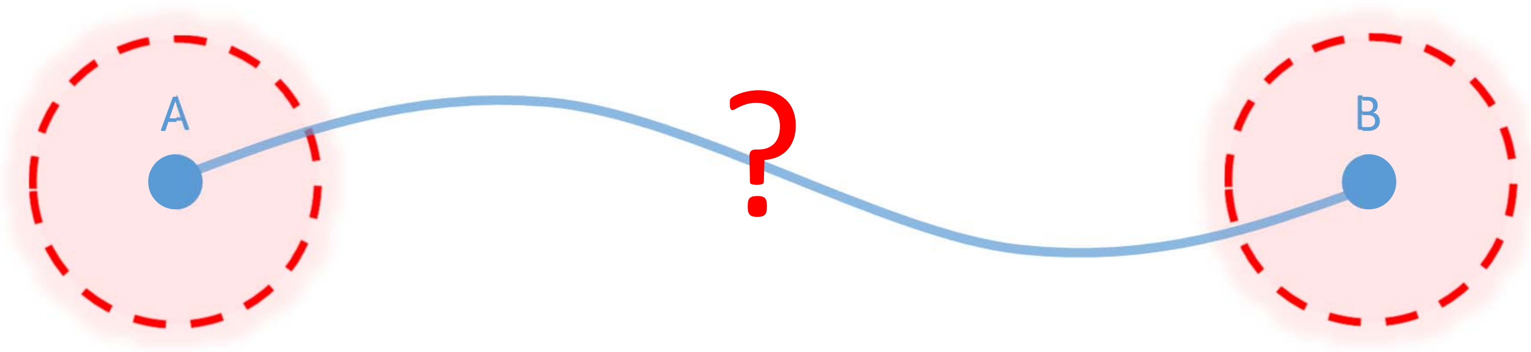


Entanglement Preserving Local Thermalization



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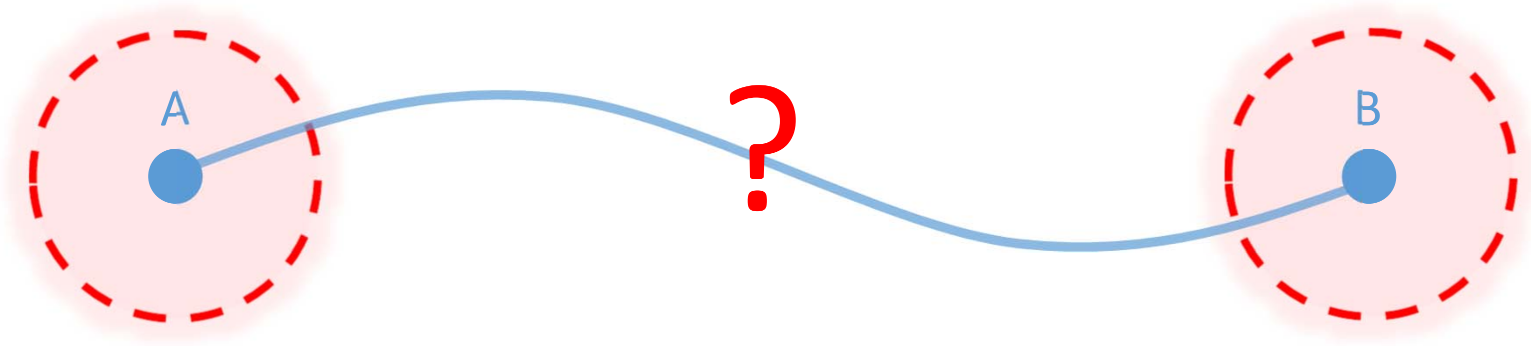
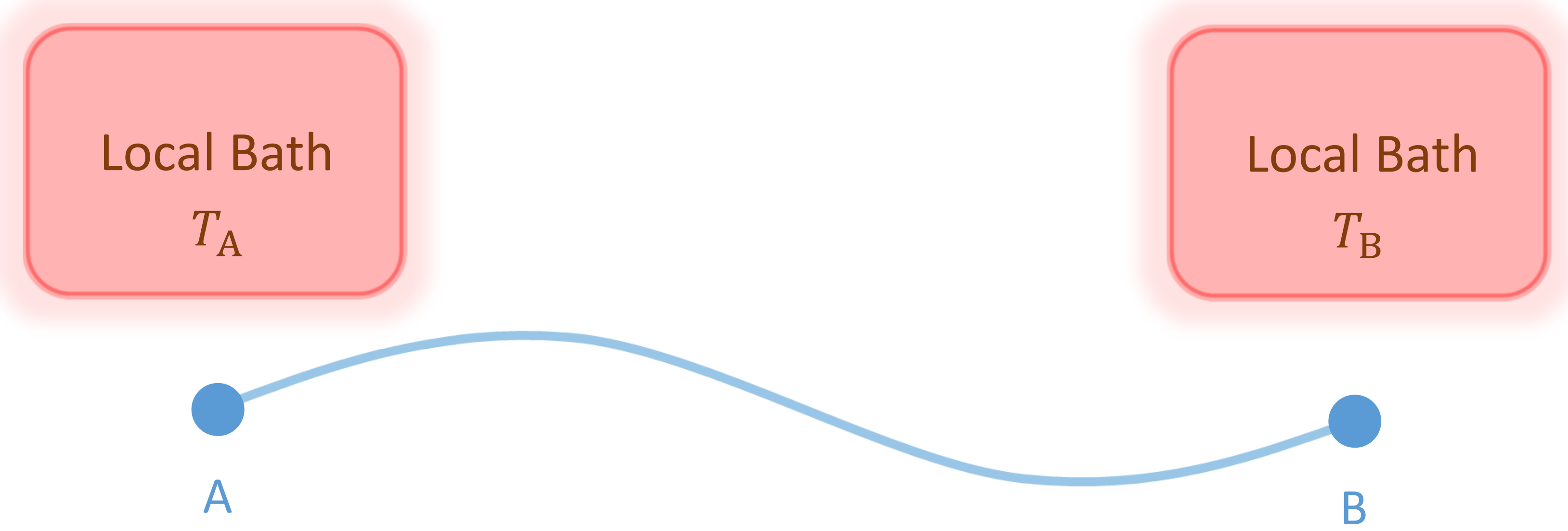


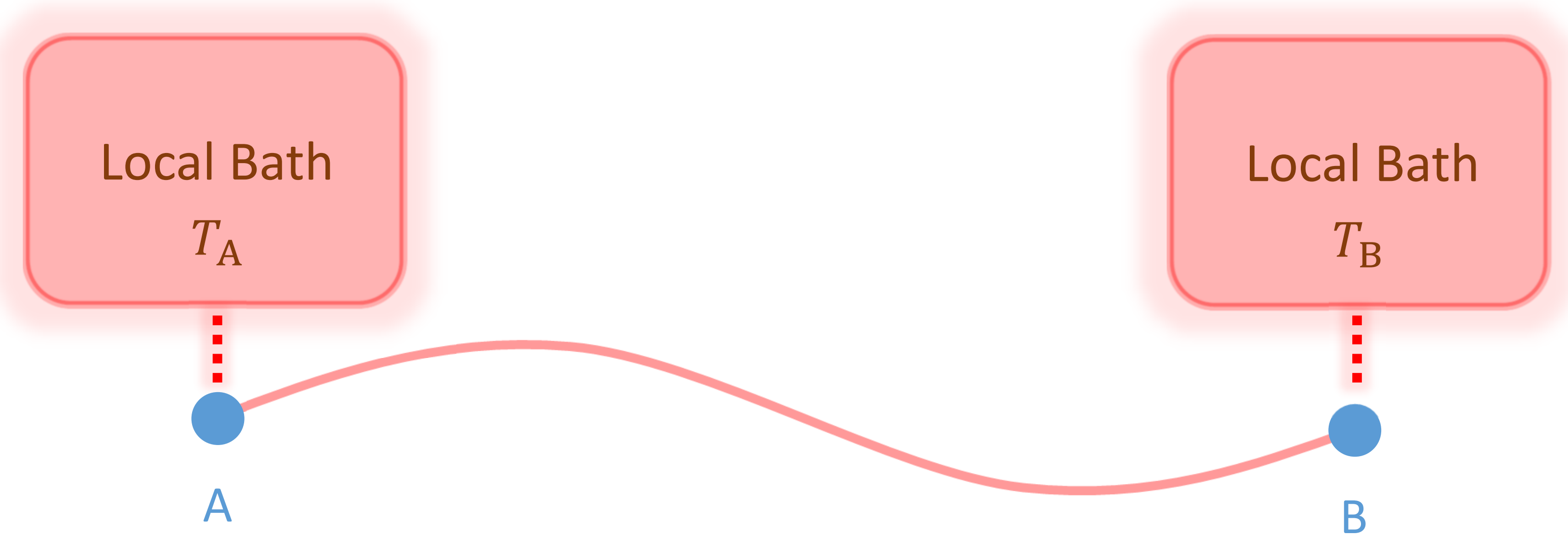
B. Morris, L. Lami, and G. Adesso, Phys. Rev. Lett 122, 130601 (2019). [Poster #42]

Formulation


A
B


Local Bath
T_A
Local Bath
T_B
A
B


Local Bath
T_A
Local Bath
T_B
A
B


Local Bath
T_A
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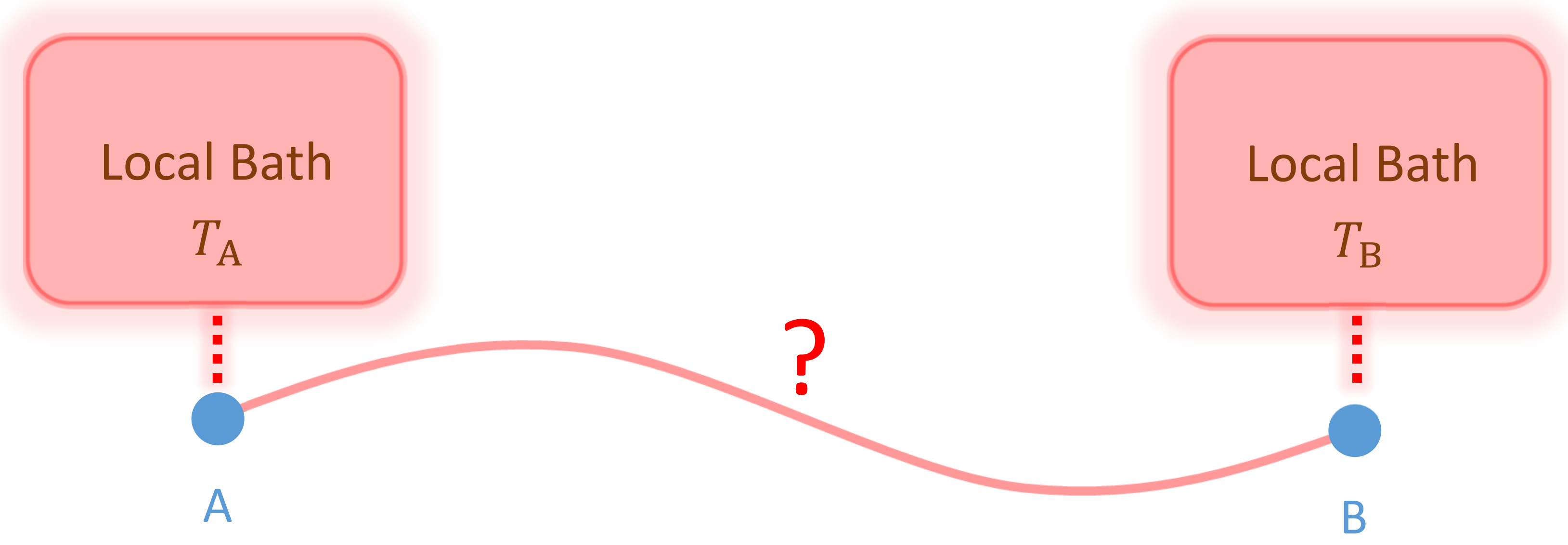
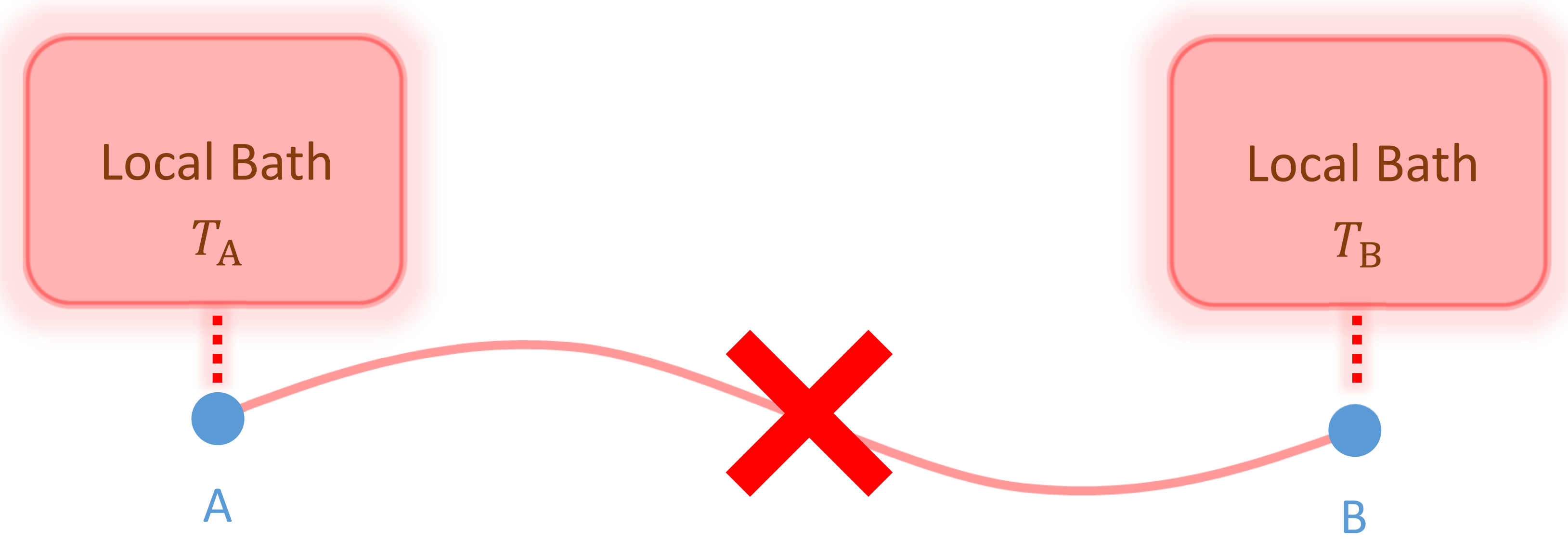
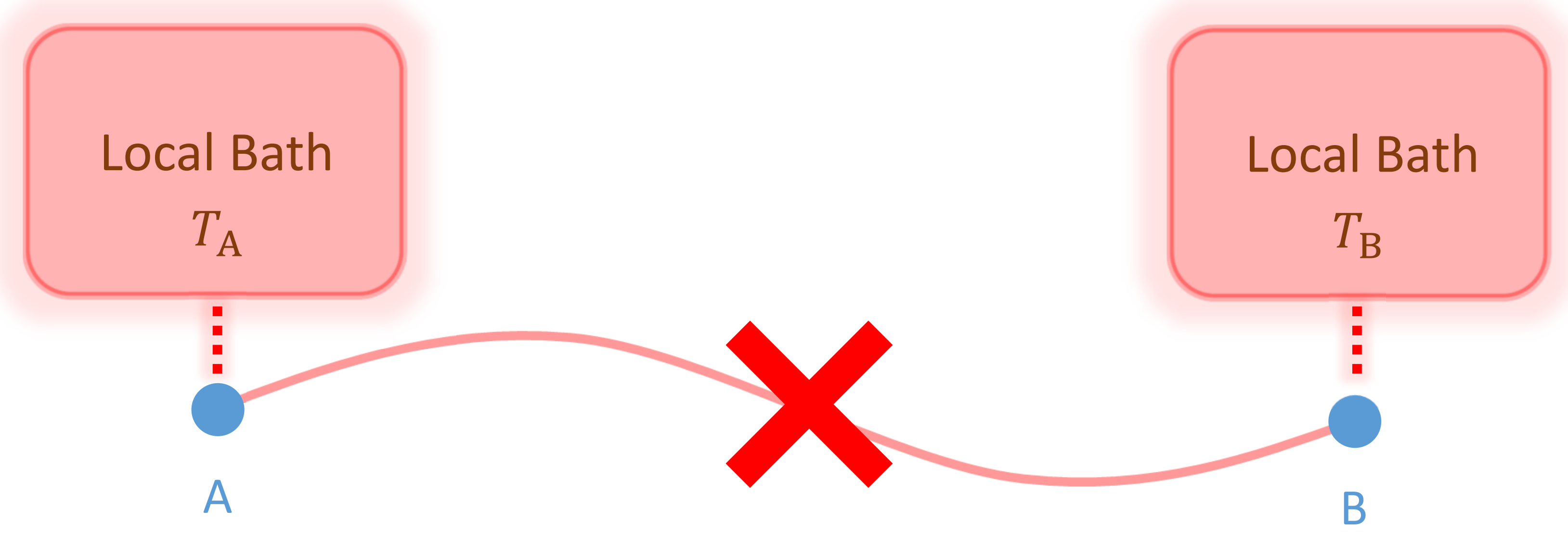
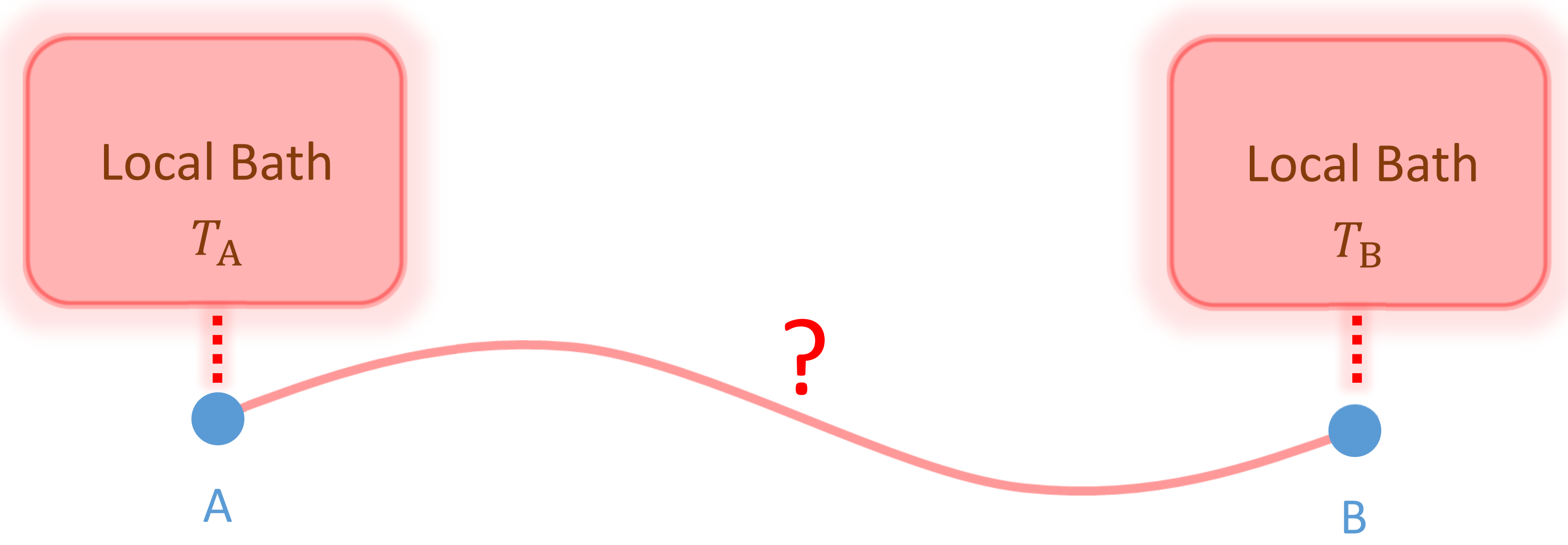
T_B
A
B


