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Bounding the resources for thermalizing many-body localized systems

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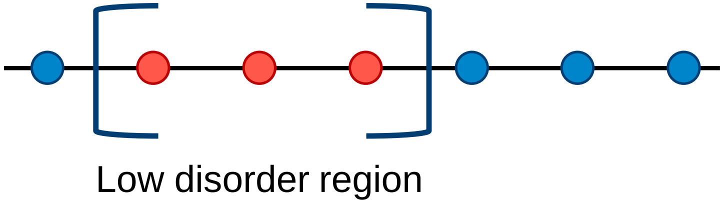
Motivation

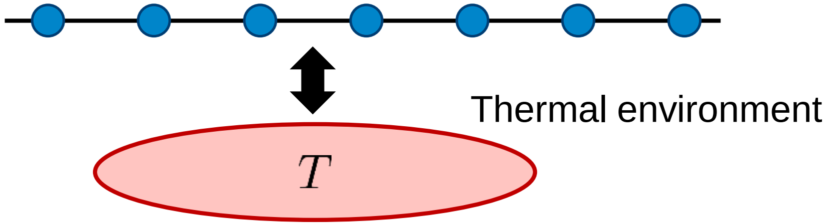
Thermalization is hindered in systems exhibiting **many-body localization** (MBL)

- How **robust** is the MBL phase? Can we get **quantitative bounds**?

Internal dynamics



External interactions



Main results: an overview

- Quantifying the robustness of MBL phase
  - Finding **upper and lower bounds** to size of external thermal reservoir
- Connecting quantum information tools to physical processes
  - Applying the **convex split lemma** to a **stochastic collision model**
- Deriving numerical results for 1D spin lattice

Index

- Many-body localization and thermalization
- The convex split lemma and stochastic collision models
- Numerical results for the disordered Heisenberg chain
- Conclusions and outlooks

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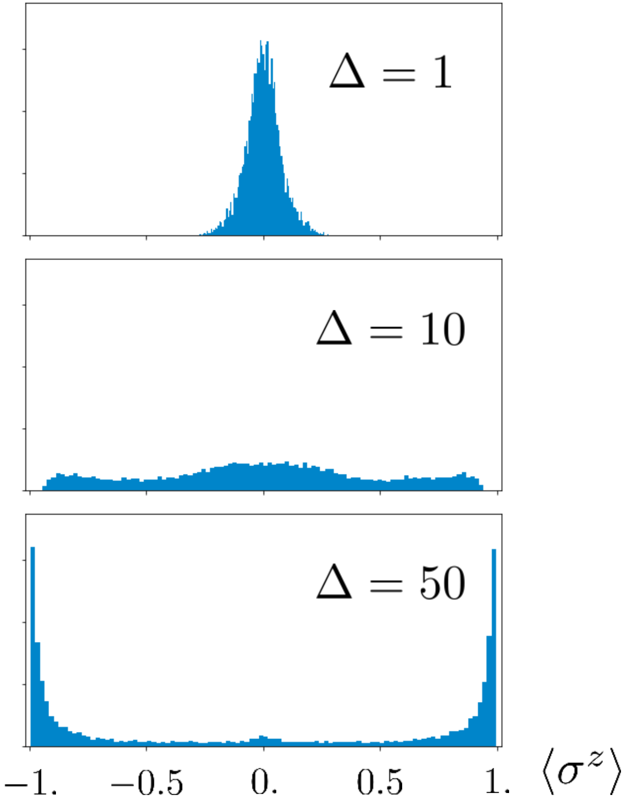
Many-body localization[1]

- Finite-temperature phase of matter exhibiting
  - **Absence** of thermalization
  - Quasi-local constant of motion
  - Entanglement **area law**
  - …
- **Example**, 1D spin chain of L sites with

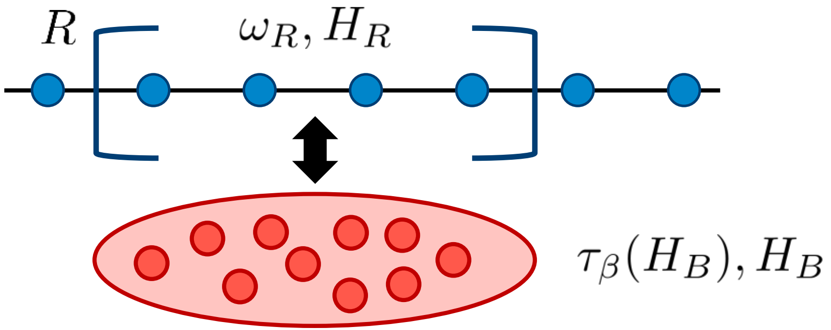
$$H = \sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + \Delta \sum_{i=1}^{L} h_i \, \sigma_i^z$$



