Correlations as a resource in single-shot quantum thermodynamics

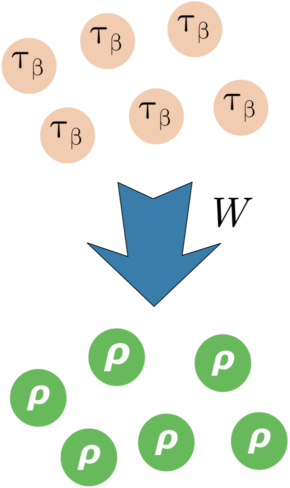
Federico Cerisola

University of Buenos Aires

QTD2019, Espoo, Finland, June 2019

departamento de física
universidad de buenos aires - exactas

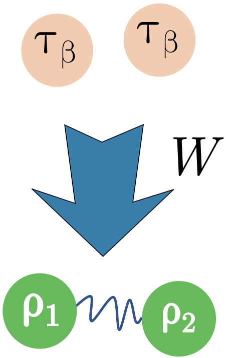
Task: create N copies of state ρ through thermodynamic transformations



Can this provide an advantage?

Correlations in standard thermodynamics

Work: $\langle W \rangle = \Delta F$



$$H = H_1 + H_2, \quad \tau_\beta^{(12)} = \tau_\beta^{(1)} \otimes \tau_\beta^{(2)}$$

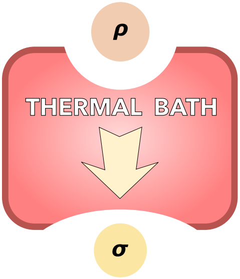
$$\langle W \rangle = \Delta F_1 + \Delta F_2 + k_B T \mathcal{I}_{12} \geq \Delta F_1 + \Delta F_2$$

Total correlations:

$$\mathcal{I}_{12} = D\left(\rho || \rho_1 \otimes \rho_2\right) = S(\rho_1) + S(\rho_2) - S(\rho_{12})$$

Correlations are costly

Thermodynamics as a resource theory



State (athermal states are resources)

$$\rho,\ H_S.$$

Thermal bath (thermal states are free)

$$\tau_B,\ H_B,$$

$$H_{SB} = H_S + H_B,\ \rho_{SB} = \rho \otimes \tau_S.$$

Energy preserving unitary
(strict energy conservation)

$$U(\rho \otimes \tau_S),\ [U, H_S + H_B] = 0.$$

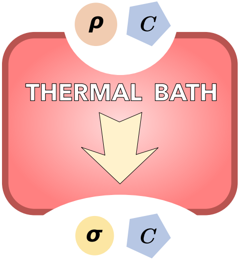
Thermal operation

$$\sigma = \mathrm{tr}_B \left[U(\rho \otimes \tau_B) \right]$$

Necessary and sufficient (for block-diagonal) condition:
thermo-majorization

M. Horodecki & J. Oppenheim. Nat. Commun. 4, 2059 (2013)

Thermodynamics as a resource theory



$$\rho \otimes c \rightarrow \sigma \otimes c$$

Necessary and sufficient (for block-diagonal states) conditions:

Set of inequalities (family of free energies)

$$F_\alpha(\rho) \geq F_\alpha(\sigma), \quad \forall \alpha \in \mathbb{R}$$

$$F_\alpha(\rho) = k_B T D_\alpha\left(\rho || \tau\right) - k_B T \log Z$$

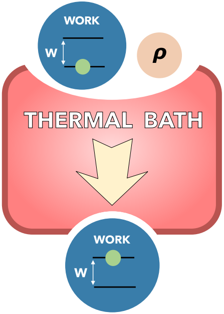
$$\tau = e^{-\beta H}/Z$$

$\alpha = 1 \longrightarrow$ standard free energy

F. Brandao, et al. PNAS 112, 3275 (2015)

Deterministic work in the single-shot regime

Work extraction

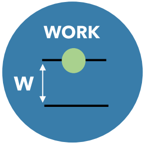


$$\rho \otimes |0\rangle\langle 0|_W \to \tau \otimes |W\rangle\langle W|_W$$

Extractable work

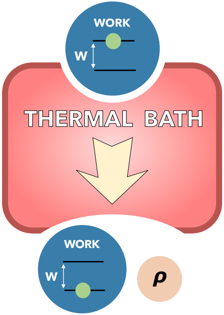
$$W_{\text{ext}}(\rho) = F_0(\rho) - F(\tau)$$