Local Bath
T_A
Local Bath
T_B
?
A
B


Local Bath
T_A
Local Bath
T_B
A
B



RESULT 1 | Impossible for product channels.


Local Bath
T_A
Local Bath
T_B
?
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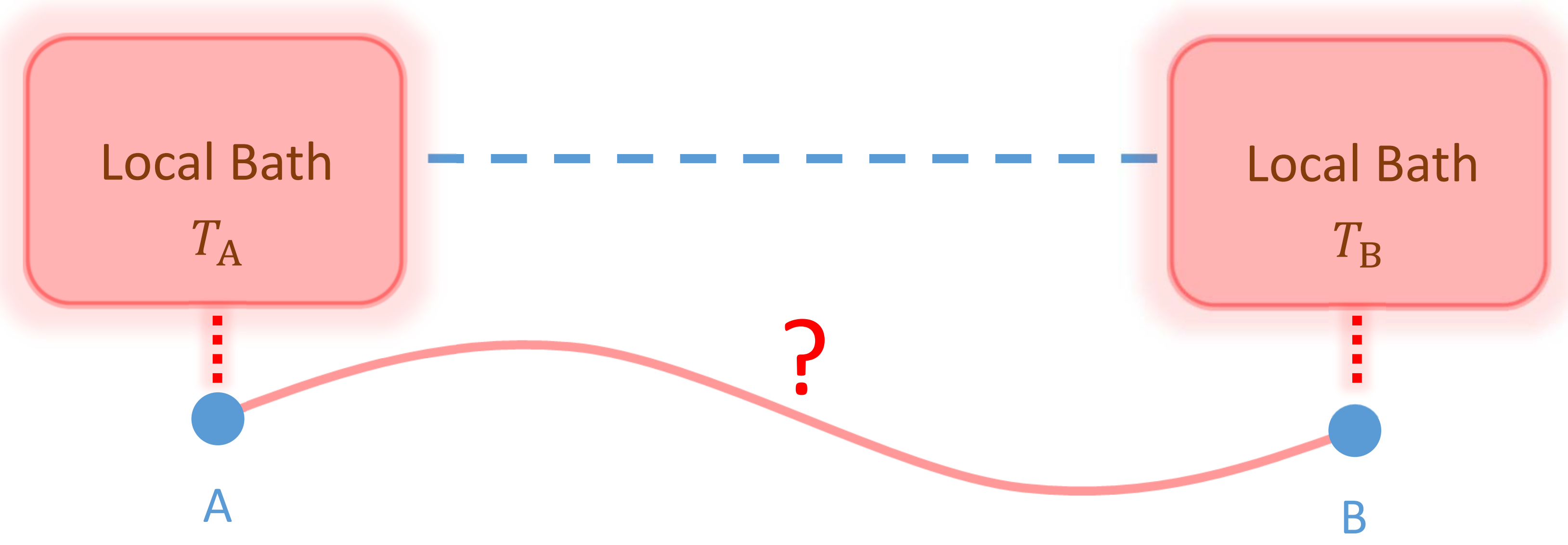
B

Entanglement Preserving Local Thermalization



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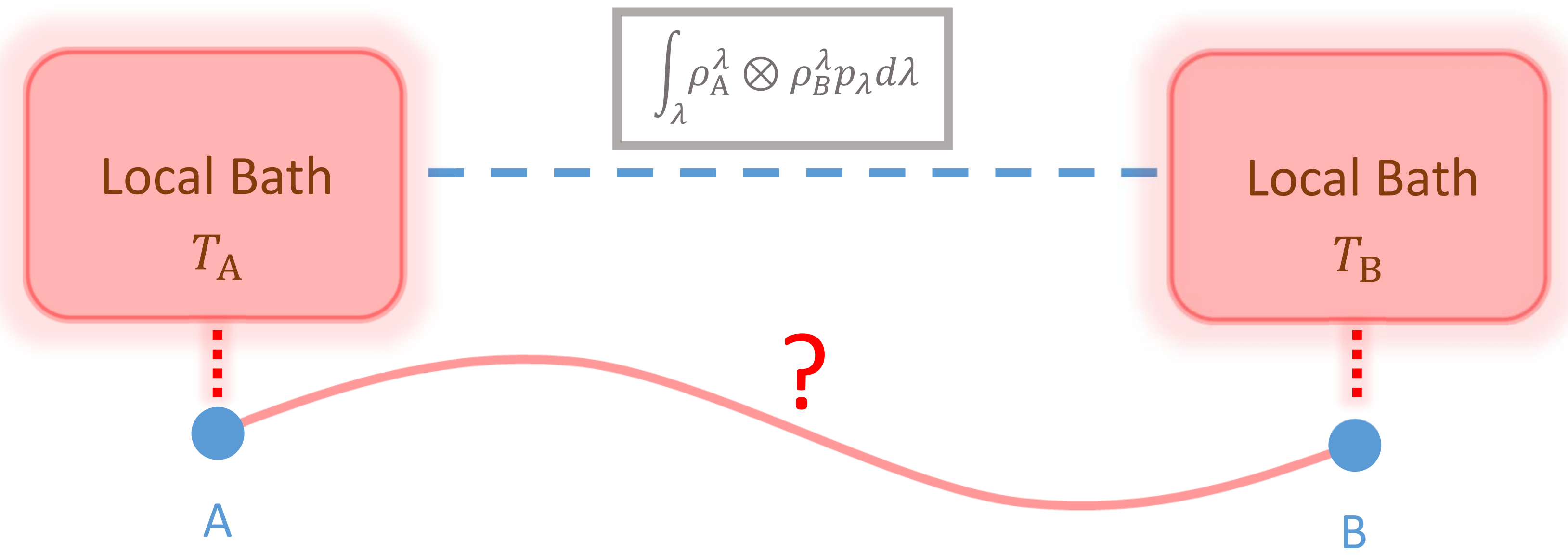
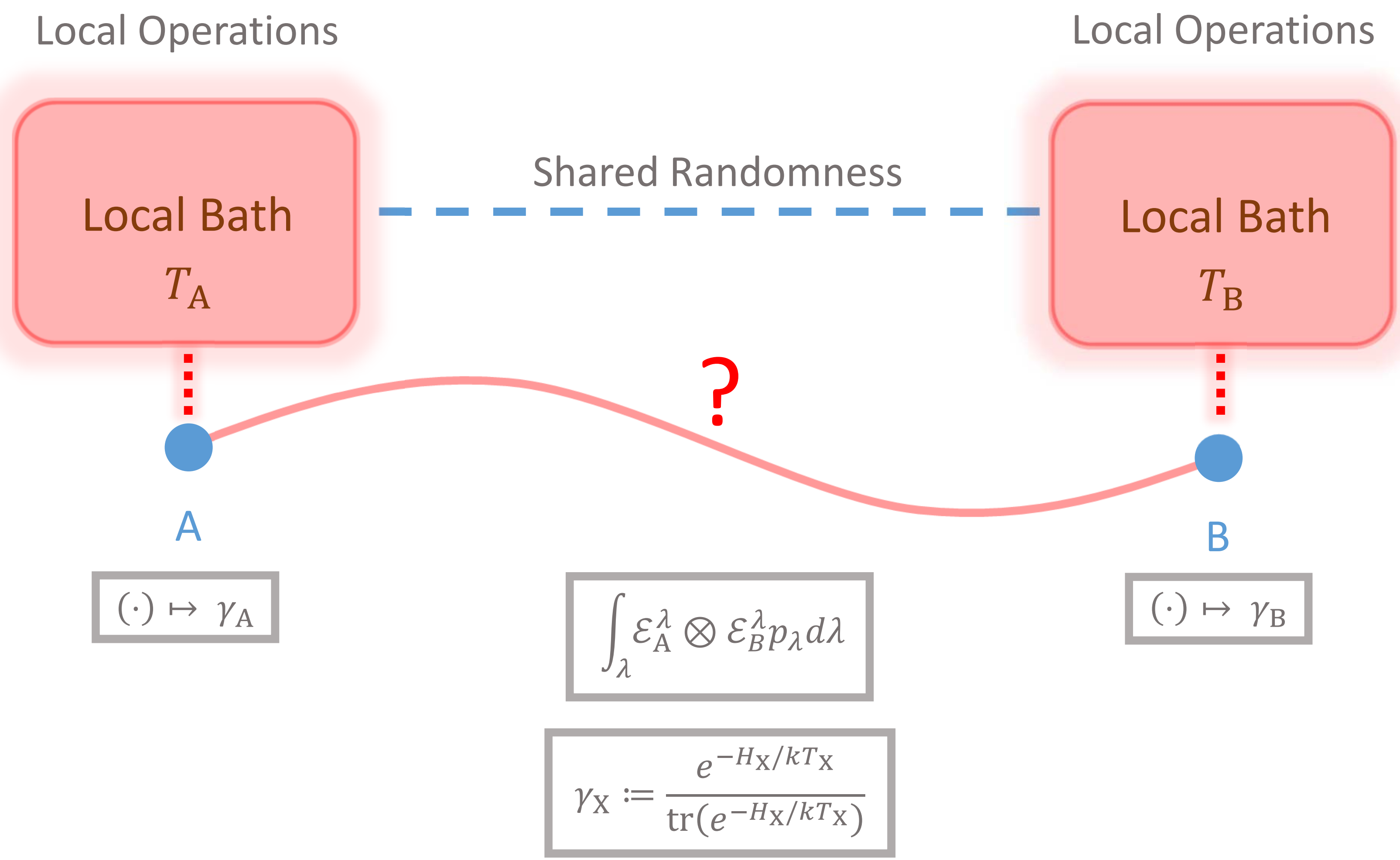