[1] Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn, *Rev. Mod. Phys.* **91**, 021001 (2019)

Thermalization setting

Consider a finite sized region R of a spin lattice exhibiting MBL



Allowed operations

Unitaries U or mixture of unitaries s.t. $[U, H_R + H_B] = 0$

How big is the thermal bath we use to **ϵ-thermalize** the region R?

$$\left\| \mathcal{E}\left(\omega_R \otimes \tau_\beta\left(H_B\right)\right) - \tau_\beta\left(H_R\right) \otimes \tau_\beta\left(H_B\right) \right\|_1 \leq \epsilon$$

Index

Convex split lemma[2-3]

Consider a Hilbert space $\mathcal{H}$, the quantum states $\rho, \sigma \in \mathcal{S}(\mathcal{H})$, and the state

$$\tau_{S_1 S_2 \ldots S_n} = \sum_{i=1}^{n} \frac{1}{n} \sigma_{S_1} \otimes \ldots \otimes \sigma_{S_{i-1}} \otimes \rho_{S_i} \otimes \sigma_{S_{j+1}} \ldots \otimes \sigma_{S_n}$$

For any $\epsilon, \delta > 0$, if $n = \frac{1}{\epsilon^2} 2^{D^{\delta}_{\max}(\rho \| \sigma)}$ then,

$$\| \tau_{S_1 S_2 \ldots S_n} - \sigma_{S_1} \otimes \sigma_{S_2} \otimes \ldots \otimes \sigma_{S_n} \|_1 \leq \epsilon + \delta$$

- Quantum max-divergence : $D_{\max}(\rho \| \sigma) := \inf\{\lambda : \rho \leq 2^{\lambda} \sigma\}$
- Smoothed quantum max-divergence : $D^{\delta}_{\max}(\rho \| \sigma) := \sup_{\tilde{\rho} \in B_\delta(\rho)} D_{\max}(\tilde{\rho} \| \sigma)$
- δ-ball : $B_\delta(\rho) = \{\tilde{\rho} \,|\, \|\rho - \tilde{\rho}\|_1 \leq \delta\}$

[2] Anurag Anshu, Min-Hsiu Hsieh, Rahul Jain, *Phys. Rev. Lett.* **121**, 190504 (2018)
[3] Anurag Anshu, Vamsi Krishna Devabathini, Rahul Jain, *Phys. Rev. Lett.* **119**, 120506 (2017)

Convex split lemma and stochastic collision models

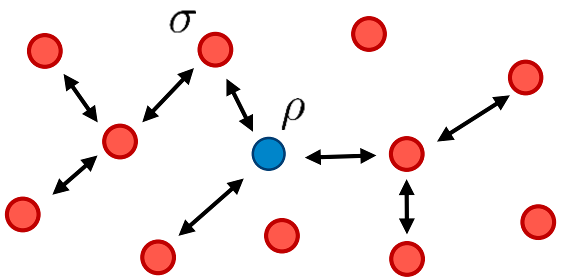
Consider the state $\tau_{S_1 S_2 \ldots S_n} = \sum_{i=1}^{n} \frac{1}{n} \sigma_{S_1} \otimes \ldots \otimes \sigma_{S_{i-1}} \otimes \rho_{S_i} \otimes \sigma_{S_{j+1}} \ldots \otimes \sigma_{S_n}$

- $\tau_{S_1 S_2 \ldots S_n}$ can be obtained from $\rho_{S_1} \otimes \left(\sigma^{\otimes n-1}\right)_{S_2 \ldots S_n}$ with the **Poisson process**

$$\frac{\partial \rho_{S_1 \ldots S_n}}{\partial t} = \frac{1}{T}\left(\sum_{i \leq j}^{n} \frac{2}{n(n+1)} U^{\text{swap}}_{S_i \leftrightarrow S_j} \rho_{S_1 \ldots S_n} U^{\text{swap}\,\dagger}_{S_i \leftrightarrow S_j} - \rho_{S_1 \ldots S_n}\right)$$