Battery



$$\leq$$

Work cost



$$\tau \otimes |W\rangle\langle W|_W \to \rho \otimes |0\rangle\langle 0|_W$$

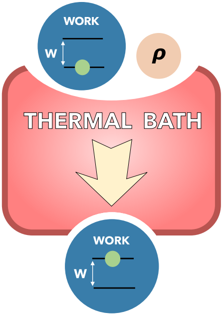
Work of formation

$$W_{\text{form}}(\rho) = F_\infty(\rho) - F(\tau)$$

Fundamental irreversibility

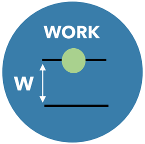
Deterministic work in the single-shot regime

Work extraction

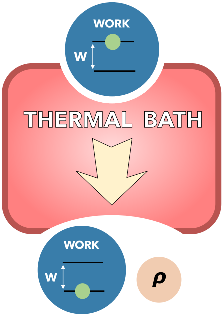


$$\rho \otimes |0\rangle\langle 0|_W \rightarrow \tau \otimes |W\rangle\langle W|_W$$

Battery



Work cost



$$\tau \otimes |W\rangle\langle W|_W \rightarrow \rho \otimes |0\rangle\langle 0|_W$$

ϵ-deterministic work extraction: $p > 1 - \epsilon$

$$W^{\epsilon}_{\text{form,extr}}(\rho) = F^{\epsilon}_{\infty,0} - F(\tau)$$

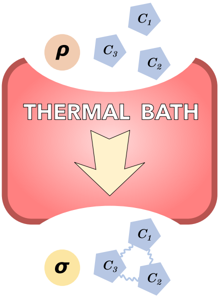
Thermodynamic limit: $\lim_{N\to\infty} \frac{1}{N} F^{\epsilon}_{0,\infty}(\rho^{\otimes N}) = F(\rho)$

Reversibility only achieved in the limit

F. Brandao, et al. PNAS 112, 3275 (2015)

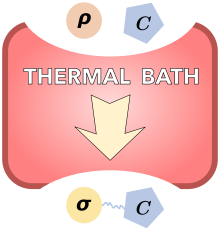
Correlations in the single-shot regime

Correlated catalysts



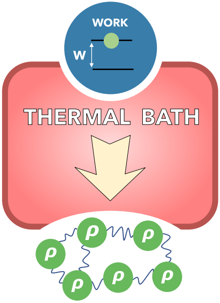
M. Lostaglio, M. P. Müller, M. Pastena, PRL 115, 150402 (2015)

Correlations with the catalyst



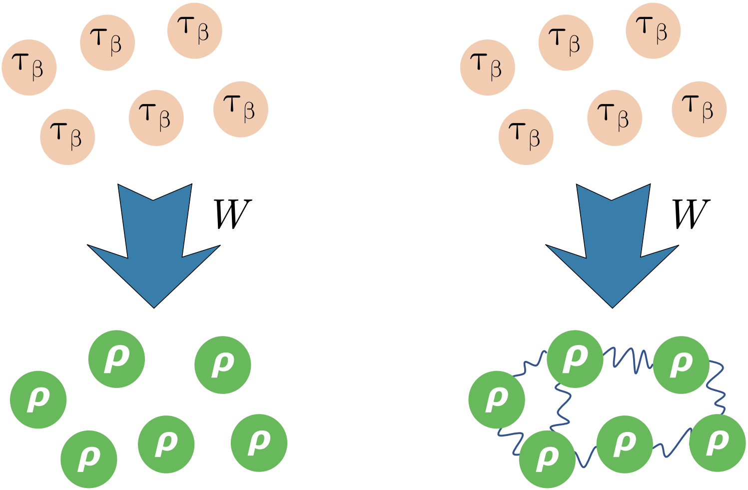
M. P. Müller, PRX 8, 041051 (2018)