Entanglement Preserving Local Thermalization

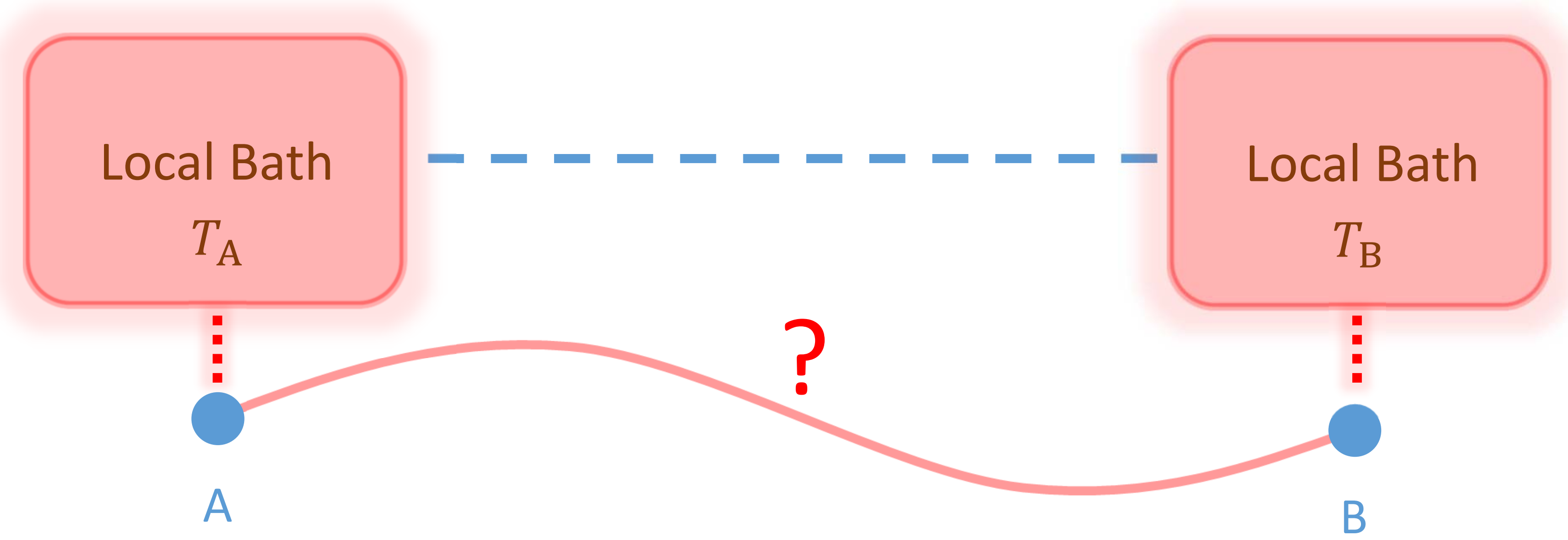
$$\int_{\lambda} \mathcal{E}_A^{\lambda} \otimes \mathcal{E}_B^{\lambda} p_{\lambda} d\lambda$$

ICFO

Entanglement Preserving Local Thermalization



Entanglement Preserving Local Thermalization

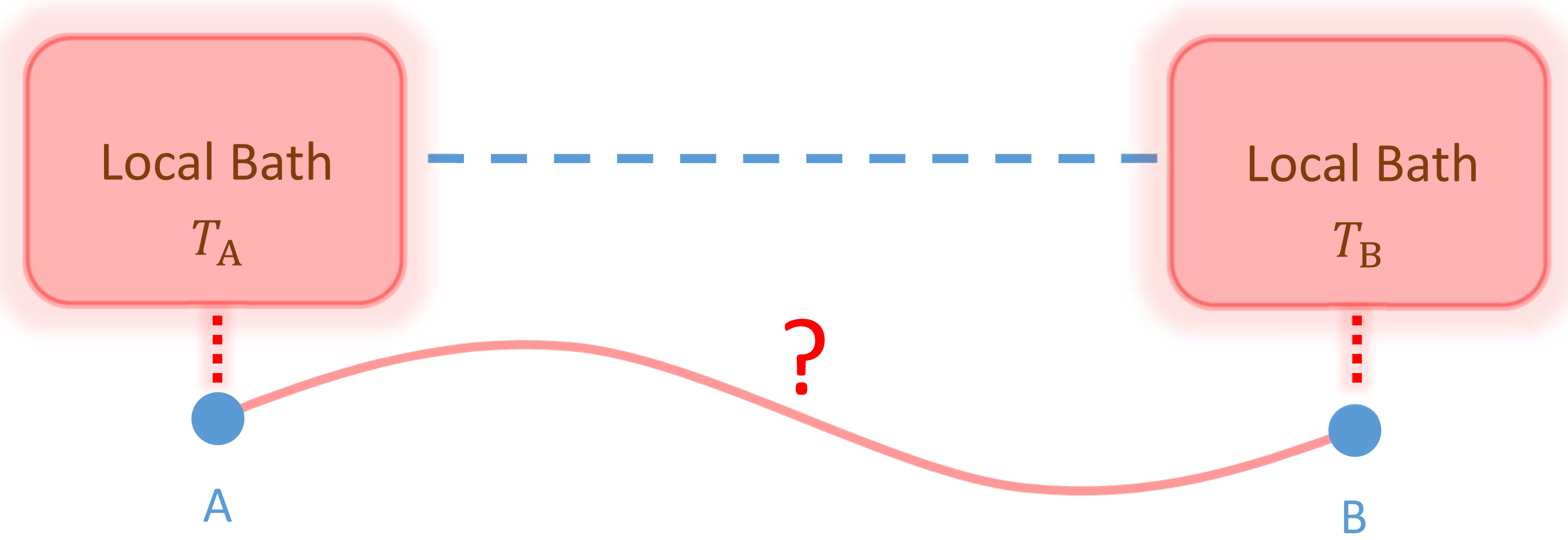


Local Thermalization

Local operations + shared randomness

Locally behaves as $(\cdot) \mapsto \gamma_A$ and $(\cdot) \mapsto \gamma_B$

Entanglement Preserving Local Thermalization



Local Thermalization | Local operations + shared randomness
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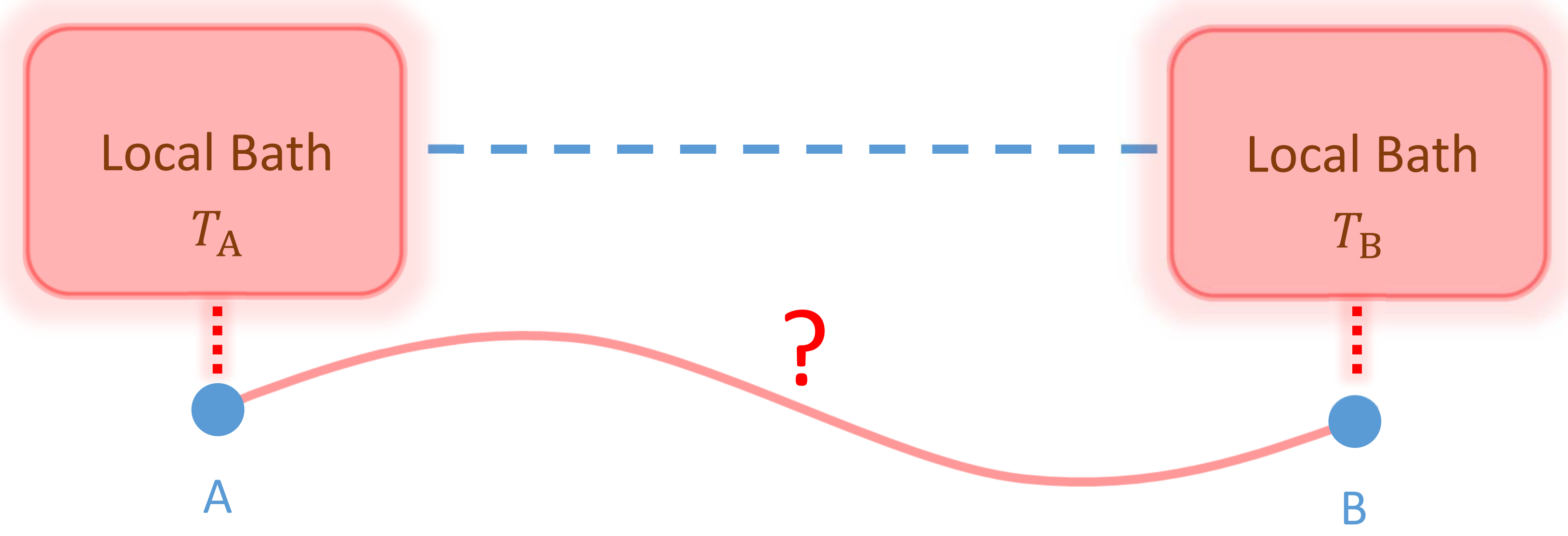


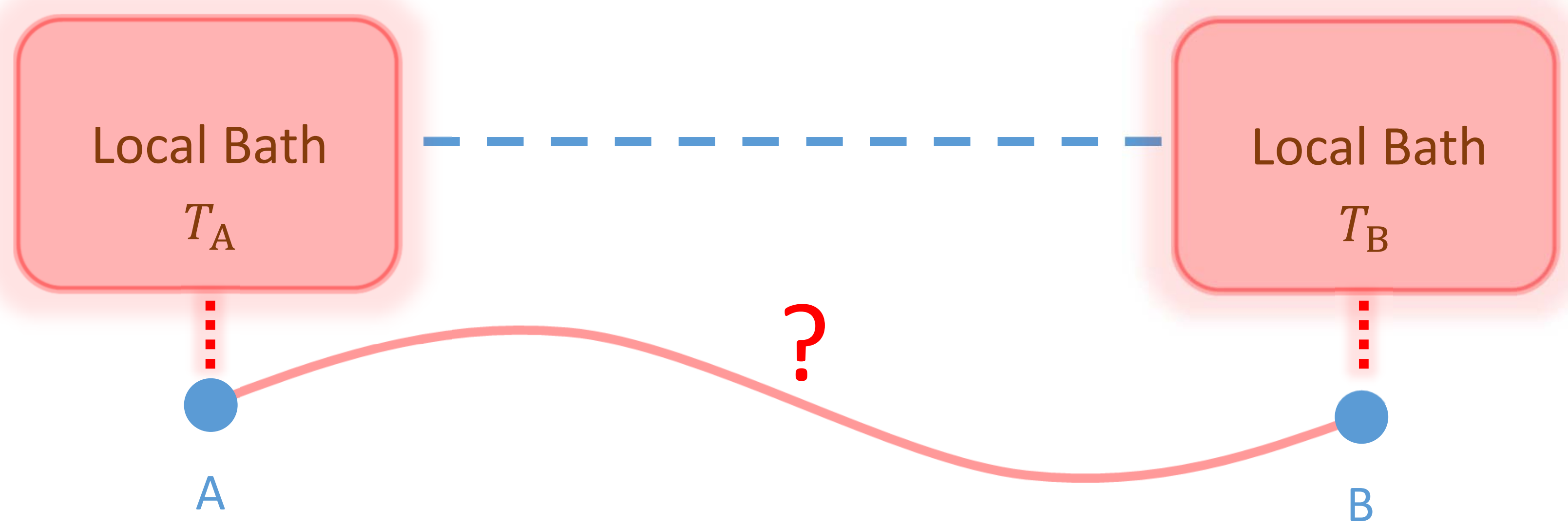
Local Thermalization

Local operations + shared randomness

Locally behaves as $(\cdot) \mapsto \gamma_A$ and $(\cdot) \mapsto \gamma_B$

RESULT 1 | Product local thermalization is identical to $(\cdot) \mapsto \gamma_A \otimes \gamma_B$.

Entanglement Preserving Local Thermalization



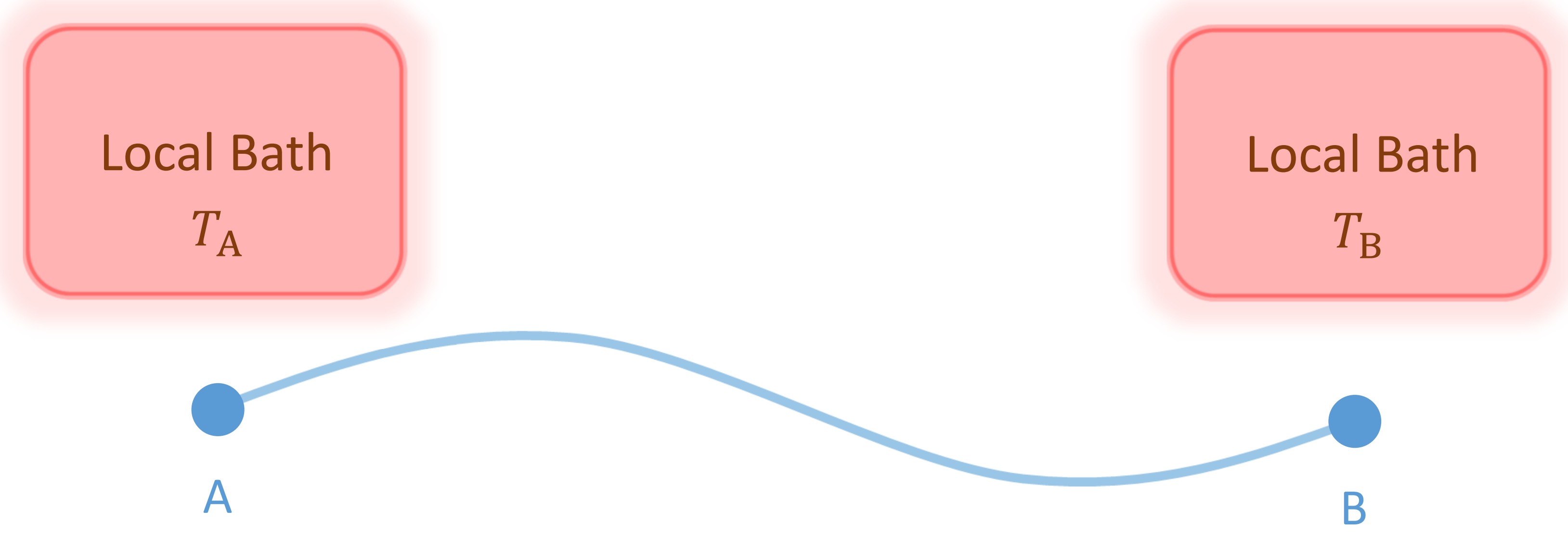
Local Thermalization

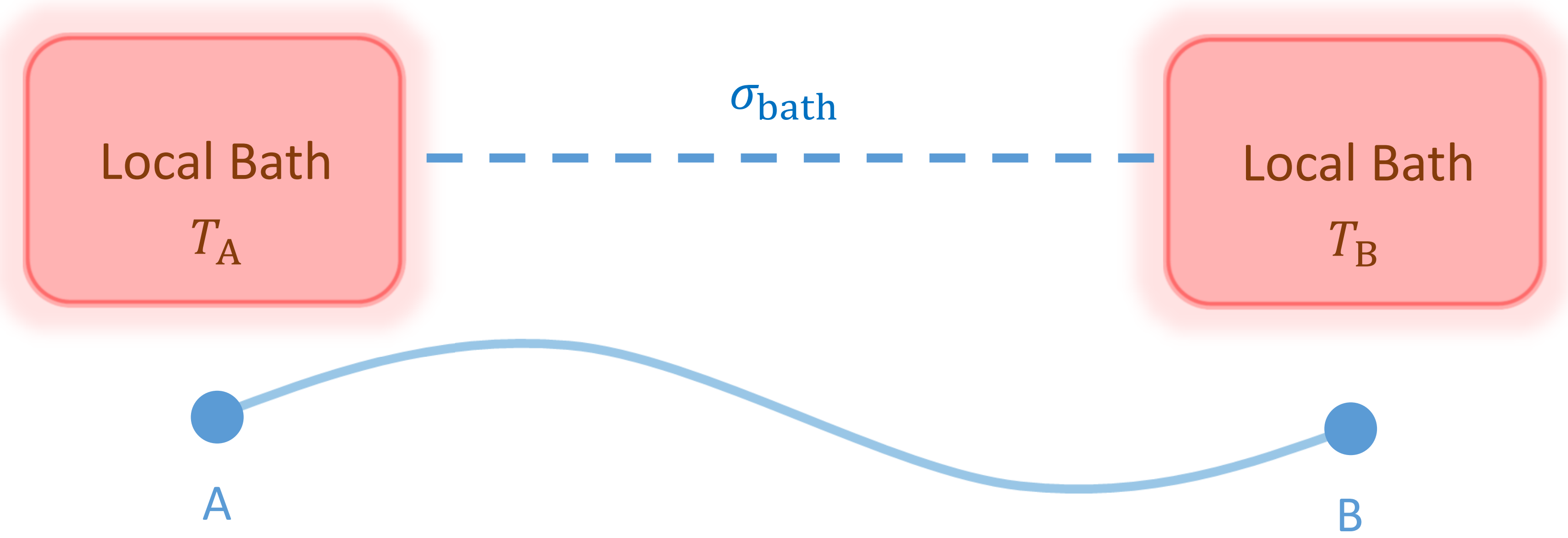
- Local operations + shared randomness
- Locally behaves as $(\cdot) \mapsto \gamma_A$ and $(\cdot) \mapsto \gamma_B$

Do entanglement preserving local thermalizations (EPLTs) exist?

