An autonomous quantum machine to measure the thermodynamic arrow of time

Juliette Monsel, Cyril Elouard and Alexia Auffèves

npj Quantum Information **4**, 59 (2018)

Espoo – June 24, 2019

Bourse de thèse
"Jean-Pierre Aguilar"
FONDATION CFM POUR LA RECHERCHE

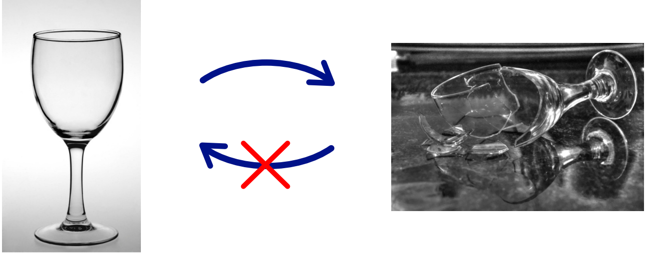
1. Reminders
   - Thermodynamic arrow of time
   - Optomechanical platform
2. Proposal of direct measurement of the arrow of time
3. Direct measurement of Jarzynski's equality
4. Relaxation toward equilibrium

Classical thermodynamics

Applied side: work extraction

Classical thermodynamics

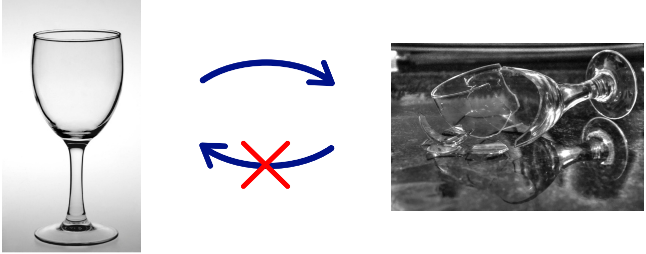
Fundamental side: irreversibility



Entropy production $\Delta_\mathrm{i} S$

Second law of thermodynamics: $\Delta_\mathrm{i} S \geq 0$

Classical thermodynamics
Fundamental side: irreversibility



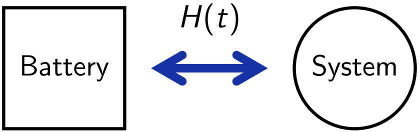
Entropy production $\Delta_\mathrm{i} S$

Second law of thermodynamics: $\Delta_\mathrm{i} S \geq 0$

Physical laws reversible at the microscopic scale:
Where does irreversibility come from?

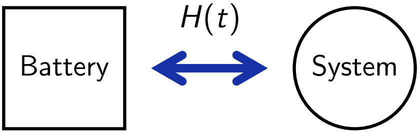
Can we reverse the movie?

Ideal case



System follows direct trajectory $\vec{\Sigma} : \Sigma_i \to \Sigma_f$

Can we reverse the movie?
Ideal case



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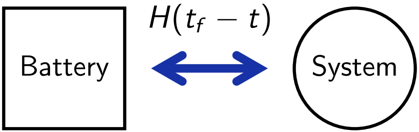
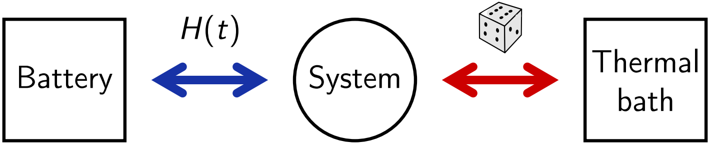


System follows reversed trajectory $\overleftarrow{\Sigma}$: $\Sigma_f \to \Sigma_i$ with certainty

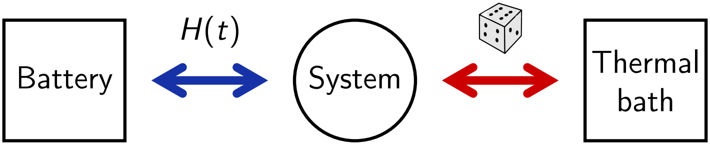
Can we reverse the movie?
Realistic case



System randomly disturbed by bath follows direct trajectory $\vec{\Sigma}$

Can we reverse the movie?