Correlated subsystems



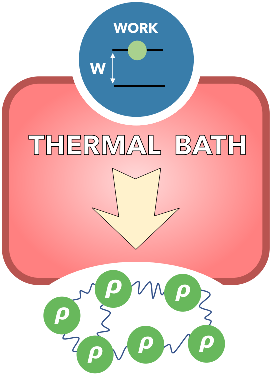
F. Sapienza, F. Cerisola, A. J. Roncaglia, Nat. Commun. 10, 2492 (2019)

Task: create N copies of state ρ through thermodynamic transformations



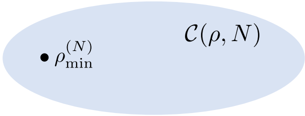
Can this provide an advantage?

Single-shot work of formation of correlated subsystems



$$\tau^{\otimes N} \otimes |W\rangle\langle W| \to \rho^{(N)} \otimes |0\rangle\langle 0|$$

$$\mathcal{C}(\rho, N) = \left\{ \rho^{(N)} \ : \ \mathrm{tr}_{-i}\left[\rho^{(N)}\right] = \rho, \quad \forall i = 1, \ldots, N \right\}$$



work of formation of correlated subsystems
(c-work of formation):

$$\mathcal{W}_{\mathrm{form}}(\rho, N) = \min_{\rho^{(N)} \in \mathcal{C}(\rho, N)} W_{\mathrm{form}}(\rho^{(N)})$$

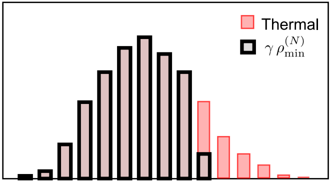
Solution for the N qubit case

$$\rho_{\min}^{(N)} = \sum_{E\in\mathcal{E}_N} \lambda_E \, |E\rangle\langle E|$$

$\mathcal{E}_N$ energy spectrum of the N qubits

$$\lambda_E = \begin{cases} \frac{1}{\gamma} e^{-\beta E} & \text{if } E \in \mathcal{E}_N^{\rho} \subset \mathcal{E}_N \\ s\frac{1}{\gamma} e^{-\beta \varepsilon} & \text{if } E = \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$$s \in [0, 1]$$



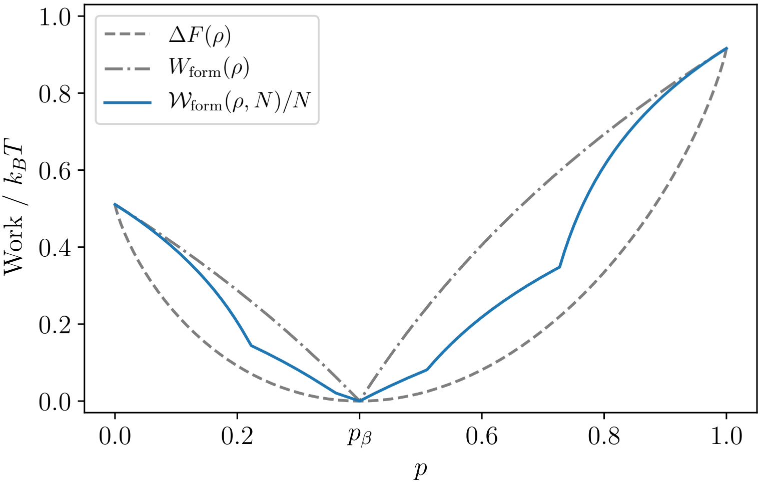
The work of formation and extractable work are

$$\mathcal{W}_{\text{form}}(\rho, N) = k_B T \log \frac{Z_S^N}{\gamma}$$

$$\mathcal{W}_{\text{ext}}(\rho, N) = k_B T \log \frac{Z_S^N}{Z}$$

Z is the partition function of a system in thermal equilibrium at temperature T with spectrum $\mathcal{E}_N^{\rho} \cup \{\varepsilon\}$.

