By-passing Fluctuation Theorems Using Catalysts

Paul Boes, Rodrigo Gallego, Nelly Ng, Jens Eisert, Henrik Wilming

FU Berlin ETH Zurich

arXiv:1904.01314

By-passing Jarzynski equality Using Catalysts

Paul Boes, Rodrigo Gallego, Nelly Ng, Jens Eisert, Henrik Wilming

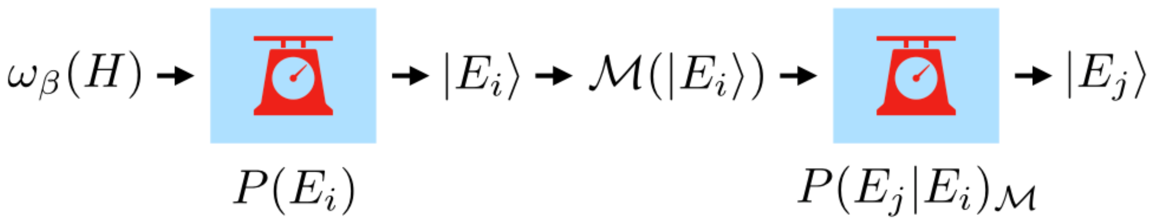
FU Berlin ETH Zurich

arXiv:1904.01314

Jarzynski equality

- Two point measurement scheme

$$W := E_i - E_j$$



- Jarzynski equality (JE)

$$\langle e^{\beta W} \rangle = e^{-\beta \Delta F}$$

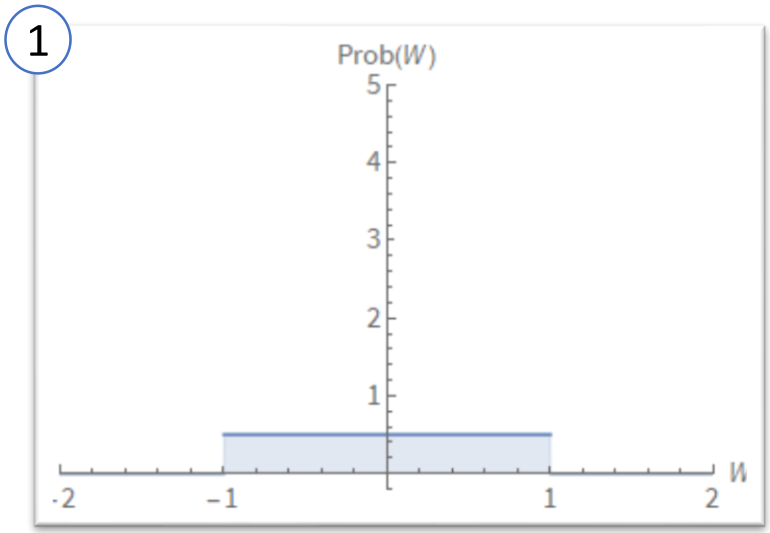
$$\Delta F = \beta^{-1}(\ln Z_i - \ln Z_f)$$

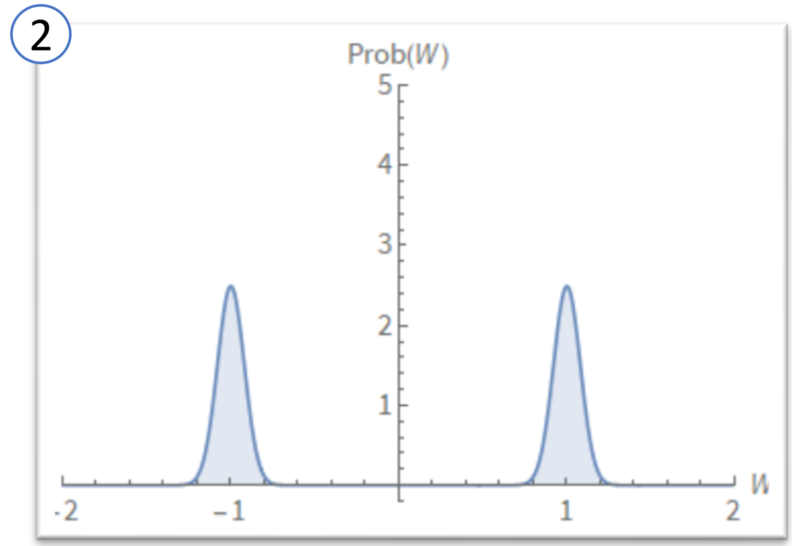
For similar initial and final Hamiltonians,