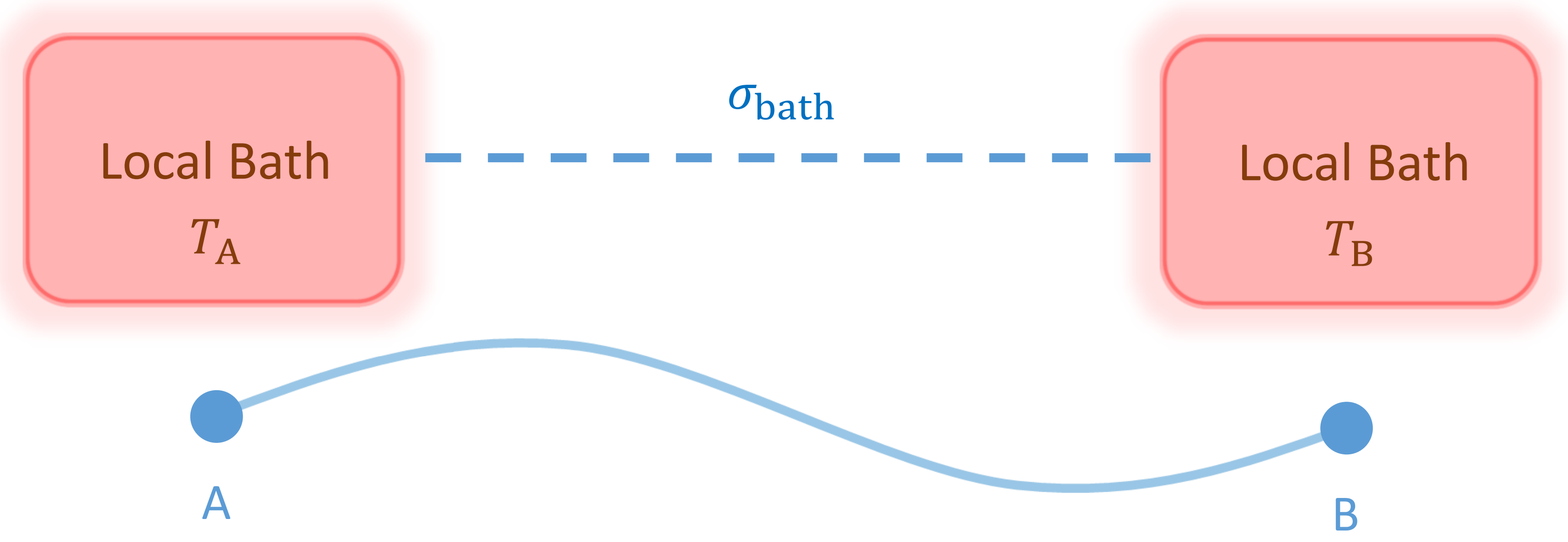
Results

Entanglement Preserving Local Thermalization

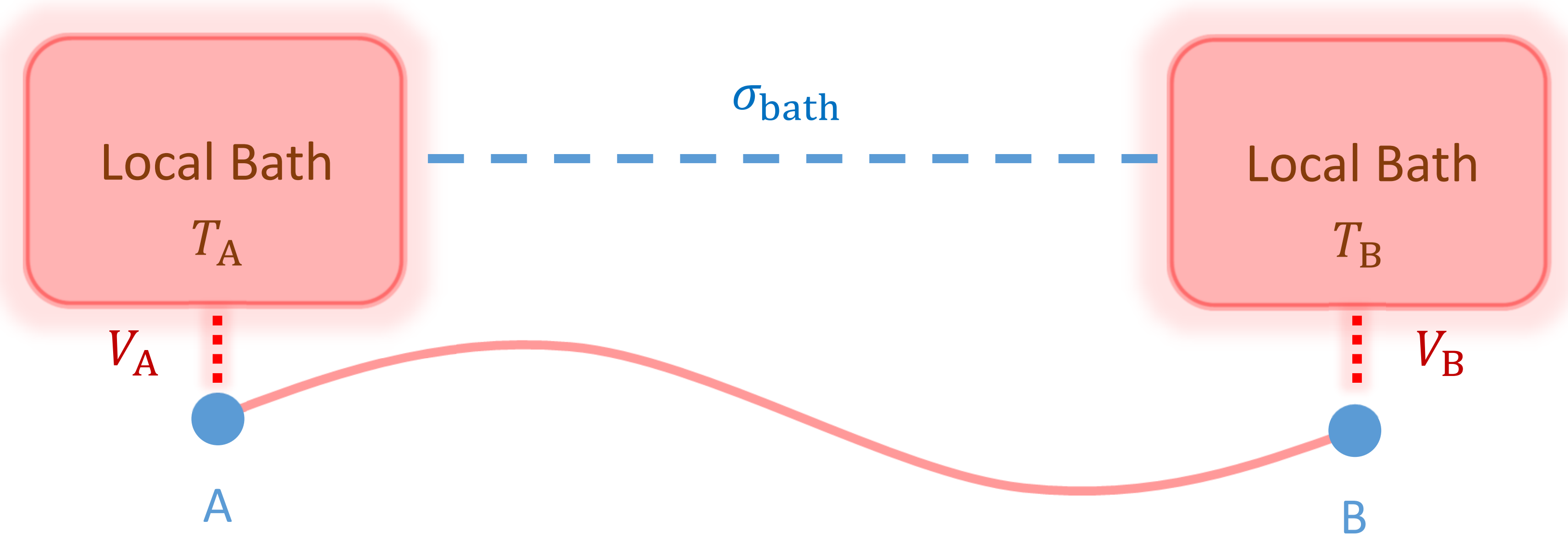



$\sigma_{\rm bath}$
Local Bath
$T_{\rm A}$
Local Bath
$T_{\rm B}$
A
B

Entanglement Preserving Local Thermalization

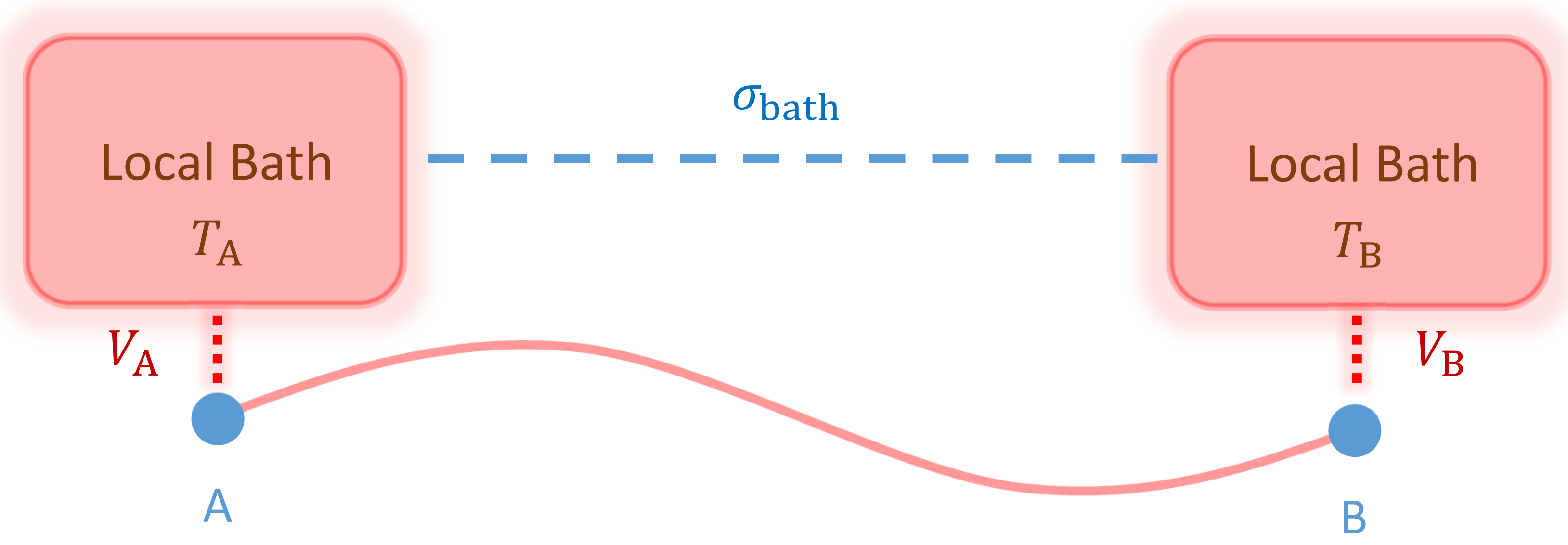


$$\sigma_{\text{bath}} = \left(\frac{1}{N_d}\sum_{k=1}^{N_d} |kk\rangle\langle kk|\right) \otimes \left(\left[\gamma_A + \frac{\epsilon}{1-\epsilon}\left(\gamma_A - \frac{\mathbb{I}}{d}\right)\right] \otimes \left[\gamma_B + \frac{\epsilon}{1-\epsilon}\left(\gamma_B - \frac{\mathbb{I}}{d}\right)\right]\right) \otimes [(1-\epsilon)|11\rangle\langle 11| + \epsilon|00\rangle\langle 00|]$$

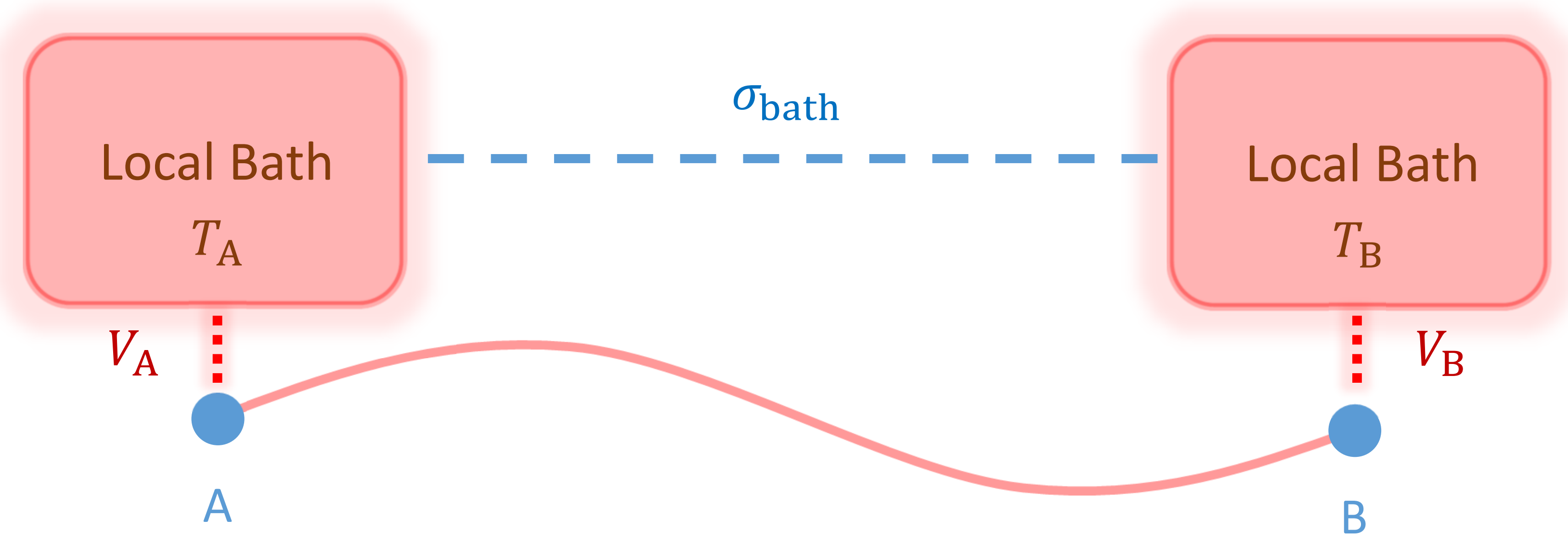


Local Bath
T_A
σ_{bath}
Local Bath
T_B
V_A
V_B
A
B

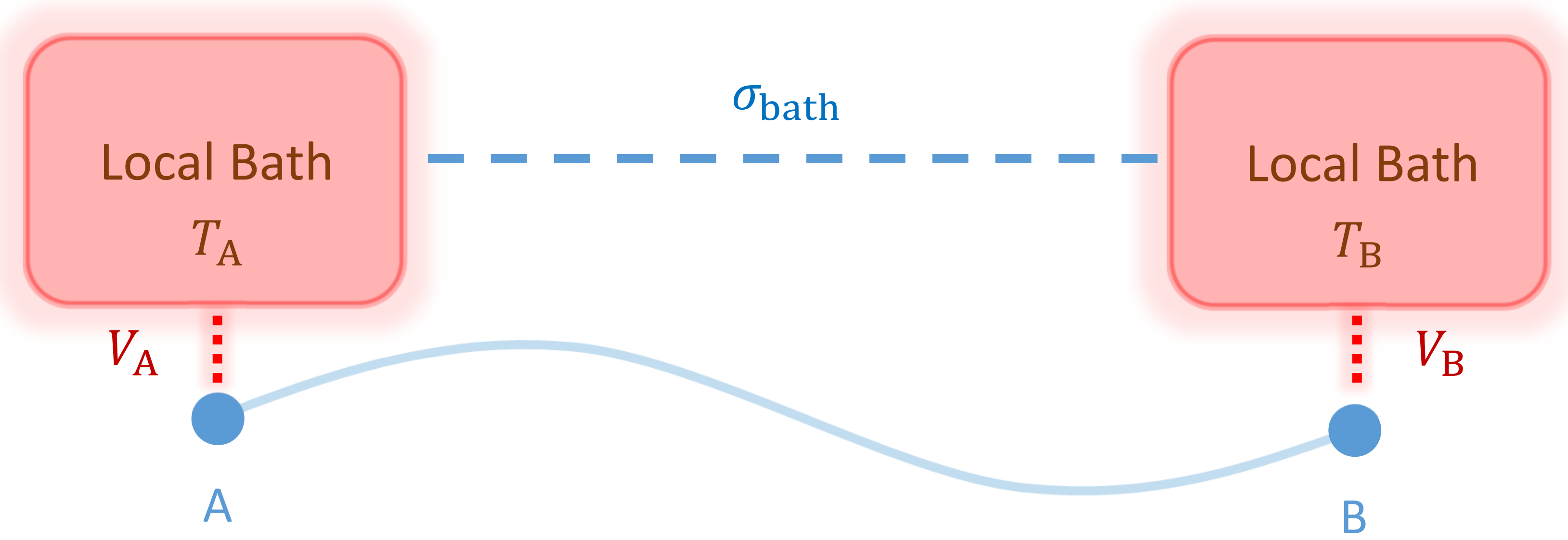
Entanglement Preserving Local Thermalization



$$V_X = \left[\mathrm{SWAP}(X, \mathrm{bath}-X2) \otimes |11\rangle\langle 11|_{\mathrm{bath}-X3} + \mathbb{I}_{X,\mathrm{bath}-X2} \otimes \left(\mathbb{I}_{\mathrm{bath}-X3} - |11\rangle\langle 11|_{\mathrm{bath}-X3}\right)\right] \times \sum_{k=1}^{N_d} U_k \otimes |k\rangle\langle k|_{\mathrm{bath}-X1}$$