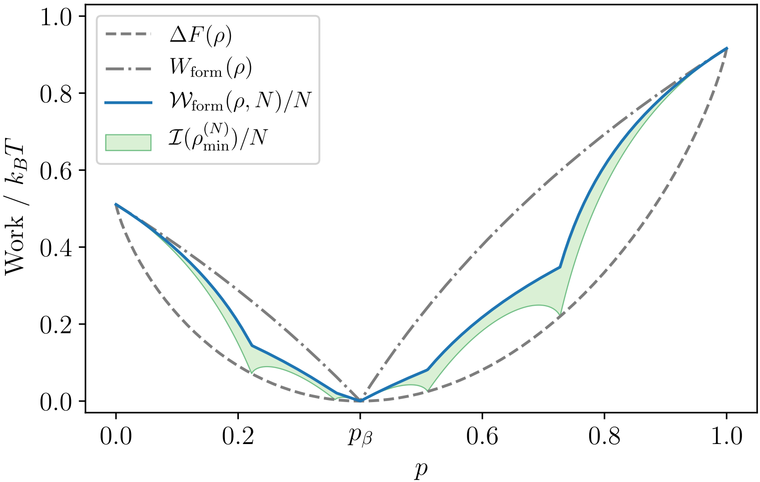
c-work of formation ($N = 3$ qubits)



$$\mathcal{W}_{\mathrm{form}}(\rho_{\mathrm{min}}^{(N)}) < N W_{\mathrm{form}}(\rho)$$

$$\rho = (1-p)\,|0\rangle\langle 0| + p\,|1\rangle\langle 1|$$

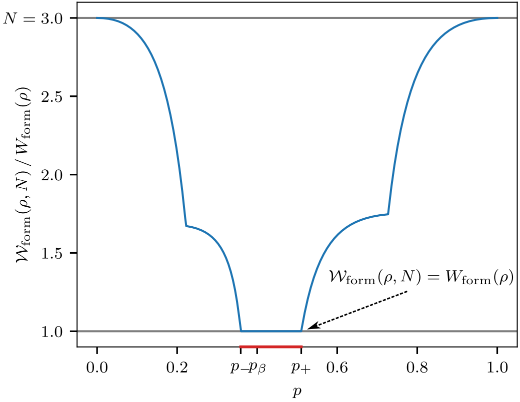
c-work of formation ($N = 3$ qubits)



Total correlations: $\mathcal{I}(\rho^{(N)}) = D\left(\rho^{(N)} \middle|\middle| \rho^{\otimes N}\right)$