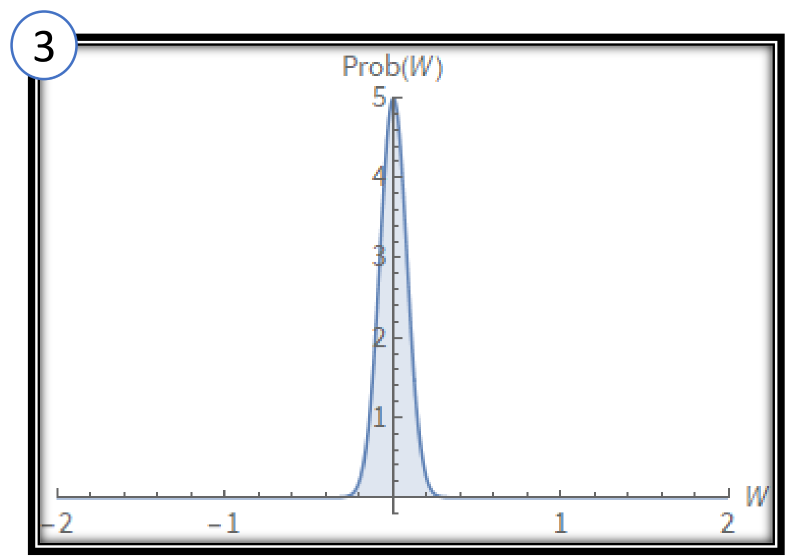
$$\langle e^{\beta W} \rangle = 1$$

+ Averaged second law satisfied
  No average work extractable from Gibbs thermal state: $\langle W \rangle \leq 0$
+ Not only so, more stringent!

Different work distributions







All have average work $\langle W \rangle \leq 0$
(satisfies second law on average)

But Jarzynski equality only allows
for distribution 3

No macroscopic work (NMW) principle

- Consider a macroscopic system with N particles, in an initially thermal state
  - Denote $p(w) := \mathrm{prob}(wN)$ as the probability of extracting an amount of work w per particle
  - NMW: probability of extracting any macroscopic work vanishes in the limit

$$\forall \epsilon > 0, \quad \lim_{N\to\infty} p(w \geq \epsilon) = 0$$

- Channels that satisfy JE also satisfy NMW

$$1 = \langle e^{\beta W} \rangle = \sum_{w} e^{\beta w N} \mathrm{prob}(wN) \geq e^{\beta \epsilon N} \sum_{w \geq \epsilon} p(w)$$

Jarzynski equality under different channels

$$\omega_\beta(H) \rightarrow [\text{measure}] \rightarrow |E_i\rangle \rightarrow \mathcal{M}(|E_i\rangle) \rightarrow [\text{measure}] \rightarrow |E_j\rangle$$

$P(E_i)$ $\qquad$ $P(E_j|E_i)_{\mathcal{M}}$

Does $\langle e^{\beta W} \rangle = 1$ hold?

Satisfied

- Arbitrary unitaries
- All unital channels

Violated

- Fully thermalizing map

$$\forall \rho, \quad \mathcal{M}_\beta(\rho) = w_\beta(H)$$

Still obeys NMW

Gibbs preserving maps

- Although “cheap” from thermodynamic perspective, they may violate JE with an arbitrarily large amount
  - Fully thermalizing map as example
- Result 0: GP maps never violate NMW

Assumption

Thermal state has weight approaching 1 within a narrow energy density window

- NMW is a good way to distinguish work from heat with the TPM scenario

The role of catalysts in quantum thermo

- Systems that are returned unchanged after a process

- Resource-theoretic setting: enable larger set of thermodynamic transformations

- Correlations between system and catalyst

Q: how does the presence of a catalyst change fluctuation relations?