Local Bath
T_A
Local Bath
T_B
σ_{bath}
V_A
V_B
A
B



Local Bath
T_A
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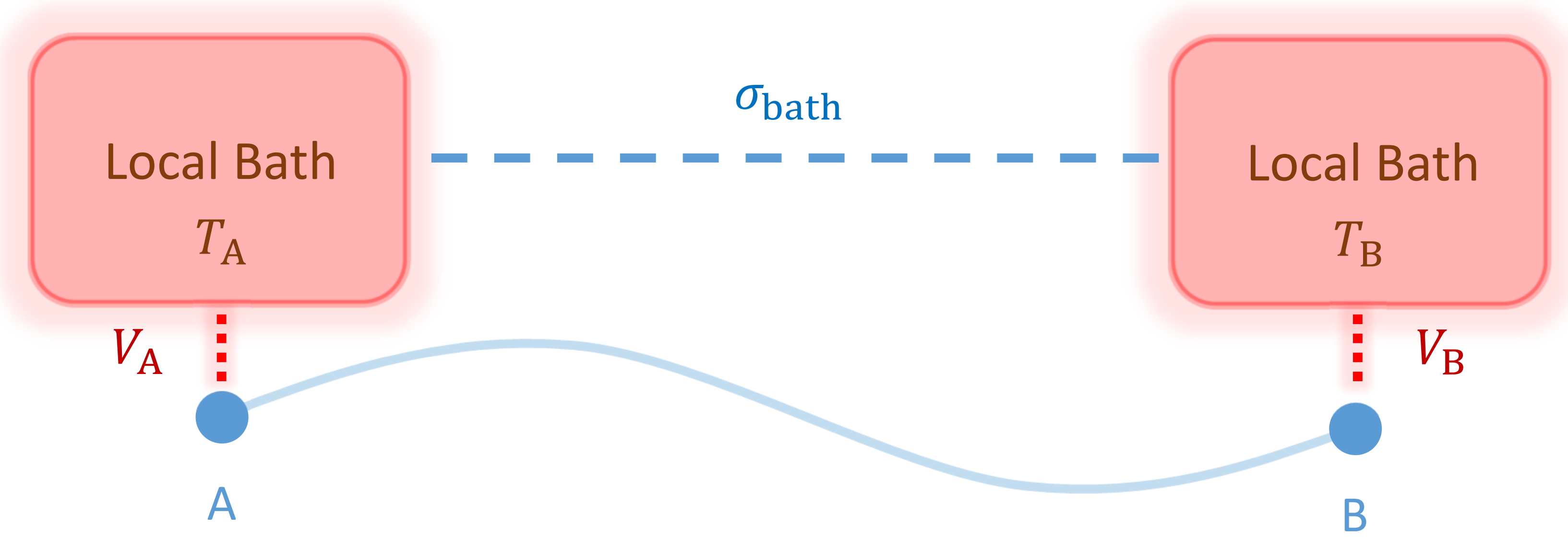
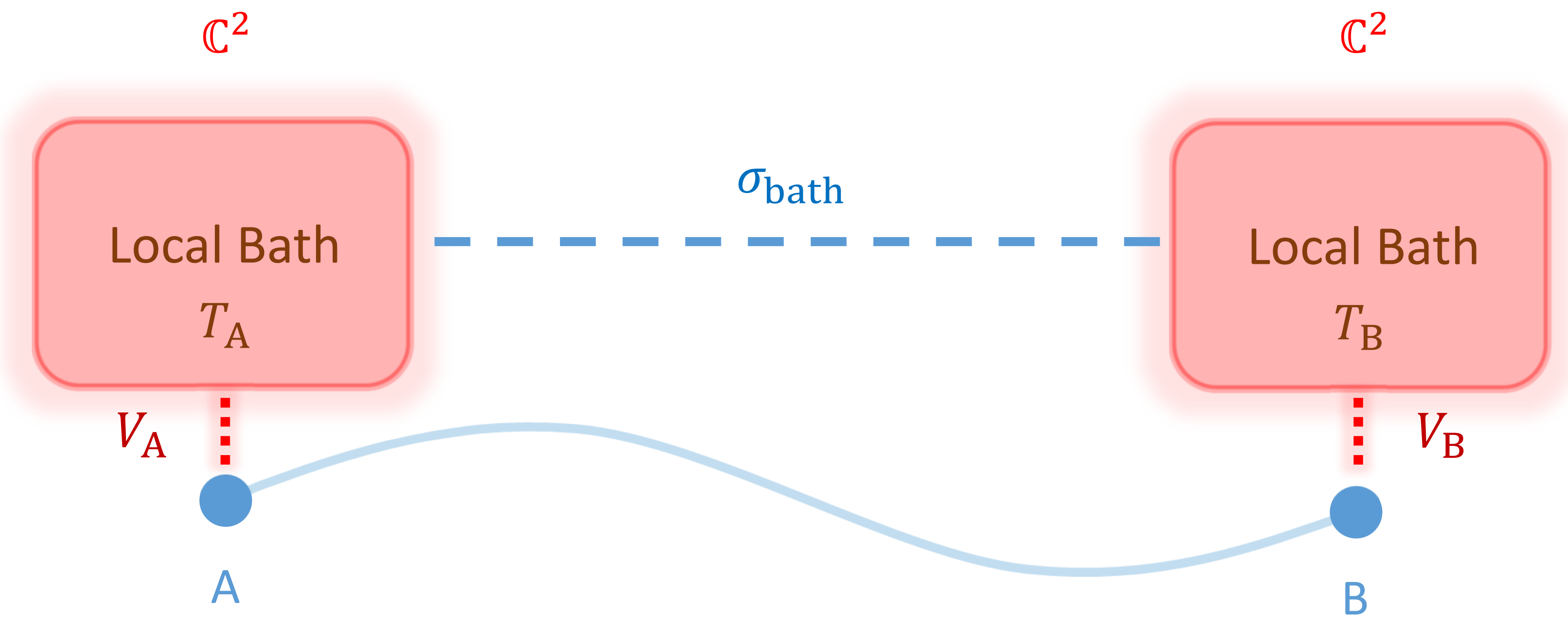
Local Bath
T_B
V_A
V_B
A
B



RESULT 2 | EPLT exists when the smallest eigenvalue among γ_{A} and $\gamma_{\text{B}} > \frac{1}{d^2}$.

Entanglement Preserving Local Thermalization



RESULT 2 | EPLT exists when the smallest eigenvalue among γ_A and $\gamma_B > \frac{1}{d^2}$.

RESULT 3 | In 2-qubit case, EPLT exists for all temperatures > 0 & finite-energy Hamiltonians.

RESULT 1 | No product EPLT.

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What is the underlying mechanism of EPLT?

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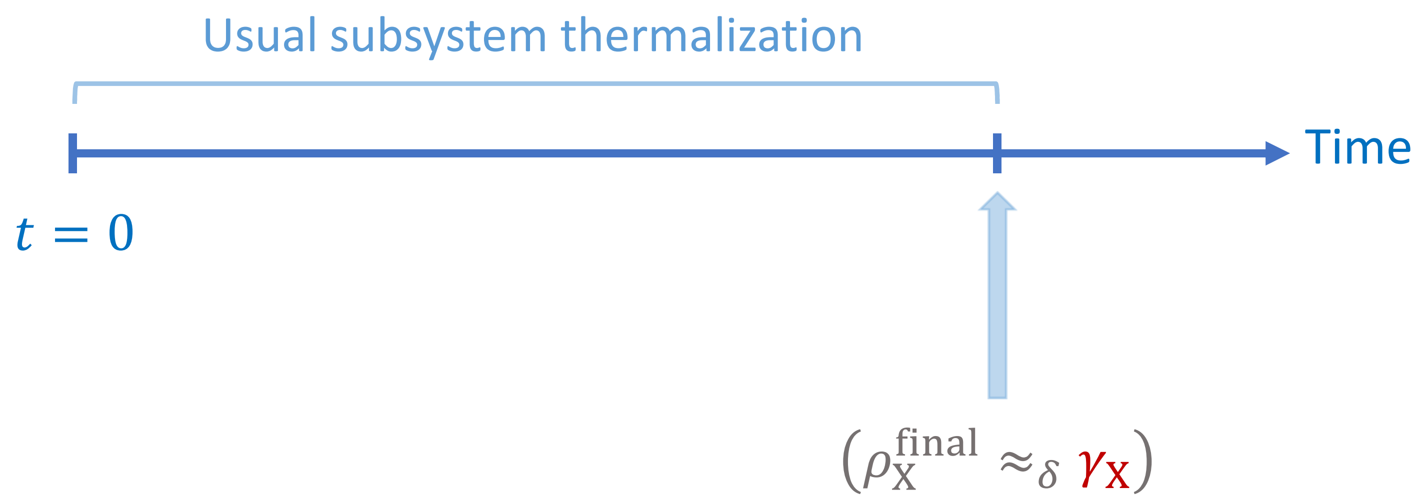

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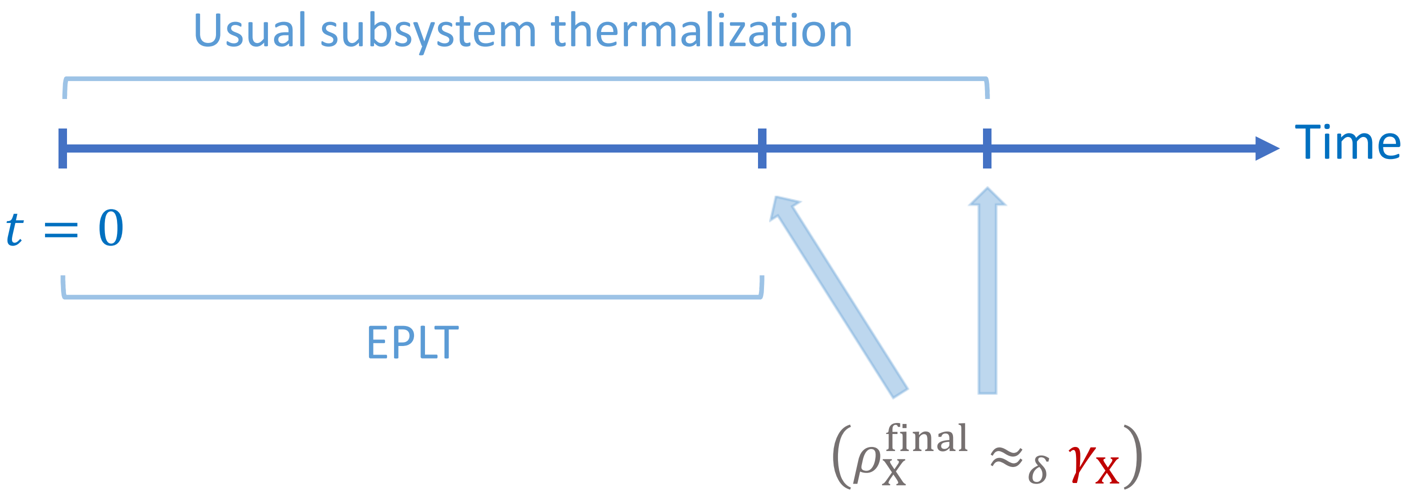

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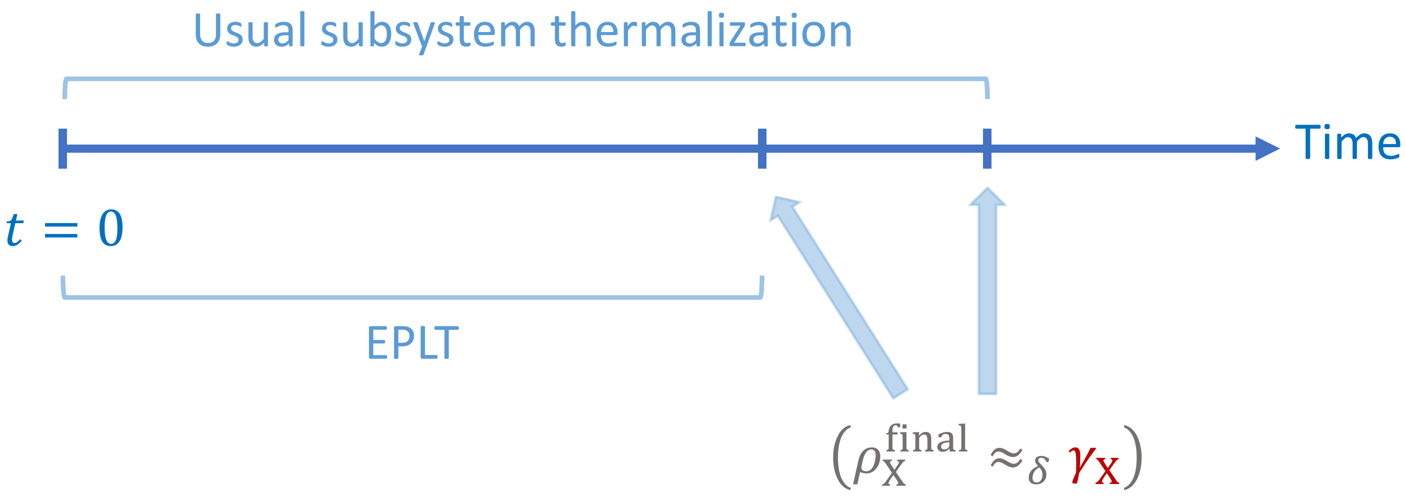
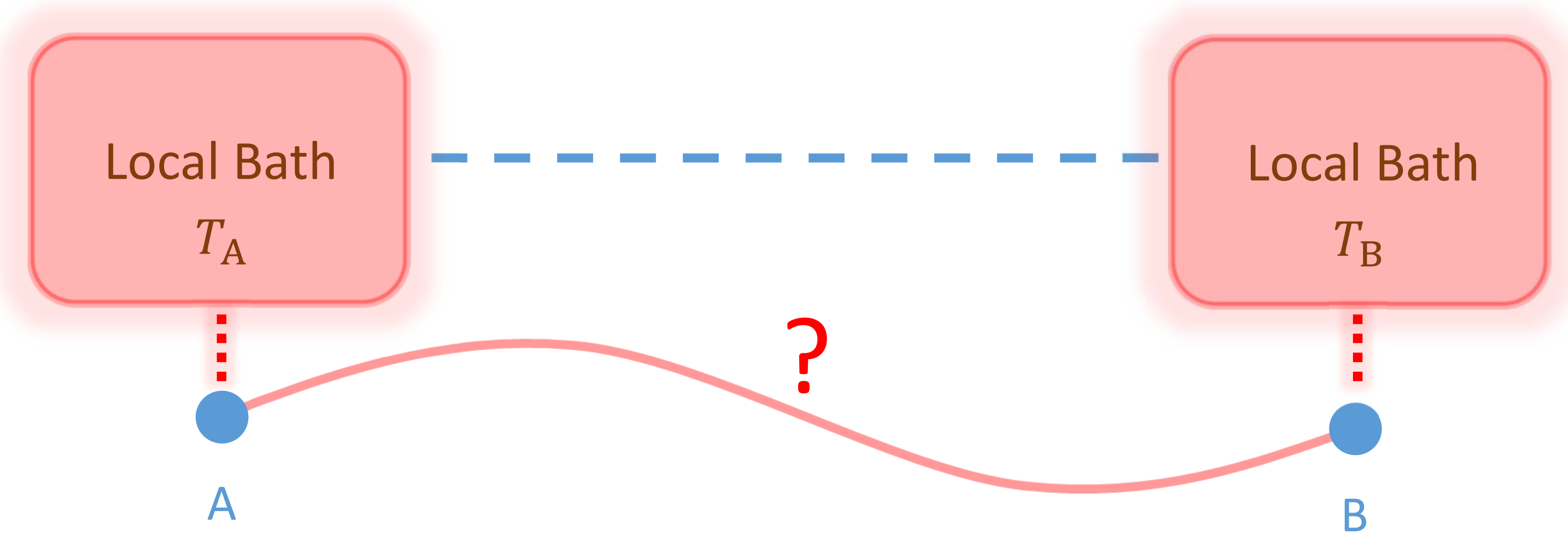