Realistic case



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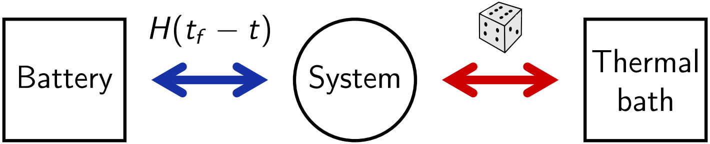

System does not follow reversed trajectory $\overleftarrow{\Sigma}$ with certainty

Central fluctuation theorem

Entropy production for a single trajectory

$$\Delta_{\mathrm{i}} s[\vec{\Sigma}] = \log \frac{P[\vec{\Sigma}]}{\tilde{P}[\overleftarrow{\Sigma}]}$$

Central fluctuation theorem

Entropy production for a single trajectory

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System at equilibrium at the start of direct and reversed transformations:

Central fluctuation theorem

$$\left\langle \mathrm{e}^{-\Delta_{\rm i} s[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = \sum_{\vec{\Sigma}} \tilde{P}[\overleftarrow{\Sigma}] = 1$$

Central fluctuation theorem

Entropy production for a single trajectory

$$\Delta_{\mathrm{i}} s[\vec{\Sigma}] = \log \frac{P[\vec{\Sigma}]}{\tilde{P}[\overleftarrow{\Sigma}]}$$

System at equilibrium at the start of direct and reversed transformations:

Central fluctuation theorem

$$\left\langle \mathrm{e}^{-\Delta_{\mathrm{i}} s[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = \sum_{\vec{\Sigma}} \tilde{P}[\overleftarrow{\Sigma}] = 1$$

Second law of thermodynamics

$$\left\langle \Delta_{\mathrm{i}} s[\vec{\Sigma}] \right\rangle_{\vec{\Sigma}} \geq 0$$

Isothermal transformation

Macroscopic

- Entropy production: $\Delta_{\mathrm{i}} S = \dfrac{1}{k_{\mathrm{B}} T}(W - \Delta F)$
- Fundamental bound: $W \geq \Delta F$

Isothermal transformation

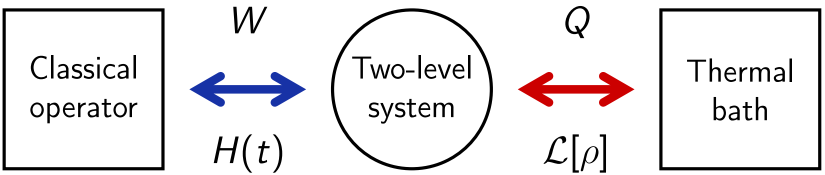
Macroscopic

- Entropy production: $\Delta_\mathrm{i} S = \frac{1}{k_\mathrm{B} T}(W - \Delta F)$
- Fundamental bound: $W \geq \Delta F$

Microscopic

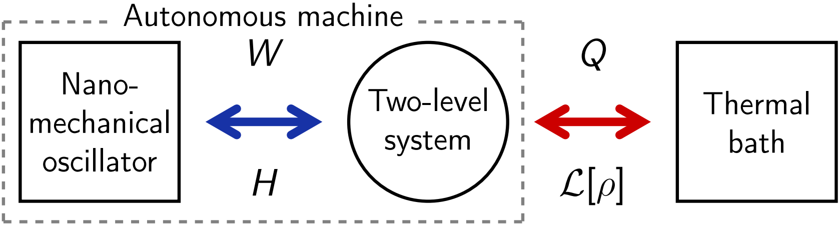
- Entropy production: $\Delta_\mathrm{i} s[\vec{\Sigma}] = \frac{1}{k_\mathrm{B} T}\left(W[\vec{\Sigma}] - \Delta F\right)$
- Jarzynski's equality: $\left\langle \mathrm{e}^{-\frac{1}{k_\mathrm{B} T} W[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = \mathrm{e}^{-\frac{1}{k_\mathrm{B} T}\Delta F}$

How to measure thermodynamic quantities for a two-level system?



