By-passing Fluctuation Theorems Using Catalysts

Paul Boes, Rodrigo Gallego, <u>Nelly Ng</u>, Jens Eisert, Henrik Wilming

FU Berlin E

ETH Zurich

arXiv:1904.01314

By-passing Jarzynski equality Using Catalysts

Paul Boes, Rodrigo Gallego, <u>Nelly Ng</u>, Jens Eisert, Henrik Wilming FU Berlin ETH Zurich

arXiv:1904.01314

Jarzynski equality

$$W := E_i - E_j$$

$$\omega_{\beta}(H) \rightarrow [E_{i}\rangle \rightarrow |E_{i}\rangle \rightarrow \mathcal{M}(|E_{i}\rangle) \rightarrow [D] \rightarrow |E_{j}\rangle$$

$$P(E_{i}) \qquad P(E_{j}|E_{i})_{\mathcal{M}}$$

• Jarzynski equality (JE)

$$\langle e^{\beta W} \rangle = e^{-\beta \Delta F}$$

$$\Delta F = \beta^{-1} (\ln Z_i - \ln Z_f)$$

For similar initial and final Hamiltonians,

$$\left\langle e^{\beta W} \right\rangle = 1$$

- + <u>Averaged second law</u> satisfied No average work extractable from Gibbs thermal state: $\langle W \rangle \leq 0$
- + Not only so, more stringent!

Different work distributions



All have average work $\langle W \rangle \leq 0$ (satisfies second law on average)

But Jarzynski equality only allows for distribution 3



No macroscopic work (NMW) principle

- Consider a macroscopic system with N particles, in an initially thermal state
 - Denote $\,p(w):={\rm prob}(wN)\,$ as the probability of extracting an amount of work $w\,$ per particle
 - NMW: probability of extracting any macroscopic work vanishes in the limit

$$\forall \epsilon > 0, \quad \lim_{N \to \infty} p(w \ge \epsilon) = 0$$

• Channels that satisfy JE also satisfy NMW

$$1 = \langle e^{\beta W} \rangle = \sum_{w} e^{\beta w N} \operatorname{prob}(wN) \ge e^{\beta \epsilon N} \sum_{w \ge \epsilon} p(w)$$

Jarzynski equality under different channels



Gibbs preserving maps

- Although "cheap" from thermodynamic perspective, they may violate JE with an arbitrarily large amount
 - Fully thermalizing map as example



NMW is a good way to distinguish work from heat with the TPM scenario

The role of catalysts in quantum thermo

Systems that are returned unchanged after a process



- Resource-theoretic setting: enable larger set of thermodynamic transformations
- Correlations between system and catalyst

Q: how does the presence of a catalyst change fluctuation relations?



Catalytic channels

Given a system S, the channel C is a β -catalytic channel iff

1. It has the following dilated form:

$$\mathcal{C}(\cdot) = \operatorname{tr}_C(U(\cdot \otimes \sigma_C)U^{\dagger})$$

2. When the channel input is $w_{\beta}(H)$, then the ancilla σ_C remains unchanged after the process

$$\operatorname{tr}_S(U(\omega_\beta(H)\otimes\sigma_C)U^\dagger)=\sigma_C$$

More about catalytic channels

- Ancilla C can be chosen to have trivial Hamiltonian (all energy is given by system S)
- Given any U, any $w_{\beta}(H)$, a catalyst σ_{C} always exists by Brouwer's fixed point theorem.

Can we bypass JE with catalysts?



Result 1a: catalytic channels violate JE

Low dimensional example: $d_s = 3$, $d_c = 2$

$$\begin{array}{c|c} \operatorname{eig}\left(w_{\beta}(H_{S})\right) = \{p_{1}, p_{2}, p_{3}\} \\ p_{1}' & p_{1} & p_{1}q_{1} & p_{1}q_{2} \\ p_{2}' & p_{2}q_{1} & p_{2}q_{2} \\ p_{3}' & p_{3}q_{1} & p_{3}q_{2} \\ s & c & q_{1} & q_{2} \\ \end{array}$$

$$\begin{array}{c|c} \operatorname{eig}\left(\sigma_{C}\right) = \{q_{1}, q_{2}\} \\ \end{array}$$

$$\begin{array}{c|c} \operatorname{System} \\ E_{1} = E_{2} = 0, \ E_{3} = \Delta \\ p_{1} = p_{2} = \frac{1}{Z}, \\ p_{3} = \frac{Z-2}{Z}, \\ Z = 2 + \exp(-\beta\Delta) \\ Z \leq Z \leq 3 \text{ for } \Delta > 0 \\ \end{array}$$

$$\begin{array}{c|c} \operatorname{Direct\ calculation:} \\ \langle e^{\beta W} \rangle = \frac{Z + 5 + 2(Z-2)(Z-1)}{Z(Z+1)} \geq 1 \\ \end{array}$$

$$\begin{array}{c|c} \operatorname{But}\left\langle W \right\rangle \leq 0 \\ \end{array}$$

Consider system S with N particles.