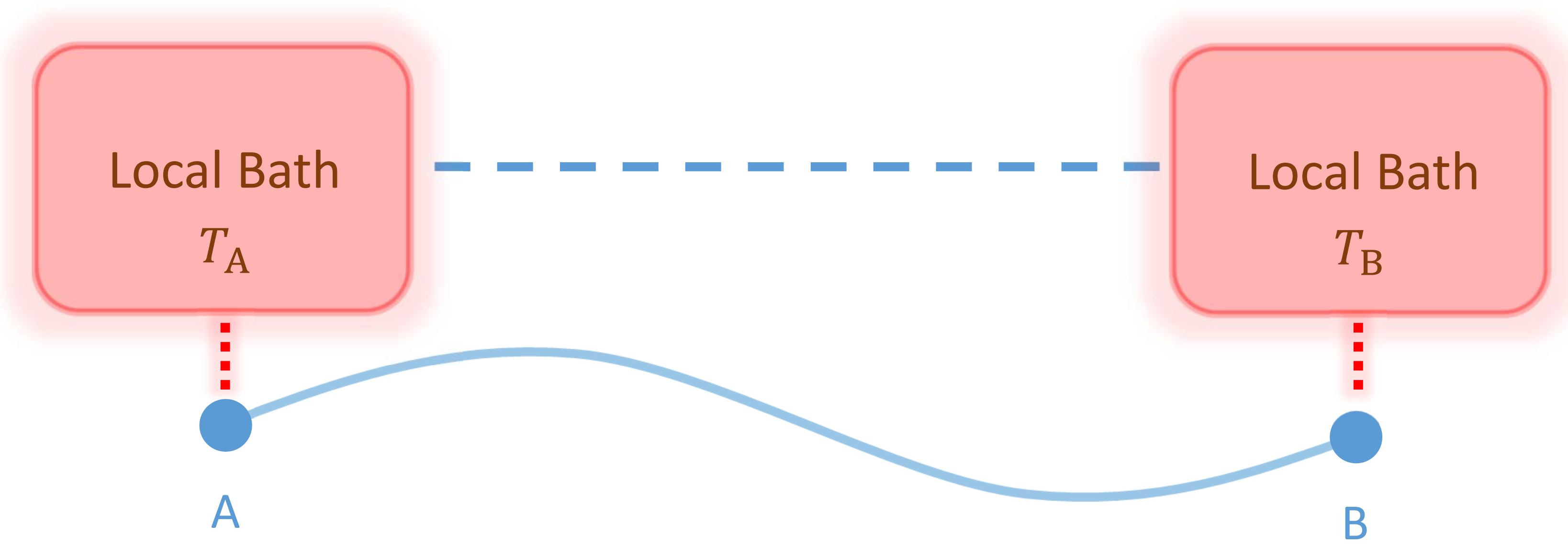
RESULT 4 | The mechanism is suggested to be a speed-up of subsystem thermalizations.

Summary

Can entanglement survive subsystem thermalization?



Can entanglement survive subsystem thermalization?



Entanglement can survive subsystem thermalization.

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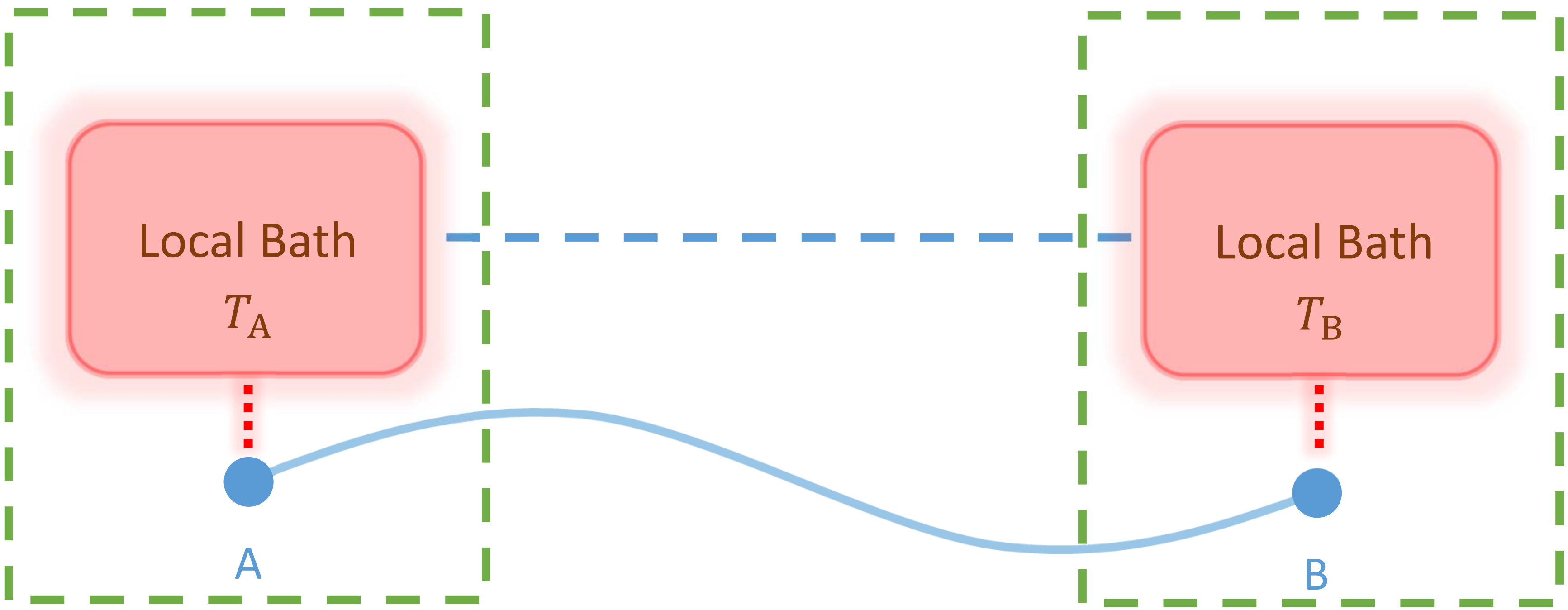

Entanglement can survive subsystem thermalization.

This is due to the speedup of subsystem thermalizations.

Acknowledgements

This project is part of the ICFOstepstone - PhD Programme for Early-Stage Researchers in Photonics, funded by the Marie Skłodowska-Curie Co-funding of regional, national and international programmes (GA665884) of the European Commission, as well as by the 'Severo Ochoa 2016-2019' program at ICFO (SEV-2015-0522), funded by the Spanish Ministry of Economy, Industry, and Competitiveness (MINECO).

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Thank you for your attention and patience!

arXiv:1904.07945

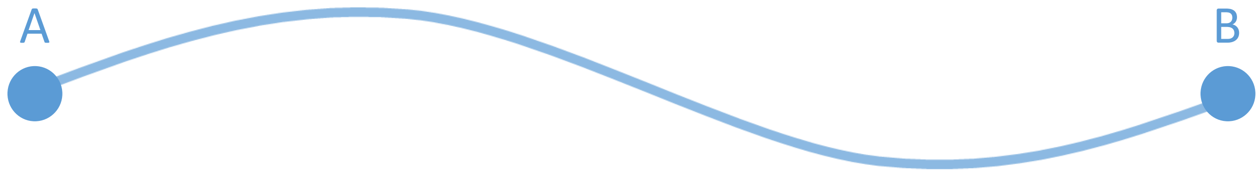
Supplementary Materials

Information-Theoretic Formulation

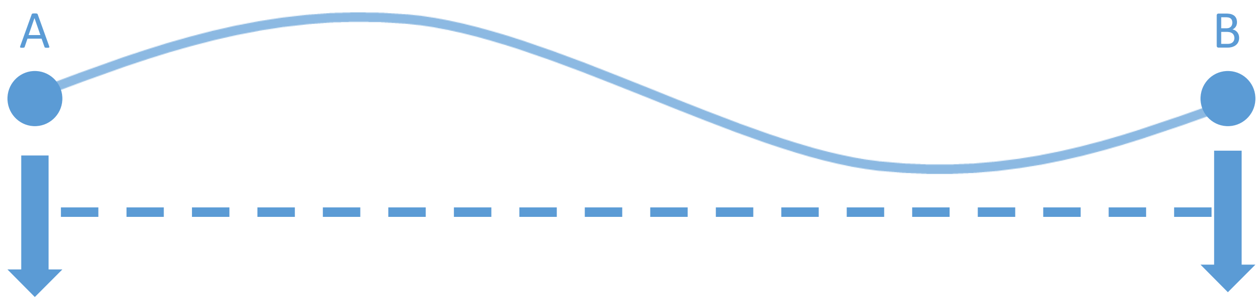
Information-theoretic formulation

Information-theoretic formulation

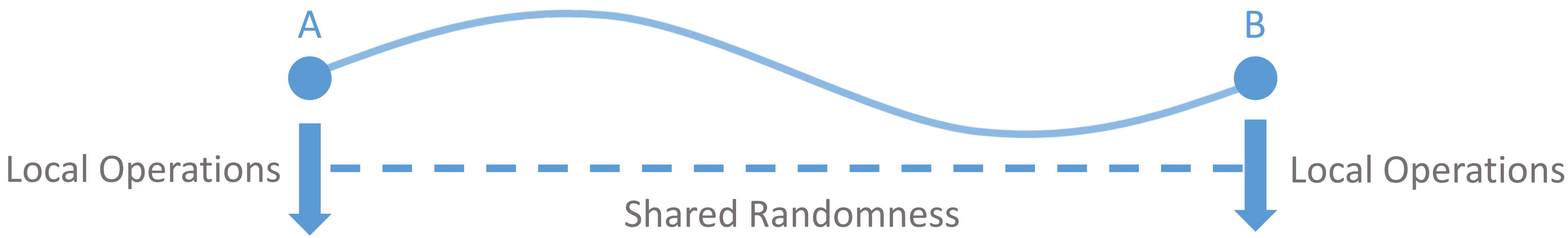
Information-theoretic formulation



Information-theoretic formulation

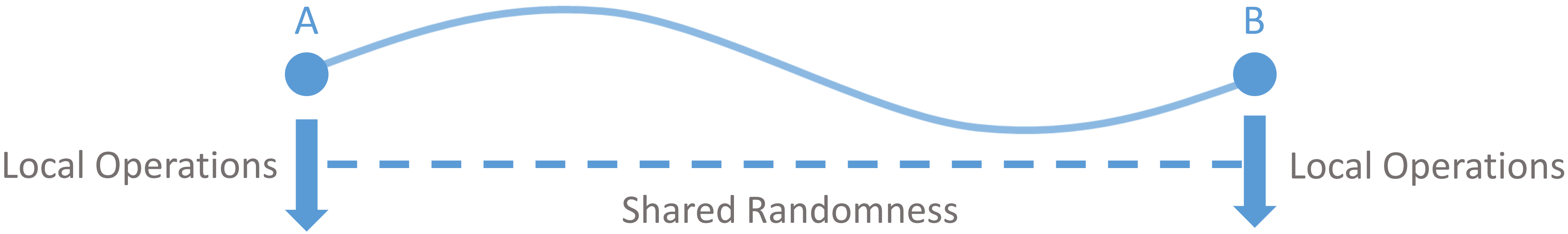


Information-theoretic formulation



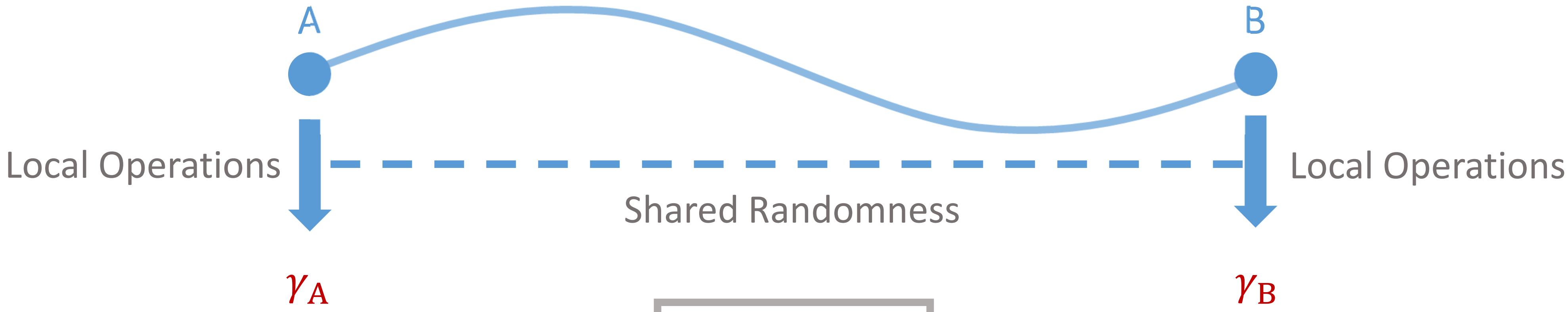
Entanglement Preserving Local Thermalization