$$\rho = (1-p)\,|0\rangle\langle 0| + p\,|1\rangle\langle 1|$$

c-work of formation

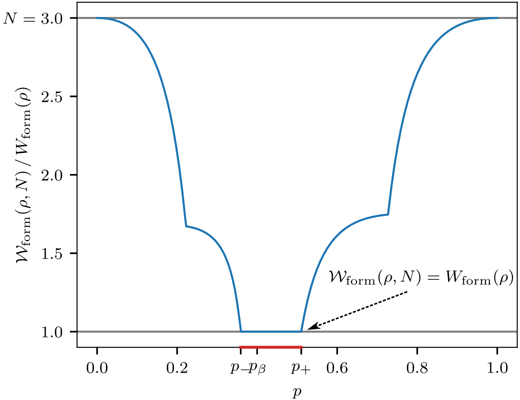


Region where

$$\mathcal{W}_{\text{form}}(\rho, N) = W_{\text{form}}(\rho)$$

N correlated copies have same cost as only one ρ

c-work of formation



Region where

$$\mathcal{W}_{\rm form}(\rho, N) = W_{\rm form}(\rho)$$

N correlated copies have same cost as only one ρ

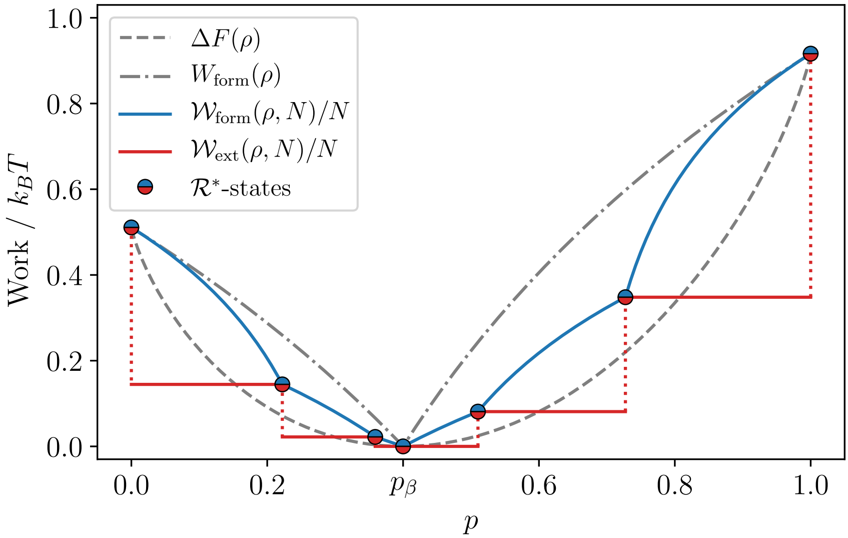
Do we always obtain an advantage with correlations?

No, correlations are **costly** if

$$\mathcal{I}(\rho^{(N)}) > N\beta(W_{\rm form}(\rho) - \Delta F(\rho)) \implies \mathcal{W}_{\rm form}(\rho, N) > N W_{\rm form}(\rho)$$

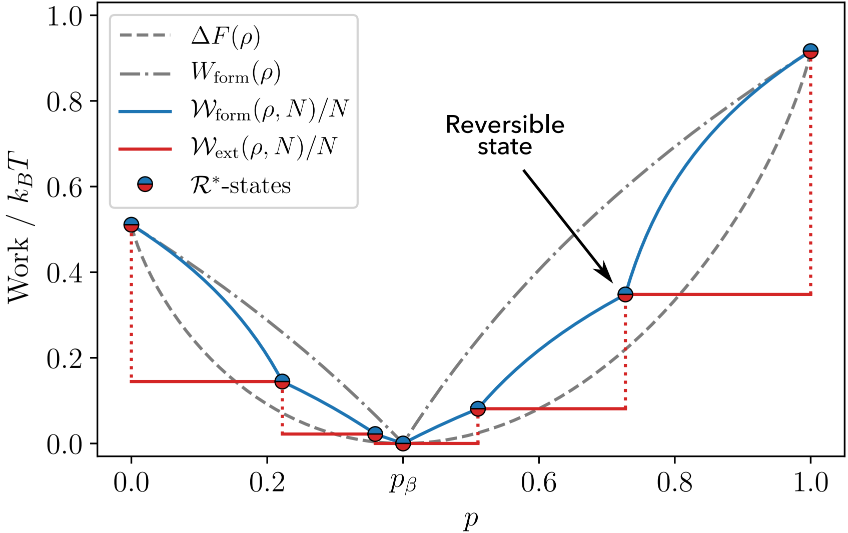
Bound on amount of useful correlations

Reversibility



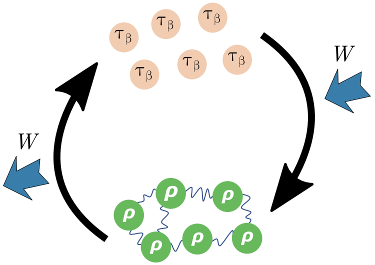
$\rho = (1-p)\,|0\rangle\langle 0| + p\,|1\rangle\langle 1|$