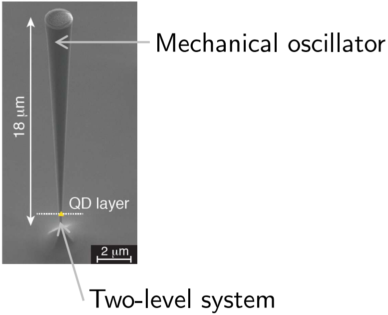
- System monitoring
- Bath monitoring

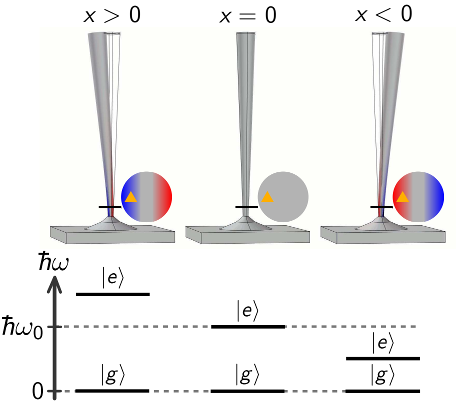
How to measure thermodynamic quantities for a two-level system?



- System monitoring
- Bath monitoring
- Measurements of the battery

Platform: Hybrid optomechanical system

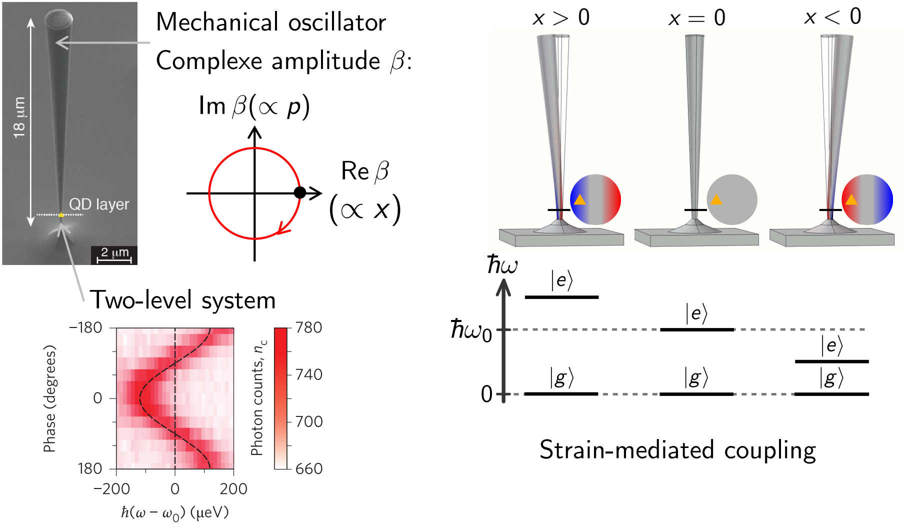




Strain-mediated coupling

I. Yeo *et al.*, *Nature Nanotechnology* **9**, 106–110 (2014)

Platform: Hybrid optomechanical system



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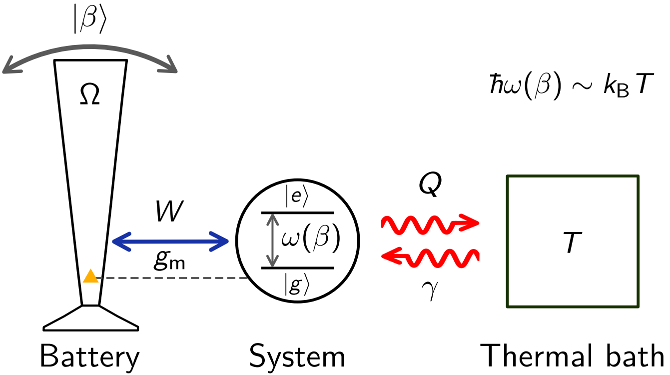
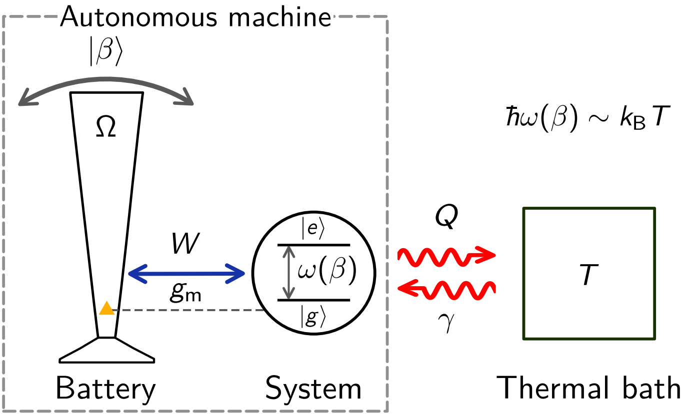
Model



Heat exchanges

- Photons
- $\mathcal{L}[\rho] = \gamma \bar{n}_\omega D[|e\rangle\langle g|]\rho + \gamma(\bar{n}_\omega + 1)D[|g\rangle\langle e|]\rho, \; D[X]\rho = X\rho X^\dagger - \frac{1}{2}\{X^\dagger X, \rho\}$

Model



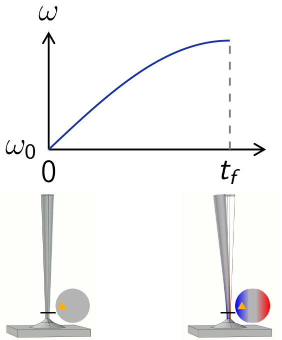
Work exchanges

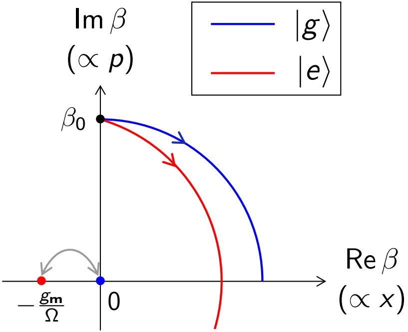
$$H = \hbar\omega_0 |e\rangle\langle e| + \hbar\Omega b^\dagger b + \hbar g_m |e\rangle\langle e| (b + b^\dagger)$$

Glimpse on work exchanges

$$H = \hbar\omega_0 \left|e\middle\rangle\middle\langle e\right| + \hbar\Omega b^\dagger b + \hbar g_{\mathrm{m}} \left|e\middle\rangle\middle\langle e\right| (b + b^\dagger)$$

Battery in a coherent state: $b\,|\beta\rangle = \beta\,|\beta\rangle$

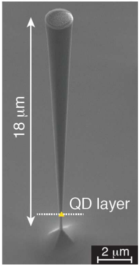




- Two-level system's frequency ω modulated by battery
- Battery's rest position depends on two-level system's state

Where do we come from?

Average energy exchanges:



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PAPER

Reversible work extraction in a hybrid opto-mechanical system

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Abstract

With the progress of nano-technology, thermodynamics also has to be scaled down, calling for specific protocols to extract and measure work. Usually, such protocols involve the action of an external, classical field (the battery) of infinite energy, that controls the energy levels of a small quantum system (the calorific fluid). Here we suggest a realistic device to reversibly extract work in a battery of finite energy : a hybrid optomechanical system. Such devices consist of an optically active two-level quantum system interacting strongly with a nano-mechanical oscillator that provides and stores mechanical work, playing the role of the battery. We identify protocols where the battery exchanges large, measurable amounts of work with the quantum emitter without getting entangled with it. When the quantum emitter is coupled to a thermal bath, we show that thermodynamic reversibility is attainable with state-of-the-art devices, paving the road towards the realization of a full cycle of information-to-energy conversion at the single bit level.

- Oscillator behaves like a battery
- Ultra-strong coupling regime $g_\mathrm{m} \gtrsim \Omega$: $\langle W \rangle = -\langle \Delta \mathcal{E}_\mathrm{m} \rangle$

Where do we come from?