Information-theoretic formulation



$$\int_\lambda \mathcal{E}_A^\lambda \otimes \mathcal{E}_B^\lambda p_\lambda d\lambda$$

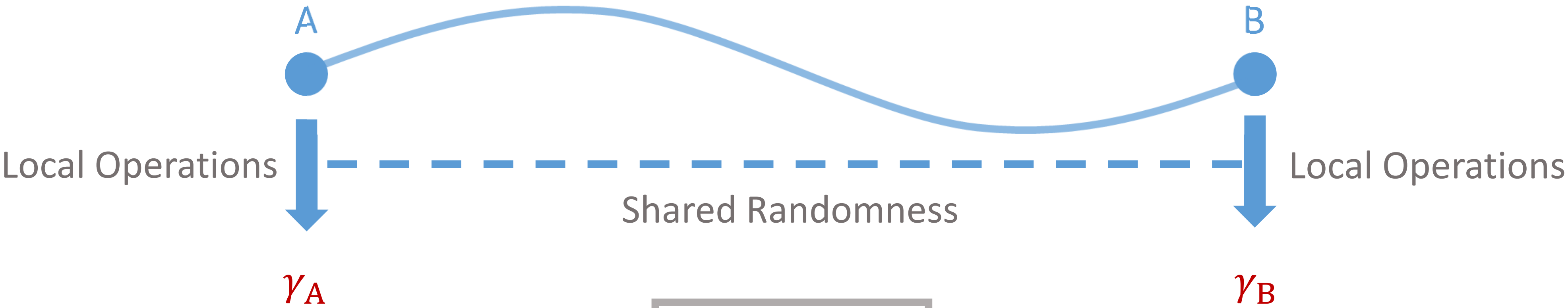
Information-theoretic formulation



$$\int_{\lambda} \mathcal{E}_{\mathrm{A}}^{\lambda} \otimes \mathcal{E}_{B}^{\lambda} p_{\lambda} d\lambda$$

Entanglement Preserving Local Thermalization

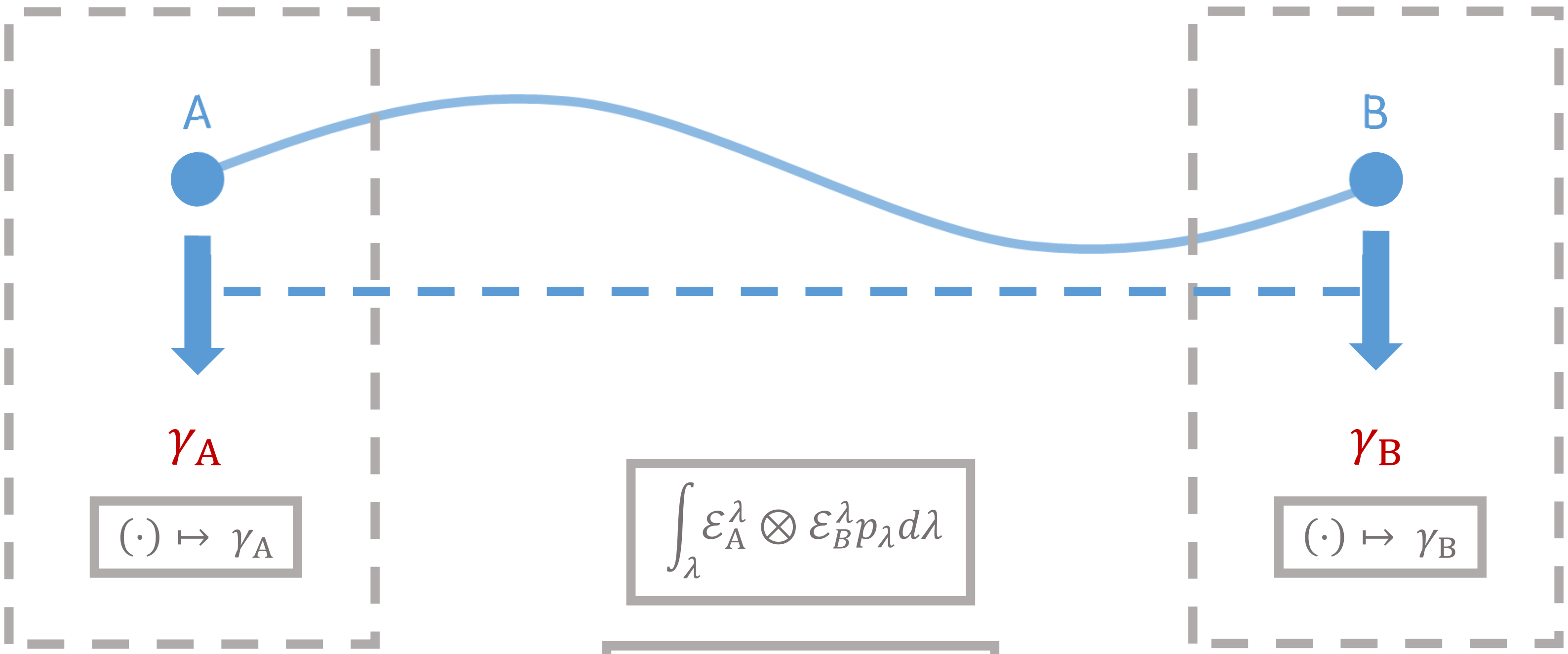
Information-theoretic formulation



$$\int_\lambda \mathcal{E}_A^\lambda \otimes \mathcal{E}_B^\lambda p_\lambda d\lambda$$

$$\gamma_X := \frac{e^{-H_X/kT_X}}{\mathrm{tr}(e^{-H_X/kT_X})}$$

Information-theoretic formulation



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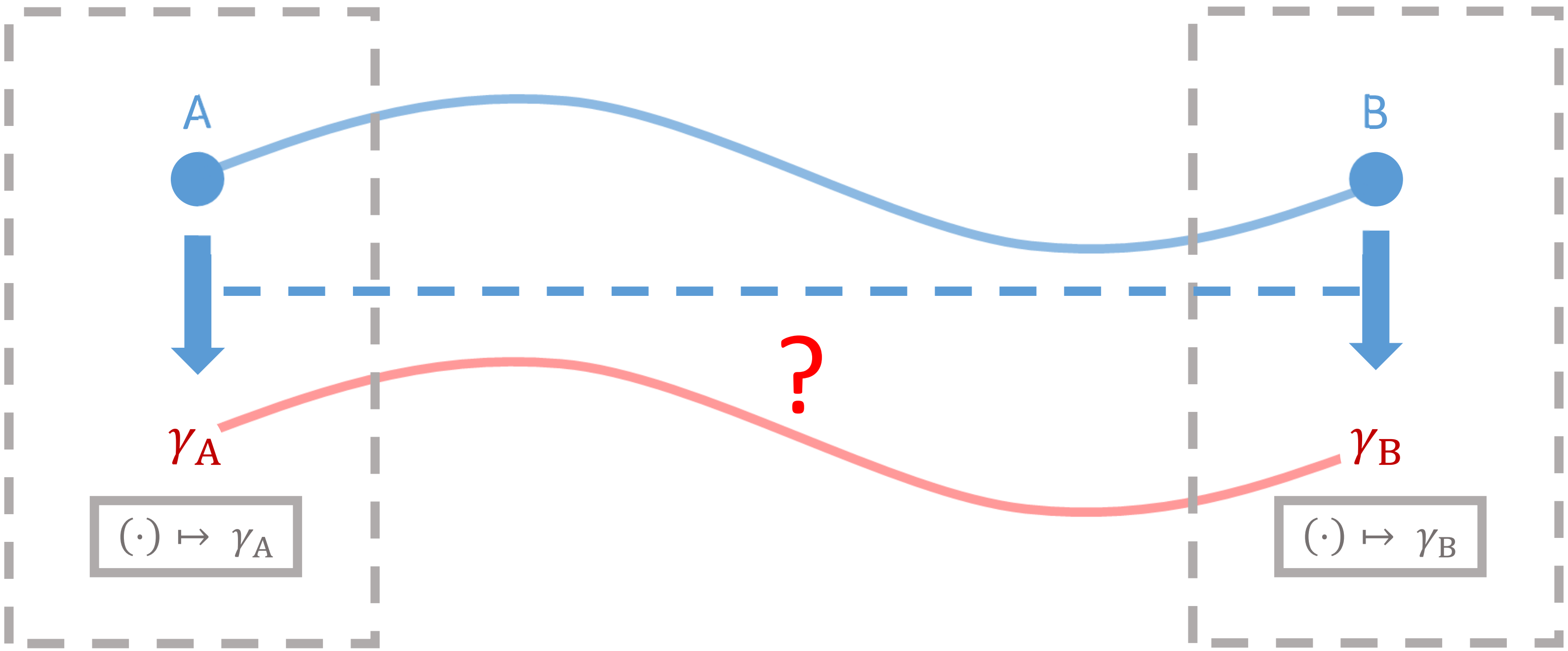

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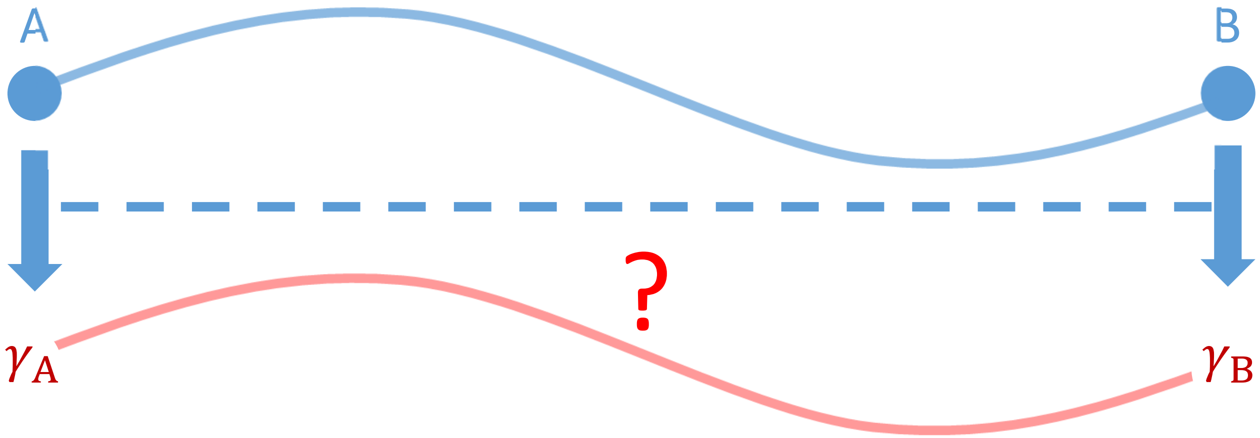
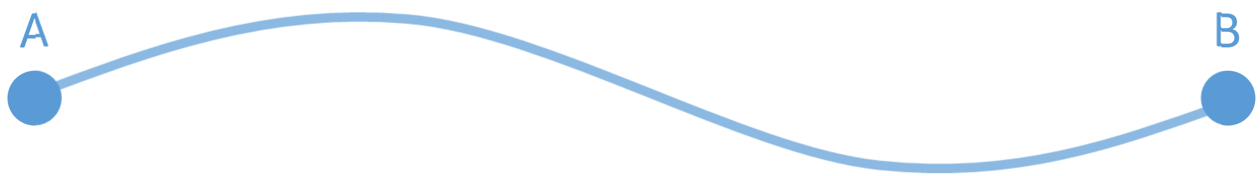


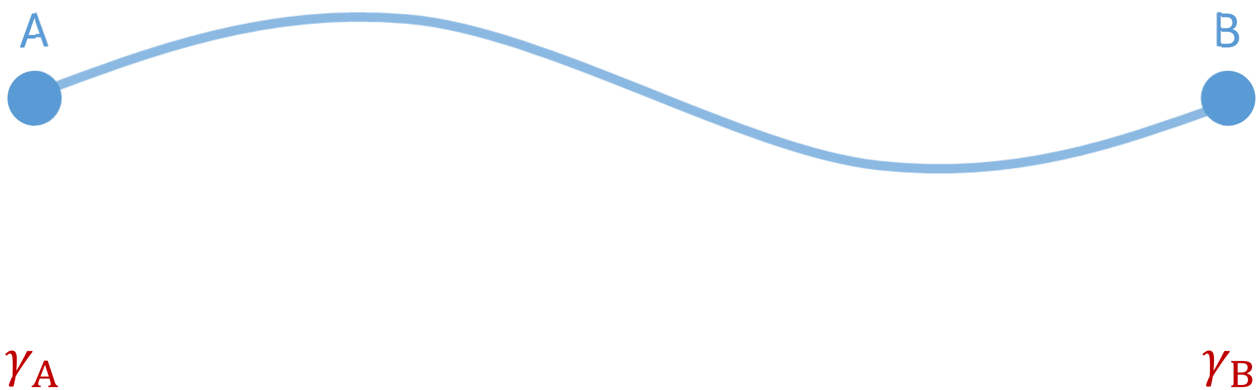
Information-Theoretic (LOSR) Formulation

EPLT in Information-Theoretic Formulation

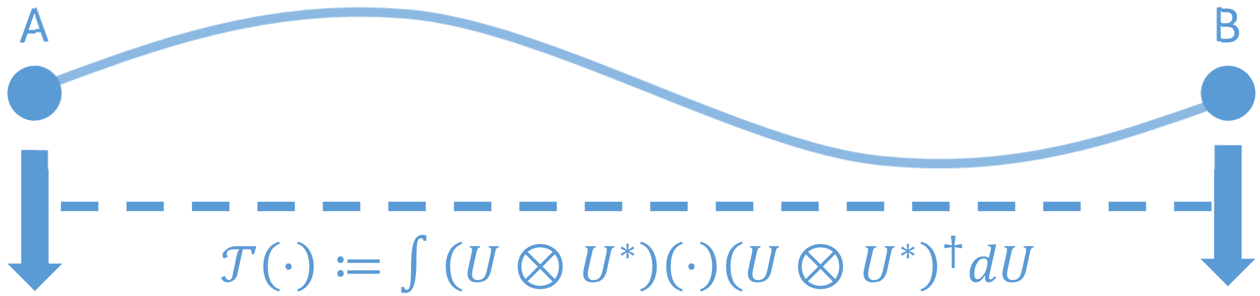
EPLT in the information-theoretic formulation



EPLT in the information-theoretic formulation



EPLT in the information-theoretic formulation



$$\mathcal{T}(\cdot) := \int (U \otimes U^*)(\cdot)(U \otimes U^*)^\dagger dU$$

C. H. Bennett *et al.*, Phys. Rev. Lett. **76**, 722 (1996); M. Horodecki & P. Horodecki, Phys. Rev. A **59**, 4206 (1999).