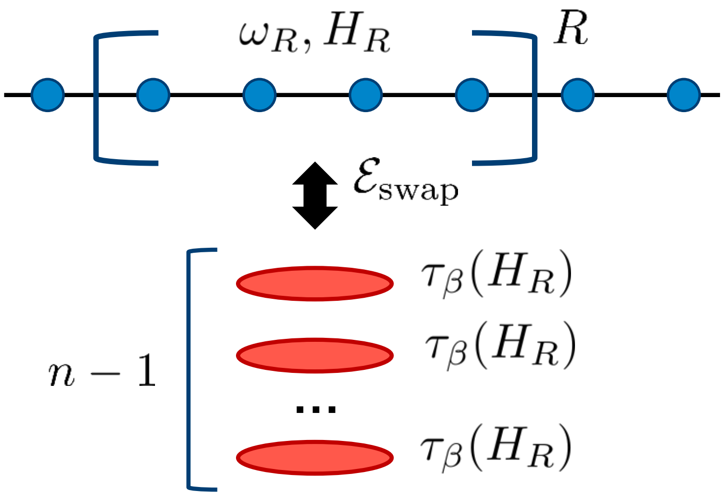
- The resulting channel is

$$\mathcal{E}_{\text{swap}}(\rho_{S_1 \ldots S_n}) = \sum_{i=1}^{n} \frac{1}{n} U^{\text{swap}}_{S_1 \leftrightarrow S_i} \rho_{S_1 \ldots S_n} U^{\text{swap}\,\dagger}_{S_1 \leftrightarrow S_i}$$



Thermalization with stochastic collision models

- Coming back to the region R of the MBL spin lattice



If the **size of the bath** is $n = \frac{1}{\epsilon^2} 2^{D_{\max}(\omega_R \| \tau_\beta(H_R))}$ then the map $\mathcal{E}_{\text{swap}}$ will ϵ-thermalizes the system,

$$\left\| \mathcal{E}_{\text{swap}} \left(\omega_R \otimes \tau_\beta \left(H_R \right)^{\otimes n-1} \right) - \tau_\beta \left(H_R \right)^{\otimes n} \right\|_1 \leq \epsilon$$

NOTE: Swaps are allowed as they preserve the energy

Converse bound on thermalization

General **stochastic collision model** $\mathcal{E}_{\mathrm{scm}}(\rho_{S_1\ldots S_n}) = \sum_{i=1}^n p_i U_{S_1 S_i} \rho_{S_1\ldots S_n} U_{S_1 S_i}^\dagger$

- $\{p_i\}_{i=1}^n$ probability distribution
- $U_{S_1 S_i}$ energy-preserving unitary

Consider the Hilbert space $\mathcal{H}$, the Hamiltonian H, and the states $\rho, \tau_\beta(H) \in \mathcal{S}(\mathcal{H})$.

If a map $\mathcal{E}_{\mathrm{scm}}$ can ϵ-thermalize the system

$$\left\| \mathcal{E}_{\mathrm{scm}}\left(\rho \otimes \tau_\beta(H)^{\otimes n-1}\right) - \tau_\beta(H)^{\otimes n} \right\|_1 \leq \epsilon$$

then the size of the bath $n \geq 2^{D_{\max}^{\delta}(\rho \| \tau_\beta(H))}$.

Index

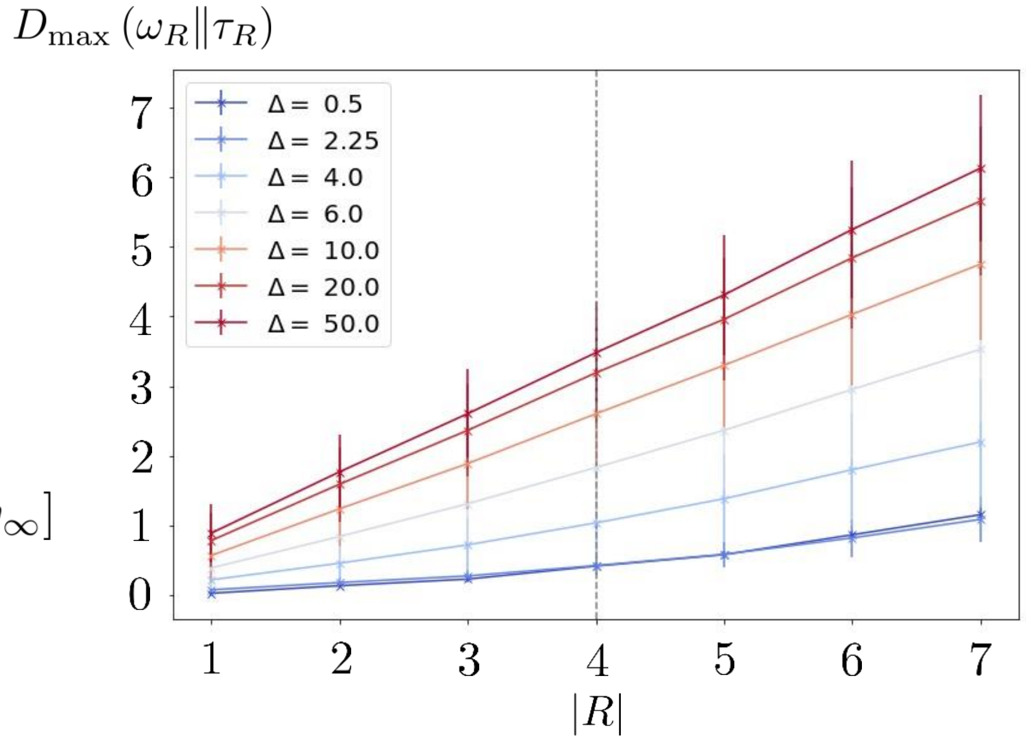
Numerical results

Lattice $L = 14$ with Hamiltonian

$$H = \sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + \Delta \sum_{i=1}^{L} h_i \, \sigma_i^z$$