Catalytic channels

Given a system S, the channel C is a β-catalytic channel iff

1. It has the following dilated form:

$$\mathcal{C}(\cdot) = \mathrm{tr}_C(U(\cdot \otimes \sigma_C)U^\dagger)$$

2. When the channel input is $w_\beta(H)$, then the ancilla σ_C remains unchanged after the process

$$\mathrm{tr}_S(U(\omega_\beta(H) \otimes \sigma_C)U^\dagger) = \sigma_C$$

More about catalytic channels

- Ancilla C can be chosen to have trivial Hamiltonian (all energy is given by system S)
- Given any U, any $w_\beta(H)$, a catalyst σ_C always exists by Brouwer's fixed point theorem.

Can we bypass JE with catalysts?

Result 1a: catalytic channels violate JE

Low dimensional example: $d_S = 3$, $d_C = 2$

$$\text{eig}\left(w_\beta(H_S)\right) = \{p_1, p_2, p_3\}$$

p'_1	p_1	$p_1 q_1$	$p_1 q_2$
p'_2	p_2	$p_2 q_1$	$p_2 q_2$
p'_3	p_3	$p_3 q_1$	$p_3 q_2$
	S / C	q_1	q_2

$$\text{eig}\left(\sigma_C\right) = \{q_1, q_2\}$$

System

$$E_1 = E_2 = 0,\ \ E_3 = \Delta$$

$$p_1 = p_2 = \frac{1}{Z},$$

$$p_3 = \frac{Z-2}{Z},$$

$$Z = 2 + \exp(-\beta\Delta)$$

$2 \leq Z \leq 3$ for $\Delta > 0$

Direct calculation:

$$\langle e^{\beta W} \rangle = \frac{Z + 5 + 2(Z-2)(Z-1)}{Z(Z+1)} \geq 1 \quad \text{when } Z < 3$$

But $\langle W \rangle \leq 0$

Result 1b: catalytic channels violate NMW for microcanonical initial state

Consider system S with N particles.

Let $I_e \in R$ be an energy window around energy density e

$$I_e = [e - O(\sqrt{N}), e]$$

The microcanonical state in this energy window:

$$\Omega_S(I_e) = \frac{1}{g(I_e)} \sum_{E_i \in I_e} |E_i\rangle\langle E_i|$$

Instead of β-catalytic channels, we consider a channel $\mathcal{C}_{\mathrm{micro}}$ that is catalytic for the microcanonical ensemble:

$$\mathcal{C}_{\mathrm{micro}} = \mathrm{tr}_C(U(\cdot \otimes \sigma_C)U^\dagger),$$

$$\text{s.t. } \mathrm{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^\dagger) = \sigma_C$$

Result 1b: catalytic channels violate NMW for microcanonical initial state

Consider system S with N particles.

Let $I_e \in R$ be an energy window around energy density e

$$I_e = [e - O(\sqrt{N}), e]$$

T

How can we achieve a non-zero probability of extracting macroscopic work?

Instead of β-catalytic channels, we consider a channel $\mathcal{C}_{\rm micro}$ that is catalytic for the microcanonical ensemble:

$$\mathcal{C}_{\rm micro} = \mathrm{tr}_C(U(\cdot \otimes \sigma_C)U^\dagger),$$

$$\text{s.t. } \mathrm{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^\dagger) = \sigma_C$$

Result 1b: catalytic channels violate NMW for microcanonical initial state

Consider system S with N particles.