Average energy exchanges:

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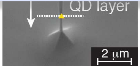
Where do we want to go?

- Extend this result to the single trajectory level
- Direct measurement of Jarzynski's equality in a quantum open system



classical field (the battery) of infinite energy, that controls the energy levels of a small quantum system (the calorific fluid). Here we suggest a realistic device to reversibly extract work in a battery of finite energy: a hybrid optomechanical system. Such devices consist of an optically active two-level quantum system interacting strongly with a nano-mechanical oscillator that provides and stores mechanical work, playing the role of the battery. We identify protocols where the battery exchanges large, measurable amounts of work with the quantum emitter without getting entangled with it. When the quantum emitter is coupled to a thermal bath, we show that thermodynamic reversibility is attainable with state-of-the-art devices, paving the road towards the realization of a full cycle of information-to-energy conversion at the single bit level.

- Oscillator behaves like a battery
- Ultra-strong coupling regime $g_{\mathrm{m}} \gtrsim \Omega$: $\langle W \rangle = -\langle \Delta \mathcal{E}_{\mathrm{m}} \rangle$

Quantum trajectories

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H, \rho] + \mathcal{L}[\rho] \xrightarrow[\text{unraveling}]{\text{Quantum jump}}$$ Trajectory $\vec{\Sigma}$ of the hybrid optomechanical system

Hybrid optomechanical system always factorized: $\vec{\Sigma} = (\vec{\epsilon}, \vec{\beta}[\vec{\epsilon}])$

Quantum trajectories

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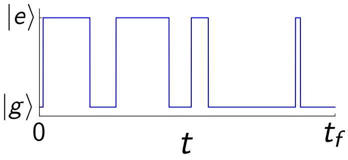
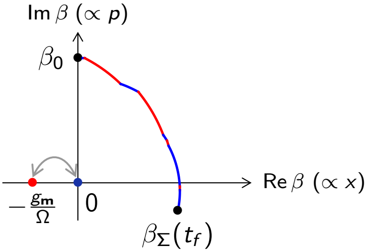
Hybrid optomechanical system always factorized: $\vec{\Sigma} = (\vec{\epsilon}, \vec{\beta}[\vec{\epsilon}])$

Two-level system: $\mathcal{E}_{\mathrm{q}}(t) = \hbar\omega(\beta(t))\delta_{\epsilon(t),e}$

$$\omega(\beta) = \omega_0 + 2g_{\mathrm{m}}\,\mathrm{Re}\,\beta$$



Battery: $\mathcal{E}_{\mathrm{m}}(t) = \hbar\Omega|\beta(t)|^2$

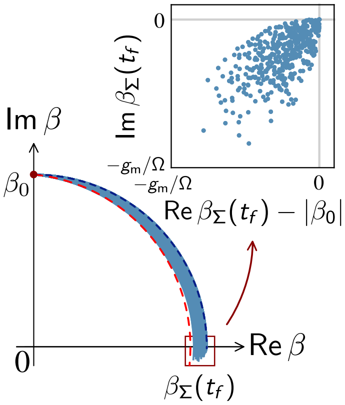


Stochastic work obtained by measuring the battery

$$W[\vec{\Sigma}] = -\Delta\mathcal{E}_{\mathrm{m}}[\vec{\Sigma}] = \hbar\Omega\left(|\beta_0|^2 - |\beta_\Sigma(t_f)|^2\right)$$

Stochastic entropy production

Battery's trajectories
(direct protocol)



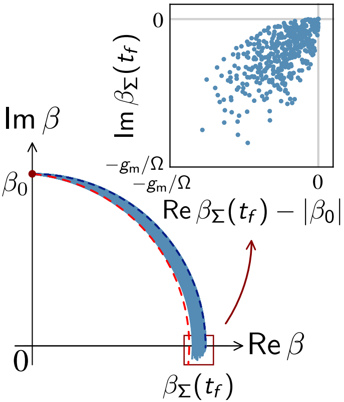
$$\Delta_\mathrm{i} s[\vec{\Sigma}] = \log \frac{P[\vec{\Sigma}]}{\tilde{P}[\overleftarrow{\Sigma}]}$$