Reversibility



$\rho = (1-p)\,|0\rangle\langle 0| + p\,|1\rangle\langle 1|$

Reversibility



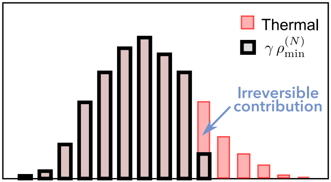
$$\mathcal{W}_{\text{form}} = \mathcal{W}_{\text{ext}}$$

which implies

$$F_\alpha = F_1 \quad \forall\alpha$$

only one free energy (standard one)

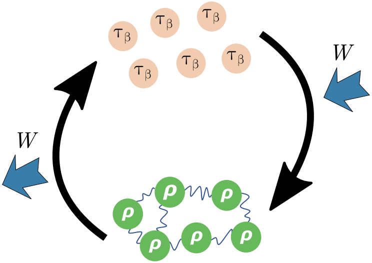
$$\lambda_E = \begin{cases} \frac{1}{\gamma} e^{-\beta E} & \text{if } E \in \mathcal{E}_N^\rho \subset \mathcal{E}_N \\ \frac{s}{\gamma} e^{-\beta E} & \text{if } E = \varepsilon \\ 0 & \text{otherwise} \end{cases}$$



Irreversible work: $\mathcal{W}_{\text{irr}} = \mathcal{W}_{\text{form}} - \mathcal{W}_{\text{ext}} = (1-s)g_N(\varepsilon)e^{-\beta\varepsilon}/Z$

Reversible states: $s = 1$ (thermal state over reduced support)

Reversible states



Key properties

1. $W_{\mathrm{form}} = W_{\mathrm{ext}}$
2. $F_\alpha = F_1 \quad \forall\alpha$