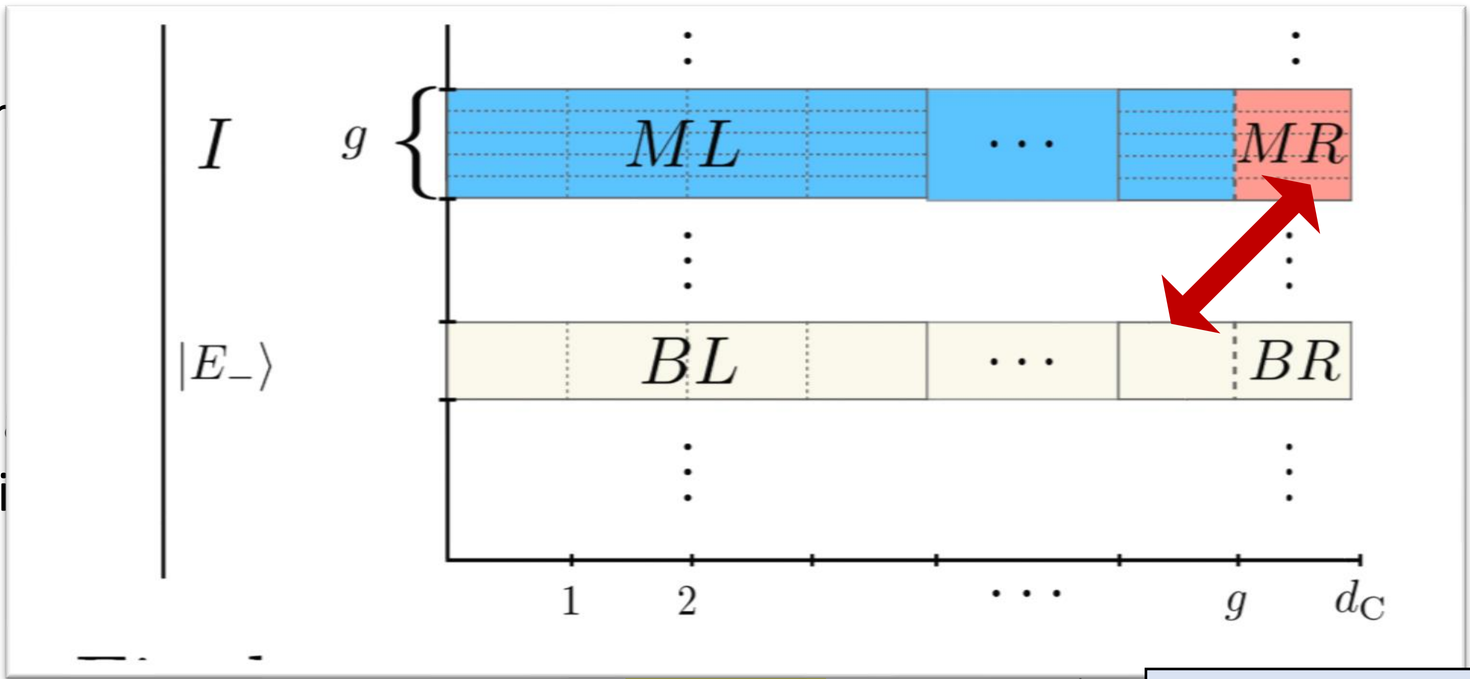
Let $I_e \in R$ be an energy window around energy density e