Prescription for reversed trajectories $\overleftarrow{\Sigma}$:

- Time reversal of unitaries
- Same stochastic map

Stochastic entropy production

Battery's trajectories
(direct protocol)



$$\Delta_\mathrm{i} s[\vec{\Sigma}] = \sigma[\vec{\Sigma}] + I_\mathrm{Sh}[\vec{\Sigma}]$$

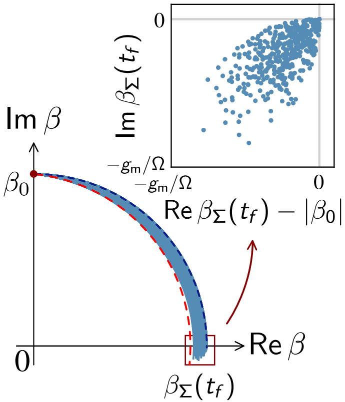
$$\sigma[\vec{\Sigma}] = -\frac{\Delta\mathcal{E}_\mathrm{m}[\vec{\Sigma}] + \Delta F[\vec{\Sigma}]}{k_\mathrm{B}T}$$

$$\Delta F[\vec{\Sigma}] = k_\mathrm{B}T \log\left(\frac{1 + e^{-\hbar\omega(\beta_0)/k_\mathrm{B}T}}{1 + e^{-\hbar\omega(\beta_\Sigma(t_f))/k_\mathrm{B}T}}\right)$$

- Thermodynamic system: two-level system
- Thermodynamic transformation: out-of-equilibrium driving
- Result: Jarzynski's equality

Stochastic entropy production

Battery's trajectories
(direct protocol)



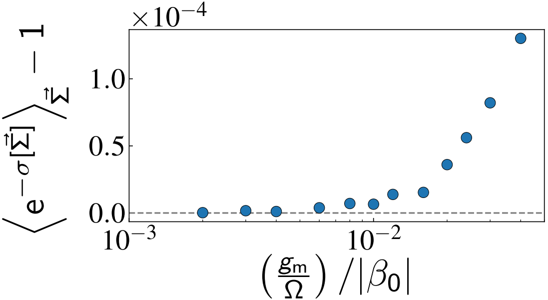
$$\Delta_\mathrm{i} s[\vec{\Sigma}] = \sigma[\vec{\Sigma}] + I_\mathrm{Sh}[\vec{\Sigma}]$$

$$I_\mathrm{Sh}[\vec{\Sigma}] = -\log(p_\mathrm{m}[\beta_\Sigma(t_f)])$$

- Thermodynamic system: whole hybrid system
- Thermodynamic transformation: relaxation toward equilibrium
- Result: irreversible transformation

Jarzynski's equality

In the regime $\frac{g_\mathrm{m}}{\Omega} \ll |\beta_0|$: ▶ $\left\langle \mathrm{e}^{-\sigma[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = 1$

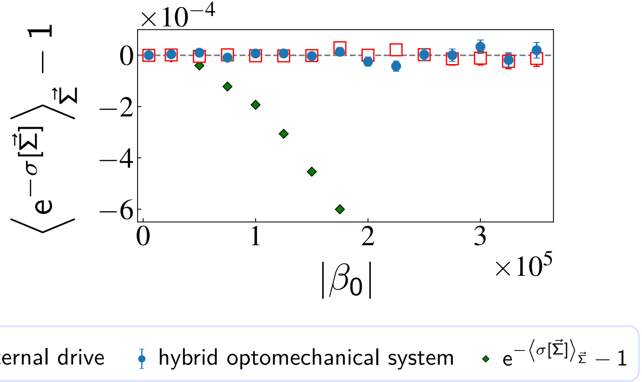


Realistic parameters: $\gamma/\Omega = 5,\ T = 80\,\mathrm{K},\ \hbar\omega_0 = 1.2 k_\mathrm{B} T$