Let $I_e \in R$ be an energy window around energy density e $I_e = [e - O(\sqrt{N}), e]$

The microcanonical state in this energy window:

$$\Omega_S(I_e) = \frac{1}{g(I_e)} \sum_{E_i \in I_e} |E_i\rangle \langle E_i|$$

Instead of β -catalytic channels, we consider a channel C_{micro} that is catalytic for the microcanonical ensemble:

$$\begin{aligned} \mathcal{C}_{\text{micro}} &= \text{tr}_C(U(\cdot \otimes \sigma_C)U^{\dagger}), \\ \text{s.t. } \text{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^{\dagger}) = \sigma_C \end{aligned}$$

Consider system S with N particles.

Let $I_e \in R$ be an energy window around energy density e



Instead of β -catalytic channels, we consider a channel C_{micro} that is catalytic for the microcanonical ensemble:

 $\mathcal{C}_{\text{micro}} = \text{tr}_C(U(\cdot \otimes \sigma_C)U^{\dagger}),$ s.t. $\text{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^{\dagger}) = \sigma_C$

Consider system S with N particles.

Let $I_e \in R$ be an energy window around energy density e







$$\sigma_{S}^{\text{initial}} = \rho_{\text{micro}}(I_{e}) = \Omega_{S}(I_{e})$$
$$\sigma_{C} = \frac{1}{2(d_{C}-1)} \sum_{i=1}^{d_{C}-1} |i\rangle\langle i|$$
$$+ \frac{1}{2} |d_{C}\rangle\langle d_{C}|$$

$$E
\begin{bmatrix}
I_{+} & g^{2} \\
I_{+} & I_{+} & I_{+} & I_{+} \\
I_{+} & I_{+} & I_{+} \\
I_{+} & I_{+} &$$

$$\rho_{SC}^{\text{final}} = \frac{1}{2} \rho_{\text{micro}}(I+) \otimes |d_C\rangle \langle d_C|$$
$$= \frac{1}{2(d_C-1)} |E_-\rangle \langle E_-| \otimes \sum_{i=1}^{d_C-1} |i\rangle \langle i|$$

$$p(w \ge e - e_-) = \frac{1}{2}$$



- Assumptions hold whenever one has exponential density of states $g(E) \propto e^{aE}$
- Violation of NMW implies violation of JE

$$p(w \ge e - e_-) = \frac{1}{2}$$

Result 1c: catalytic channels violate NMW for initial thermal states

Let $(S^{(N)})_N$ be a sequence of *N*-particle locally interacting lattice systems with Hamiltonian H(N) satisfying mild assumptions. Consider the thermal state $\omega_{\beta}(H_{S^{(N)}})$ for some $\beta > 0$.

Then, for sufficiently large N, there exist values of $\epsilon > 0$, such that one can achieve macroscopic work with non-vanishing probability, in particular

$$p(w \ge \epsilon) \approx \frac{1}{2}$$

How far can we bypass JE with catalysts?



Result 2: Bound on possible JE violation for catalytic channels

Let C be any β -catalytic channel with $d_C = \dim(H_C)$, then $\langle e^{\beta W} \rangle \leq \min\{d_C \cdot \|\sigma\|_{\infty}, d_S \cdot \|\omega_{\beta}(H)\|_{\infty}\}$ $\leq \min\{d_C, d_S\}$

Application: multi-player work extraction



System 1,2,3...n identical : N particles

Application: multi-player work extraction



There exists $\epsilon > 0$ such that

$$p(\epsilon, -\epsilon, \epsilon, -\epsilon, ...) = \lambda;$$

$$p(-\epsilon, \epsilon, -\epsilon, \epsilon, ...) = 1 - \lambda;$$

where λ can be made arbitrarily close to $\frac{1}{2}$.

Catalyst is fixed, does not depend on n

Same strategy across players

Summary

- Violation of JE (in operationally meaningful way) using a catalyst
- Thermodynamic advantage of correlations
- Practical settings of catalytic usefulness?

Average Second Law

- catalytic channels

Jarzynski Equality

Unital channels

- Gibbs preserving maps

No Macroscopic Work