$$I_e = [e - O(\sqrt{N}), e]$$

The r

Inste

that i



s.t. $\mathrm{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^\dagger)$

$$d_C = g(I_e) + 1$$

Result 1b: catalytic channels violate NMW for microcanonical initial state

Consider system S with N particles

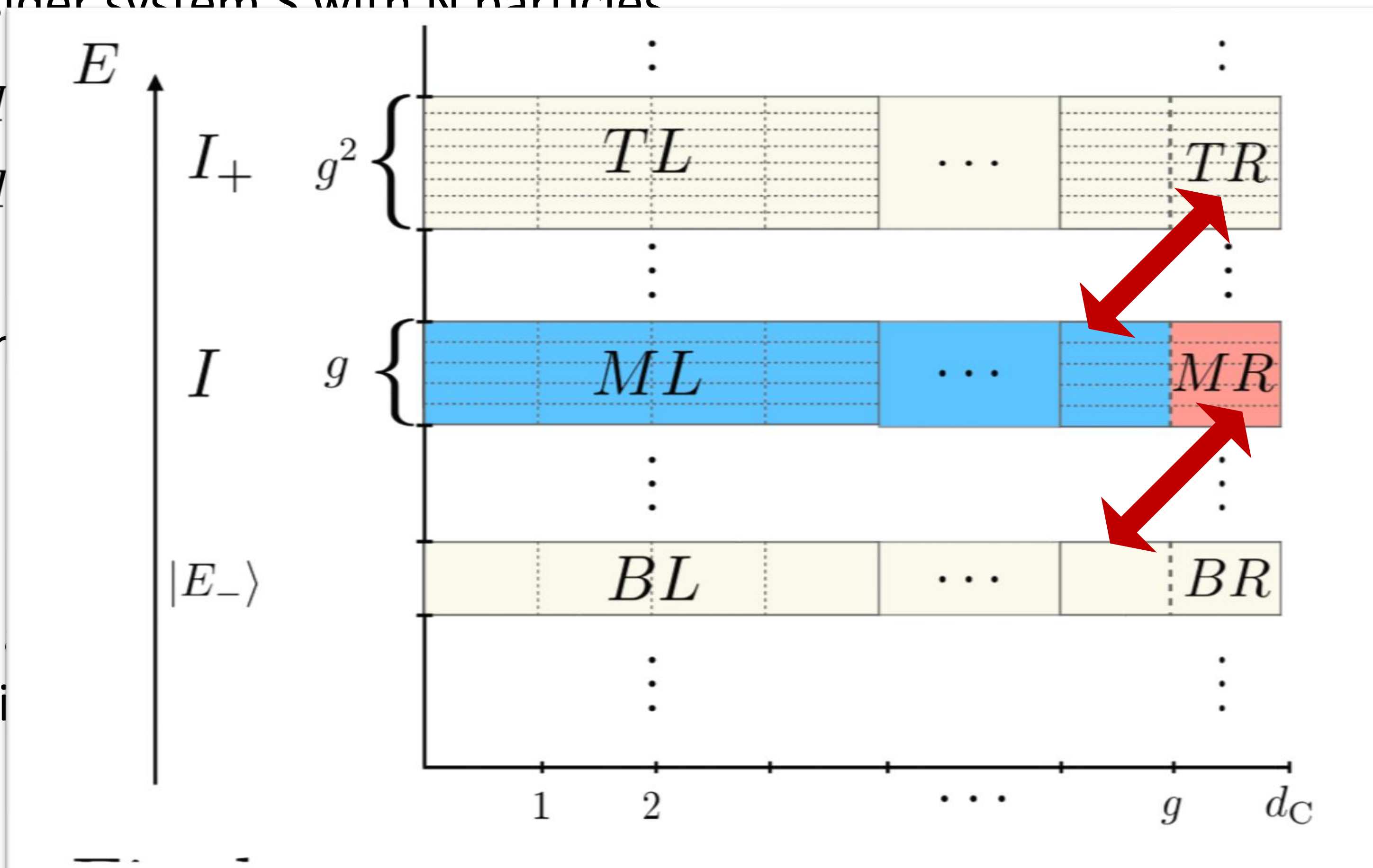
Let I

I

The r

Inste

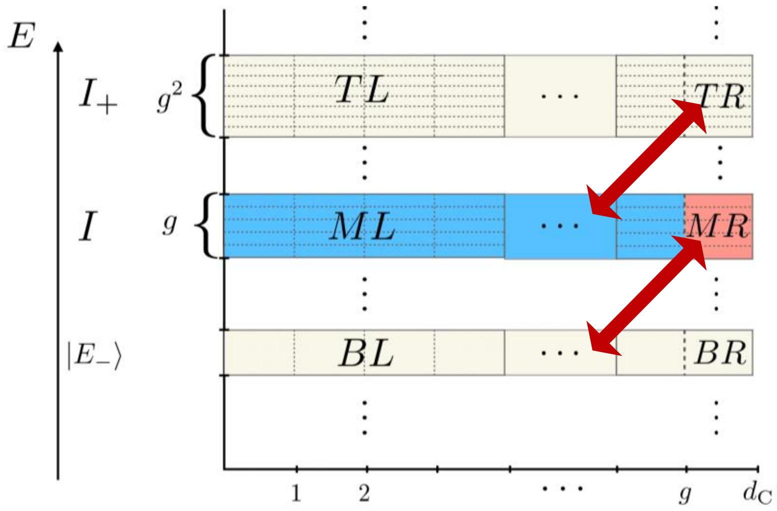
that i

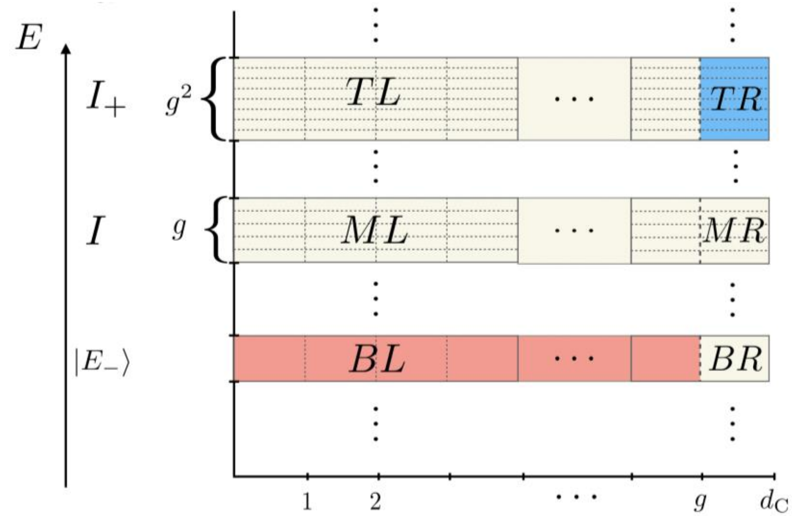