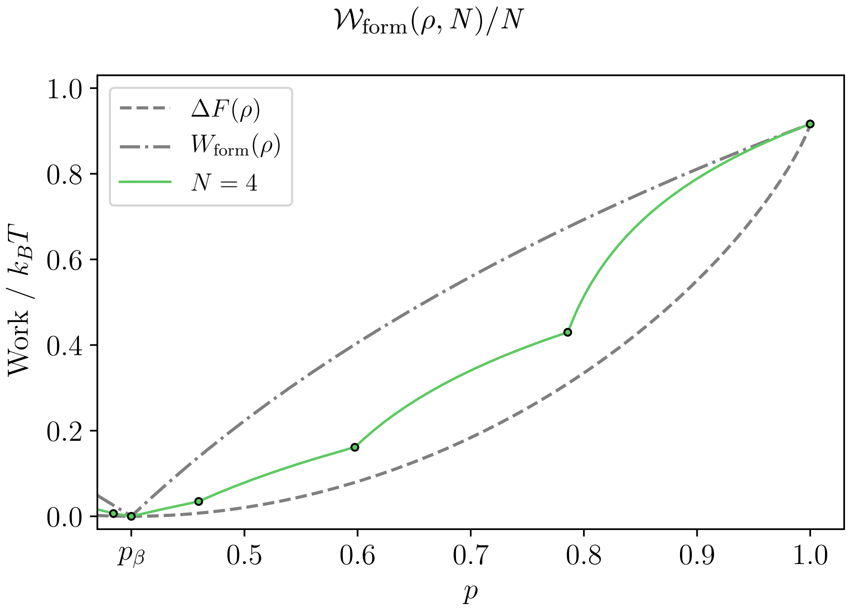
   only the standard free energy

3. Are a thermal state over a reduced support
4. Simple relation between work, free energy, and correlations:

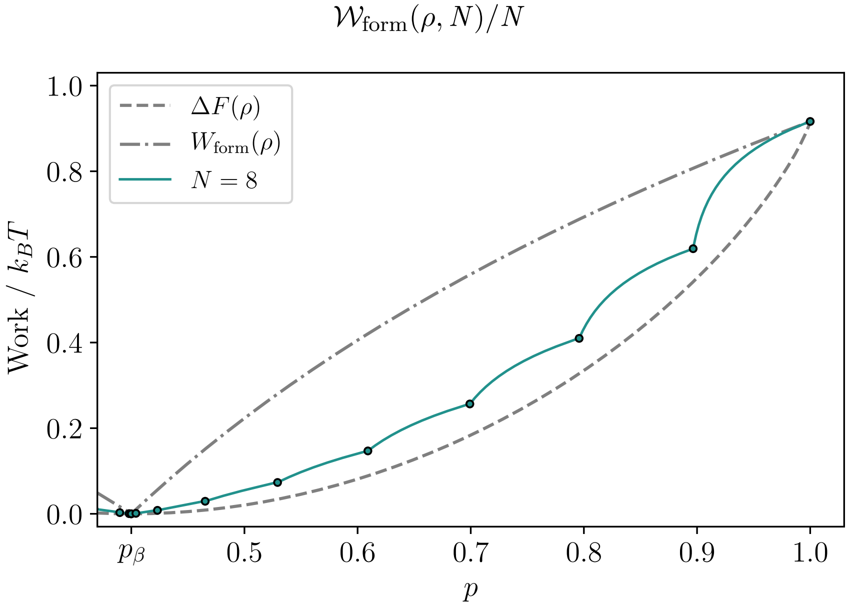
$$\mathcal{W}(\rho, N) = N\Delta F(\rho) + k_B T \mathcal{I}$$

5. Transformations between reversible states ruled only by free energy difference $\implies$ we can use this to define reversible heat engines with deterministic work extraction ... (*work in preparation*)
6. They are dense: $\epsilon > 0$, $N = \mathcal{O}(1/\epsilon)$, ρ^* rev., s.t. $\|\rho - \rho^*(\epsilon)\| < \epsilon$.

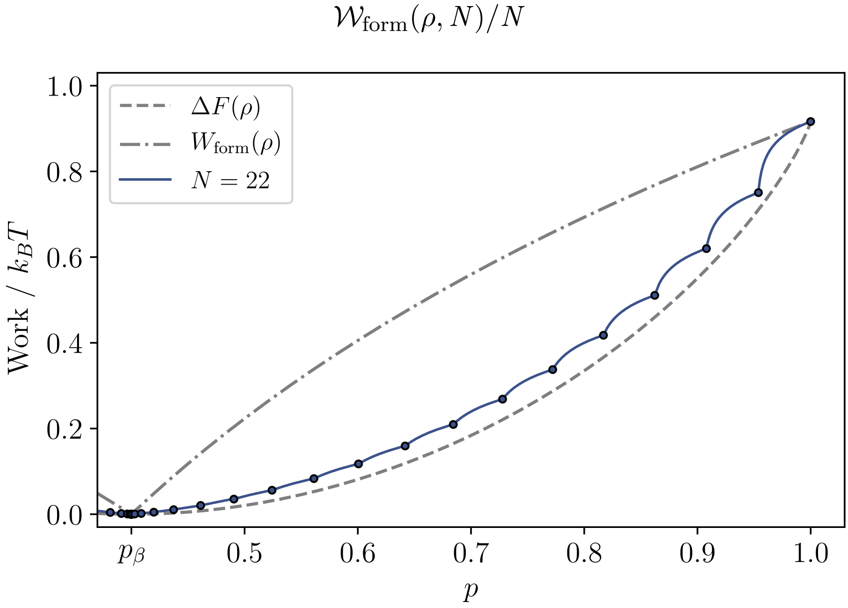
Density of reversible states & Thermodynamic limit