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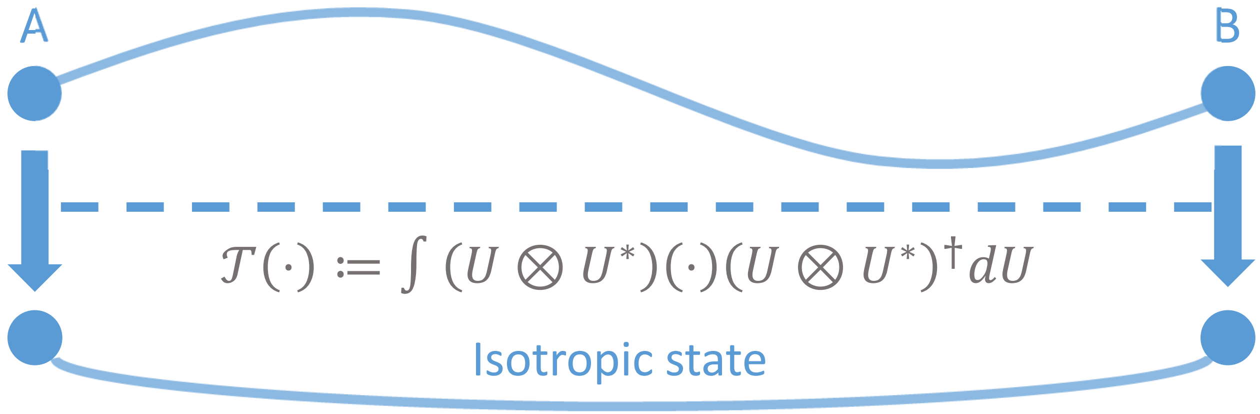
EPLT in the information-theoretic formulation



$$\mathcal{T}(\cdot) \coloneqq \int (U \otimes U^*)(\cdot)(U \otimes U^*)^\dagger dU$$

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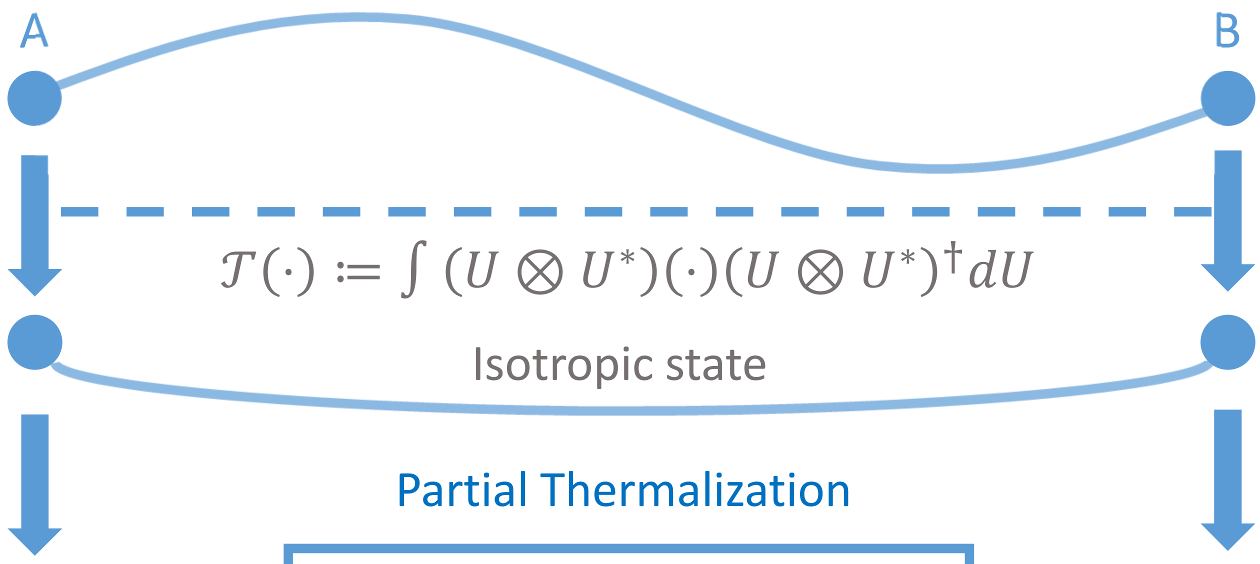
EPLT in the information-theoretic formulation



$$\mathcal{T}(\cdot) := \int (U \otimes U^*)(\cdot)(U \otimes U^*)^\dagger dU$$

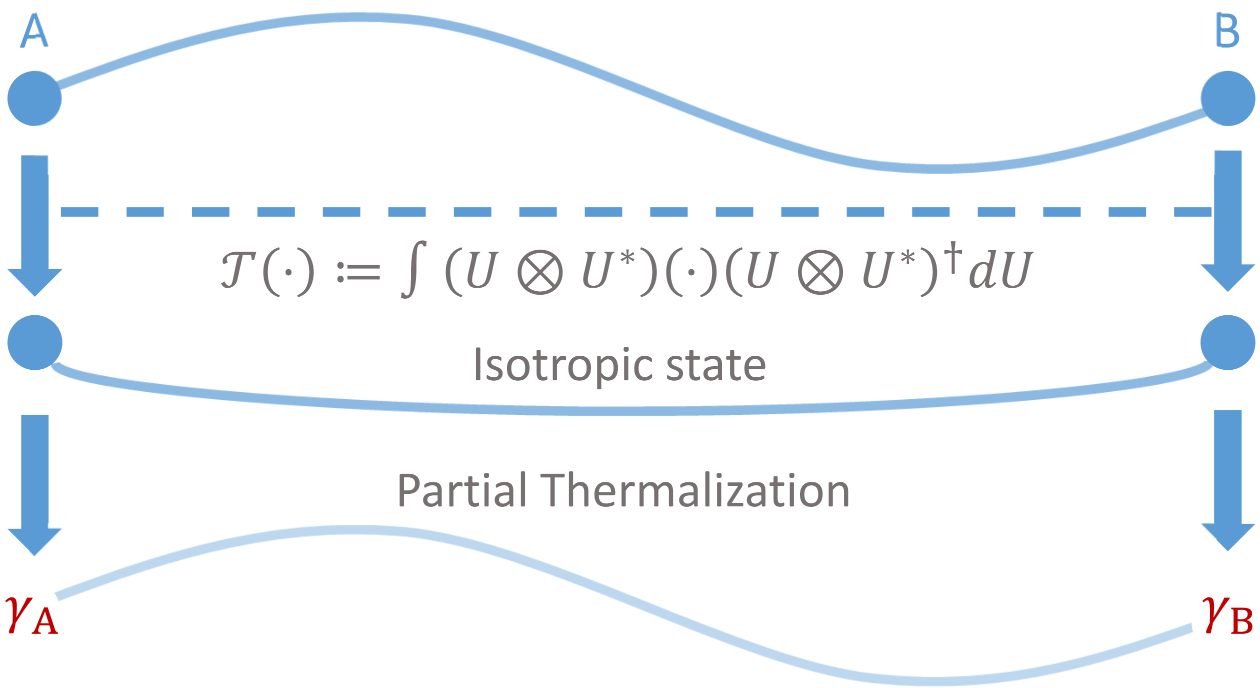
Isotropic state

Partial Thermalization

$$(\cdot) \mapsto (1-\epsilon)(\eta_A^\epsilon \otimes \eta_B^\epsilon) + \epsilon(\cdot)$$

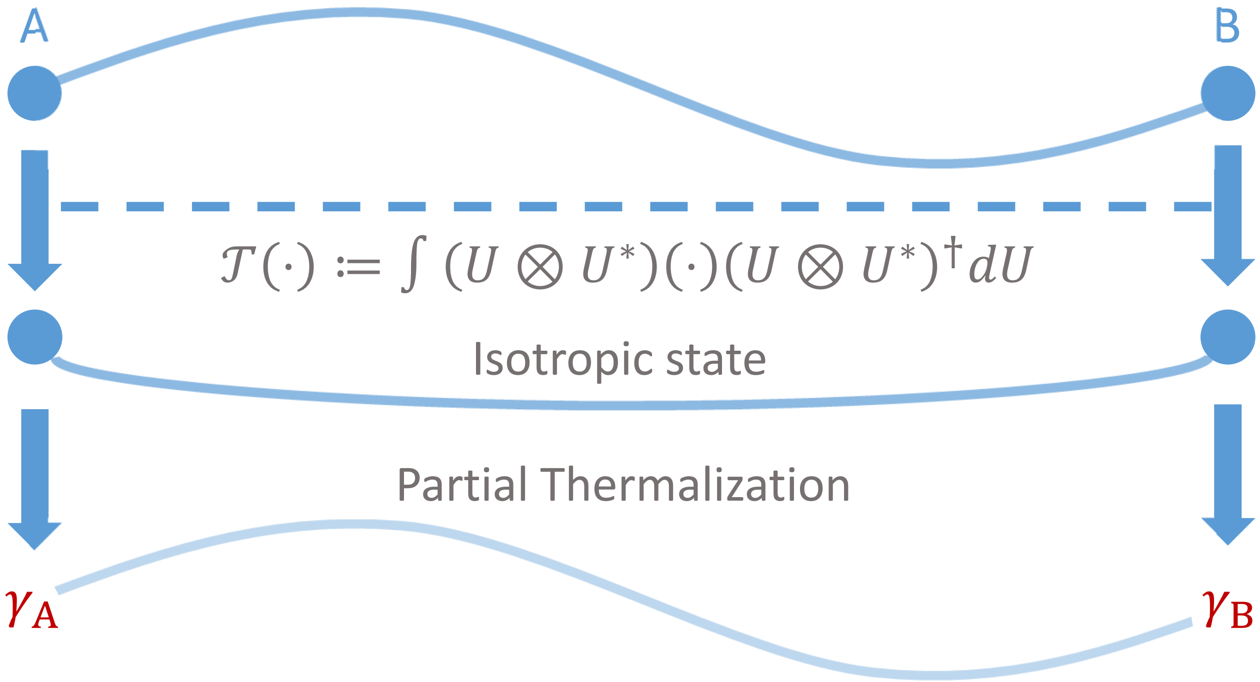
$$\eta_X^\epsilon := \gamma_X + \epsilon\left(\gamma_X - \frac{\mathbb{I}_X}{d}\right)$$

EPLT in the information-theoretic formulation



Entanglement Preserving Local Thermalization

EPLT in the information-theoretic formulation



$$\mathcal{E}^{\epsilon}_{(\gamma_{\mathrm{A}},\gamma_{\mathrm{B}})}(\cdot) \coloneqq (1-\epsilon)(\eta^{\epsilon}_{\mathrm{A}} \otimes \eta^{\epsilon}_{\mathrm{B}}) + \epsilon\mathcal{T}(\cdot)$$

EPLT in the information-theoretic formulation

THEOREM The channel

$$\mathcal{E}^{\epsilon}_{(\gamma_{\mathrm{A}},\gamma_{\mathrm{B}})}(\cdot) \coloneqq (1-\epsilon)(\eta^{\epsilon}_{\mathrm{A}} \otimes \eta^{\epsilon}_{\mathrm{B}}) + \epsilon\mathcal{T}(\cdot)$$

with $\eta^{\epsilon}_{\mathrm{X}} \coloneqq \gamma_{\mathrm{X}} + \frac{\epsilon}{1-\epsilon}\left(\gamma_{\mathrm{X}} - \frac{\mathbb{I}_{\mathrm{X}}}{d}\right)$ is a local thermalization to $(\gamma_{\mathrm{A}}, \gamma_{\mathrm{B}})$ for all $0 \leq \epsilon \leq dP_{\min}$, where $P_{\min}$ is the smallest eigenvalue among γ_{A} and γ_{B}. Moreover, for all input ρ_{AB}, we have

$$\mathcal{F}_{\max}\left[\mathcal{E}^{\epsilon}_{(\gamma_{\mathrm{A}},\gamma_{\mathrm{B}})}(\rho_{\mathrm{AB}})\right] \geq \epsilon\langle\Psi^{+}_{d}|\rho_{\mathrm{AB}}|\Psi^{+}_{d}\rangle.$$

$\mathcal{F}_{\max}(\rho) \coloneqq \max_{\Psi}\langle\Psi|\rho|\Psi\rangle$ is the fully entangled fraction.

$|\Psi^{+}_{d}\rangle \coloneqq \sum_{n=0}^{d-1}|nn\rangle$ is a maximally entangled state.

EPLT in the information-theoretic formulation

THEOREM The channel

$$\mathcal{E}^{\epsilon}_{(\gamma_A,\gamma_B)}(\cdot) \coloneqq (1-\epsilon)(\eta^{\epsilon}_{A}\otimes\eta^{\epsilon}_{B}) + \epsilon\mathcal{T}(\cdot)$$

with $\eta^{\epsilon}_{X} \coloneqq \gamma_X + \frac{\epsilon}{1-\epsilon}\left(\gamma_X - \frac{\mathbb{I}_X}{d}\right)$ is a local thermalization to (γ_A,γ_B) for all $0 \leq \epsilon \leq dP_{\min}$, where $P_{\min}$ is the smallest eigenvalue among γ_A and γ_B. Moreover, for all input ρ_{AB}, we have

$$\mathcal{F}_{\max}\left[\mathcal{E}^{\epsilon}_{(\gamma_A,\gamma_B)}(\rho_{AB})\right] \geq \epsilon\langle\Psi^{+}_{d}|\rho_{AB}|\Psi^{+}_{d}\rangle.$$

RESULT 2 EPLT exists when the smallest eigenvalue among γ_A and $\gamma_B > \frac{1}{d^2}$.

Speed-Up of Subsystem Thermalization

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$$\mathcal{T}(\cdot) := \int (U \otimes U^*)(\cdot)(U \otimes U^*)^\dagger dU$$

Entanglement Preserving Local Thermalization

Speed-up of subsystem thermalization

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$$\approx \mathcal{T}_{\mathbf{U}}^{(N)}(\cdot) := \prod_{i=1}^{N} \mathcal{T}_i(\cdot)$$

$$\mathcal{T}_i(\cdot) := \frac{1}{2}\mathcal{I}(\cdot) + \frac{1}{2}(U_i \otimes U_i^*)(\cdot)(U_i \otimes U_i^*)^\dagger$$

$\mathbf{U} := \{U_i\}_{i=1}^{N}$ is the vector of unitary arguments

G. Tóth and J. J. García-Ripoll, Phys. Rev. A 75, 042311 (2007).

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LEMMA | For every $\lambda > 0$, we have $P\left(\left\|\mathcal{T} - \mathcal{T}_{\mathbf{U}}^{(N)}\right\|_\infty^2 - \frac{1}{2^N} > \lambda\right) < \frac{1}{\lambda^2 2^N}$

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Speed-up of subsystem thermalization

THEOREM | Given the local thermal state γ_X. If

$$\tau_{\gamma_\mathrm{X}} > t_U \times \frac{8}{\ln 2}$$

then for every local input $\rho_\mathrm{X} \neq \gamma_\mathrm{X}$, there exists a value $\delta' > 0$ such that for every $\delta \in (0, \delta')$ and $N_\delta := \left\lceil 8 \log_2 \frac{d^2 P_{\min}^{\mathrm{X}} \sqrt{2}}{\delta} \right\rceil$, N_δ-EPLT can demonstrate a speed-up of δ-thermalization for ρ_X to γ_X with success probability

$$1 - \left[\frac{\delta}{d^2 P_{\min}^{\mathrm{X}} \sqrt{2}}\right]^4$$

$P_{\min}^{\mathrm{X}}$: smallest eigenvalue of γ_X

N-EPLT: an EPLT realized by N sequential random unitaries.

Speed-up: EPLT thermalization time is shorter than normal one.