$$\text{s.t. } \mathrm{tr}_S(U(\Omega_S(I_e) \otimes \sigma_C)U^\dagger)$$

$$d_C = g(I_e) + 1$$

Result 1b: catalytic channels violate NMW for microcanonical initial state





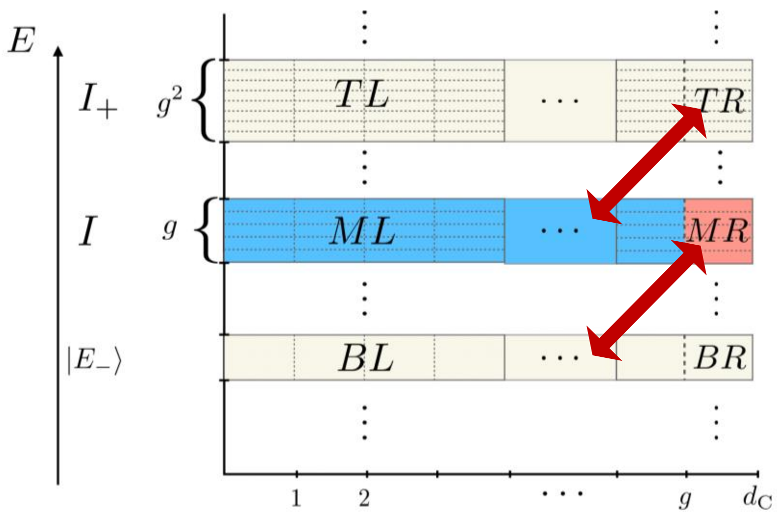
$$\rho_S^{\text{initial}} = \rho_{\text{micro}}(I_e) = \Omega_S(I_e)$$

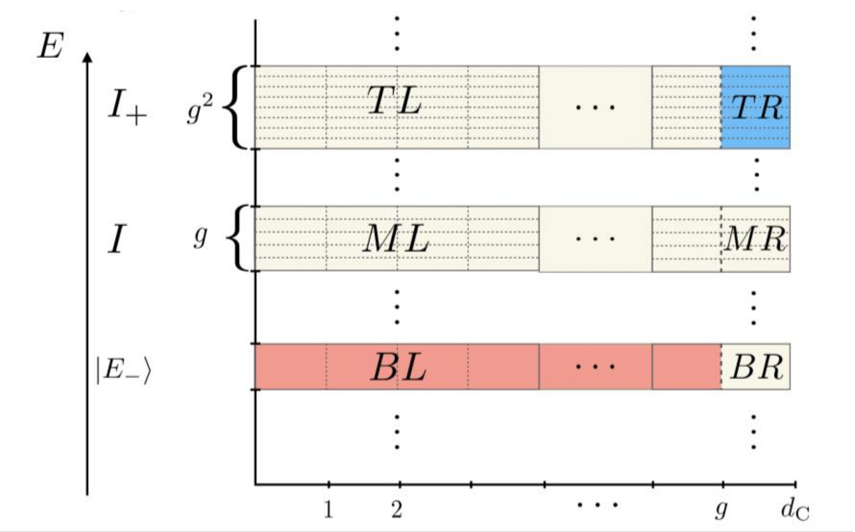
$$\sigma_C = \frac{1}{2(d_C-1)} \sum_{i=1}^{d_C-1} |i\rangle\langle i| + \frac{1}{2}|d_C\rangle\langle d_C|$$