Density of reversible states & Thermodynamic limit

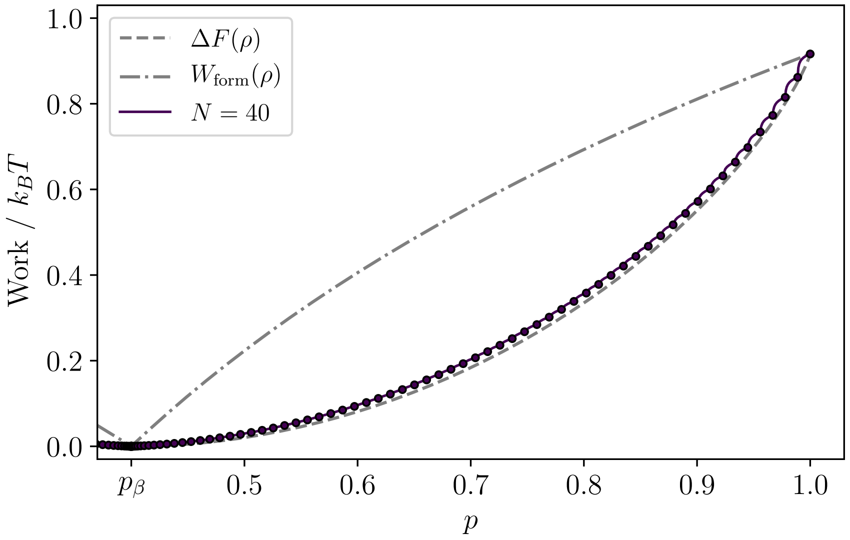


Density of reversible states & Thermodynamic limit



Density of reversible states & Thermodynamic limit

$\mathcal{W}_{\mathrm{form}}(\rho, N)/N$

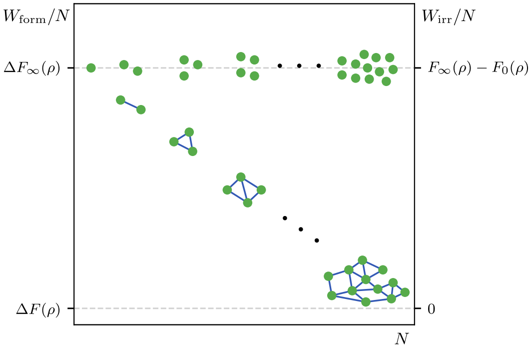


Thermodynamic limit

Let ρ be any local state of a system S. Then

$$\frac{\mathcal{W}(\rho, N)}{N} \xrightarrow{N\to\infty} \Delta F(\rho)$$

where $\mathcal{W}$ either $\mathcal{W}_{\text{form}}$ or $\mathcal{W}_{\text{ext}}$ of $\rho_{\min}^{(N)}$. Convergence $\mathcal{O}(\log N/N)$.



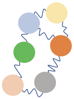
We recover standard results with total correlations

$$\mathcal{I}(\rho_{\min}^{(N)}) \sim \mathcal{O}(\log N)$$

correlations per particle vanish as $N \to \infty$

Generalizations

- Local states of arbitrary dimension D.
- Arbitrary different local states (i.e. not only copies).
- Generalization of the thermodynamic limit:



Let $(p^{(1)}, E^{(1)}), \dots, (p^{(N)}, E^{(N)}) \in \mathbb{R}^{2D}_{\geq 0}$ an i.i.d. sample with arbitrary distribution $\mathcal{D}$. Let $\mathcal{W}_N$ be the c-work of formation of a system with local states ρ_i defined by population $p^{(i)}$ and spectrum $E^{(i)}$. Then

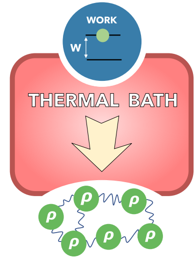
$$\frac{\mathcal{W}_N}{N} \xrightarrow{N\to\infty} \langle \Delta F \rangle_{\mathcal{D}}$$

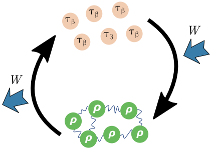
where

$$\langle \Delta F \rangle_{\mathcal{D}} = \int_{\Omega} \Delta F(p, E) f(p, E) \, \mathrm{d}p \, \mathrm{d}E$$

Summary [Nat. Commun. 10, 2492 (2019)]

- Work of formation of correlated systems in the single shot regime.
- Correlations can reduce the work cost in the creation of multipartite states with local constraints, but they are costly if they are bigger than a given upper bound.
- Reversibility appears naturally when the work of formation of correlated systems is studied.
- We can recover standard results in the thermodynamic limit with correlations per particle $\mathcal{O}(\log N/N)$.





Further research

- Can we extend these ideas to states with coherence?
- Reversible heat engine with deterministic single-shot work extraction.