Given sub-region R, we **compare**

- Infinite time-average $\omega_R = \mathrm{Tr}_{R^C}\left[\omega_\infty\right]$
- Gibbs state $\tau_R = \mathrm{Tr}_{R^C}\left[\tau_\beta(H)\right]$

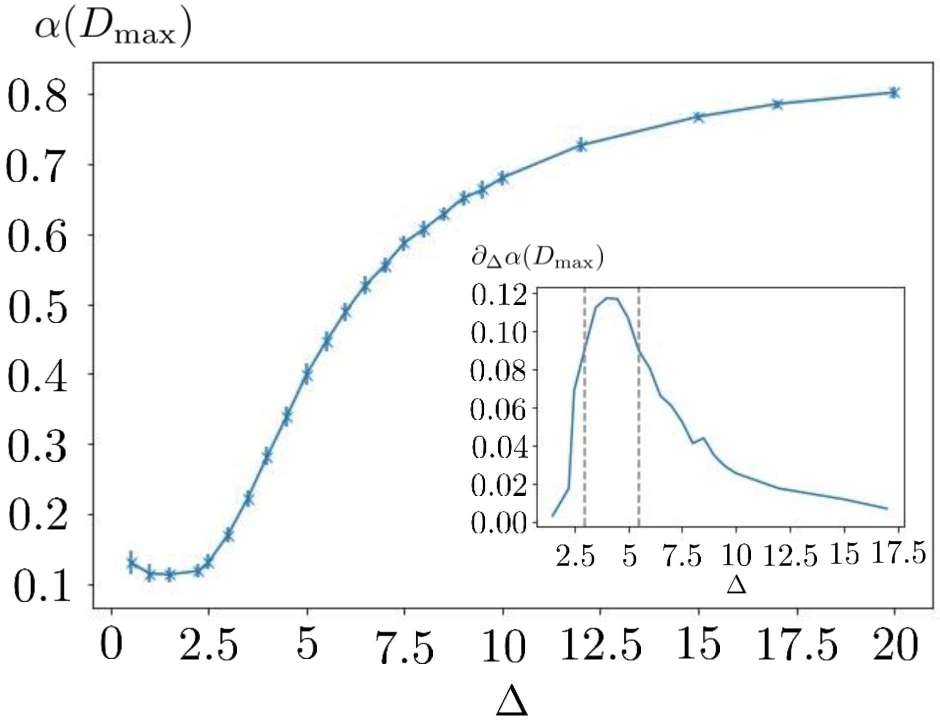


Numerical results

- $D_{\max}(\omega_R \| \tau_R)$ scales linearly in $|R|$
- The linear coefficient highlights the phase transition
- The **critical value** of the disorder is

$$\Delta_C^{L=14} \approx 4.5$$

in line with known results[4-5]



[4] David J. Luitz, Nicolas Laflorencie, Fabien Alet, *Phys. Rev. B* **91**, 081103 (2015)
[5] Johnnie Gray, Sougato Bose, Abolfazl Bayat, *Phys. Rev. B* **97**, 201105 (2018)

Index

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Conclusions

- Connection between **convex split lemma** and thermalization

$$\frac{\partial \rho_{S_1 \ldots S_n}}{\partial t} = \frac{1}{T}\left(\sum_{i \leq j}^{n} \frac{2}{n(n+1)} U^{\text{swap}}_{S_i \leftrightarrow S_j} \rho_{S_1 \ldots S_n} U^{\text{swap}\,\dagger}_{S_i \leftrightarrow S_j} - \rho_{S_1 \ldots S_n}\right)$$

- **Quantitative bounds** to the robustness of the MBL phase
  - For the models of interaction considered, the **size of the bath** n is

$$\log n \propto D_{\max}\left(\omega_R \| \tau_\beta(H_R)\right)$$

- Linear scaling of $D_{\max}$ for the disordered Heisenberg chain of $L = 14$ sites
  - Hinting at **robustness of the MBL phase**

Outlooks

- Adapting other quantum information tools to obtain better bounds
  - Results in **randomness extraction**, bounds on the seed
- Exploit the analytical feature of the states describing an MBL system
  - Entropic properties of **matrix product states** (MPS)
- **Extending** the class of models for which the lower bound applies