$$\rho_{SC}^{\text{final}} = \frac{1}{2}\rho_{\text{micro}}(I+) \otimes |d_C\rangle\langle d_C| = \frac{1}{2(d_C-1)}|E_-\rangle\langle E_-| \otimes \sum_{i=1}^{d_C-1} |i\rangle\langle i|$$

$$p(w \geq e - e_-) = \tfrac{1}{2}$$

Result 1b: catalytic channels violate NMW for microcanonical initial state





- Assumptions hold whenever one has exponential density of states $g(E) \propto e^{aE}$
- Violation of NMW implies violation of JE

$$p(w \geq e - e_-) = \frac{1}{2}$$

Result 1c: catalytic channels violate NMW for initial thermal states

Let $(S^{(N)})_N$ be a sequence of N-particle locally interacting lattice systems with Hamiltonian $H(N)$ satisfying mild assumptions. Consider the thermal state $\omega_\beta(H_{S^{(N)}})$ for some $\beta > 0$.

Then, for sufficiently large N, there exist values of $\epsilon > 0$, such that one can achieve macroscopic work with non-vanishing probability, in particular

$$p(w \geq \epsilon) \approx \frac{1}{2}$$

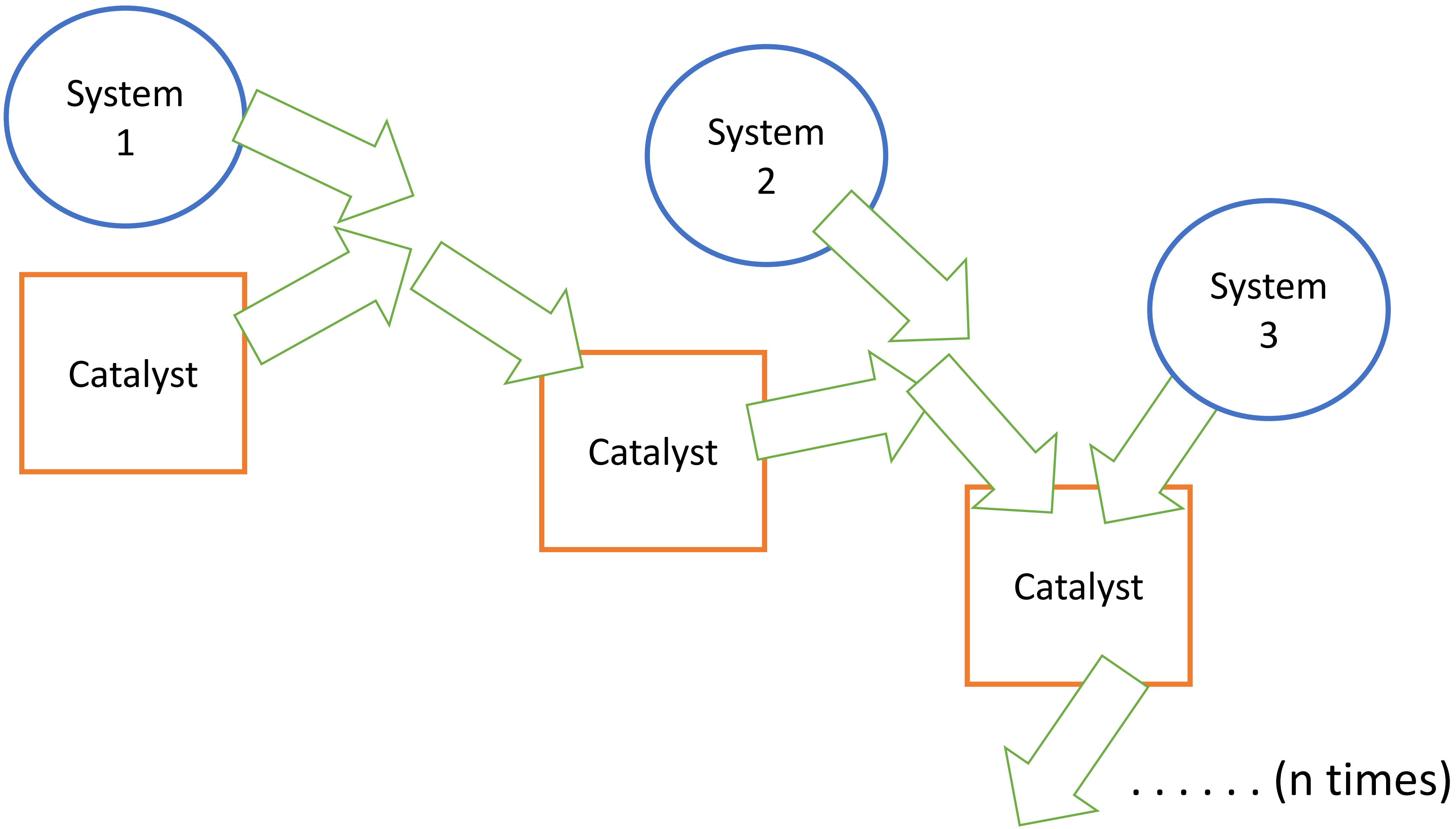
How far can we bypass JE with catalysts?

Result 2: Bound on possible JE violation for catalytic channels

Let C be any β-catalytic channel with $d_C = \dim(H_C)$, then

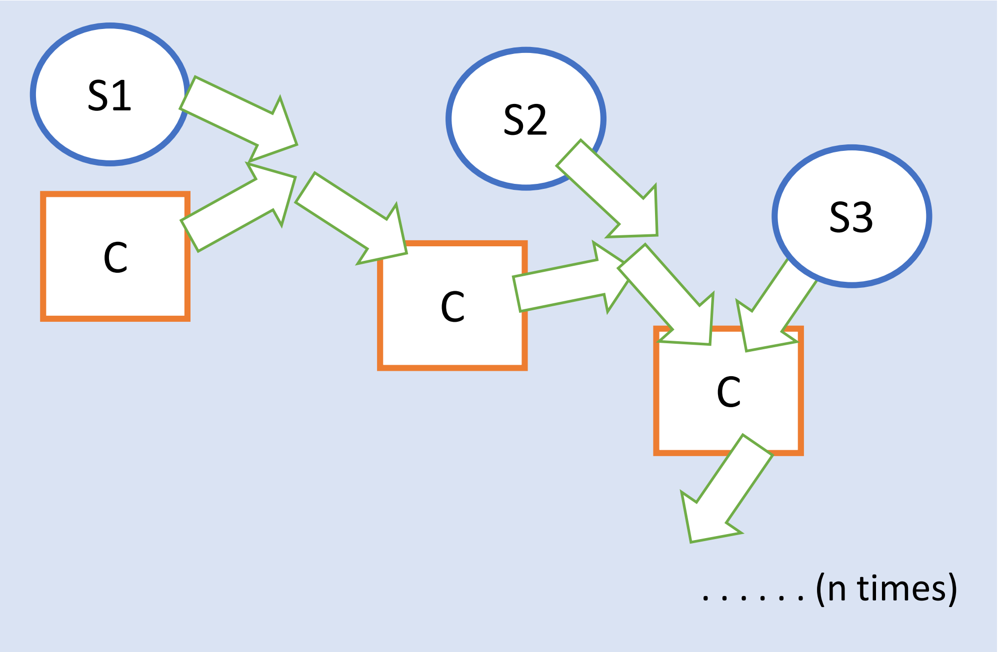
$$\begin{aligned}\langle \mathrm{e}^{\beta W} \rangle &\leq \min\{d_C \cdot \|\sigma\|_\infty, d_S \cdot \|\omega_\beta(H)\|_\infty\} \\ &\leq \min\{d_C, d_S\}\end{aligned}$$

Application: multi-player work extraction



System 1,2,3…n identical : N particles

Application: multi-player work extraction



There exists $\epsilon > 0$ such that

$$p(\epsilon, -\epsilon, \epsilon, -\epsilon, \ldots) = \lambda;$$
$$p(-\epsilon, \epsilon, -\epsilon, \epsilon, \ldots) = 1 - \lambda;$$

where λ can be made arbitrarily close to $\frac{1}{2}$.

Catalyst is fixed,
does not depend on n

Same strategy across players

Summary

- Violation of JE (in operationally meaningful way) using a catalyst
- Thermodynamic advantage of correlations
- Practical settings of catalytic usefulness?