Jarzynski's equality

In the regime $\frac{g_m}{\Omega} \ll |\beta_0|$:

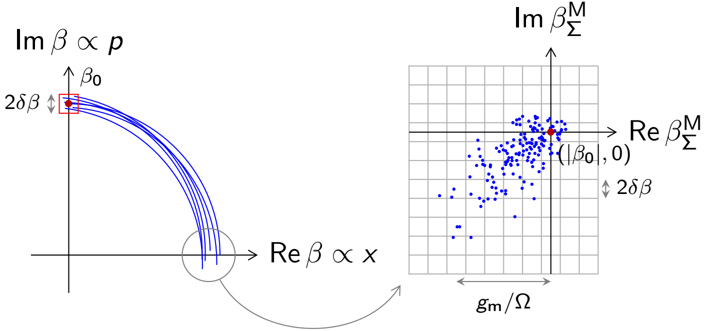
- $\left\langle e^{-\sigma[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = 1$
- Oscillator equivalent to external drive



Realistic parameters: $\gamma/\Omega = 5,\ T = 80\,\mathrm{K},\ \hbar\omega_0 = 1.2 k_B T,\ g_m/\Omega = 10$

Experimental error modeling

Re β and Im β known with a precision of $\pm\delta\beta$

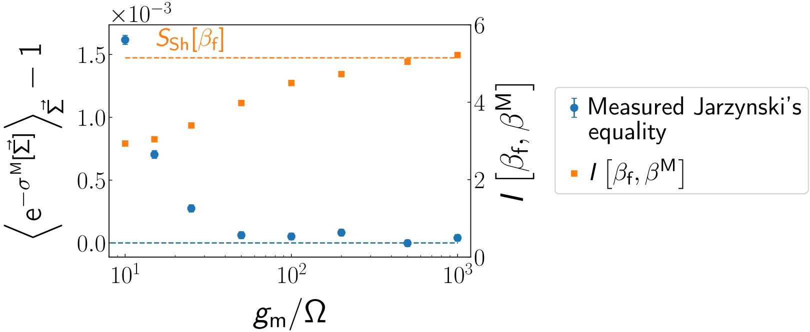


Measured entropy production: $\sigma^M[\vec{\Sigma}] = -\dfrac{\Delta\mathcal{E}_m^M[\vec{\Sigma}] + \Delta F}{k_B T}$

with $\Delta\mathcal{E}_m^M[\vec{\Sigma}] = \hbar\Omega(|\beta_0|^2 - |\beta_\Sigma^M|^2)$

Mutual information and Jarzynski's equality

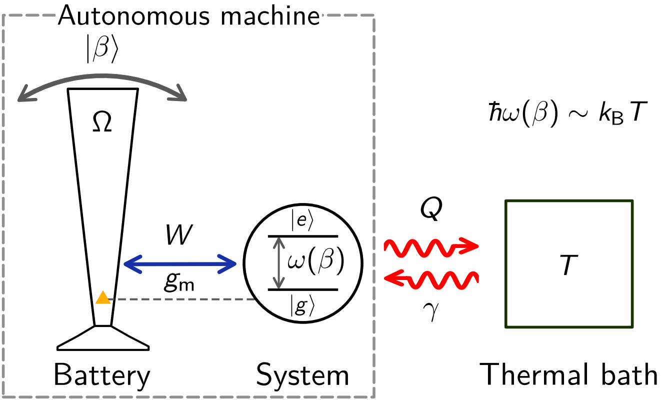
$$I[\beta_f, \beta^M] = \sum_{\beta_f, \beta^M} p(\beta_f, \beta^M) \log\left(\frac{p(\beta_f, \beta^M)}{p_m[\beta_f] p_m[\beta^M]}\right)$$



Realistic parameters: $\delta\beta = 2, \gamma/\Omega = 5,\ T = 80\,\mathrm{K},\ \hbar\omega_0 = 1.2 k_B T,\ 2g_m|\beta_0|/2\pi = 600$ GHz

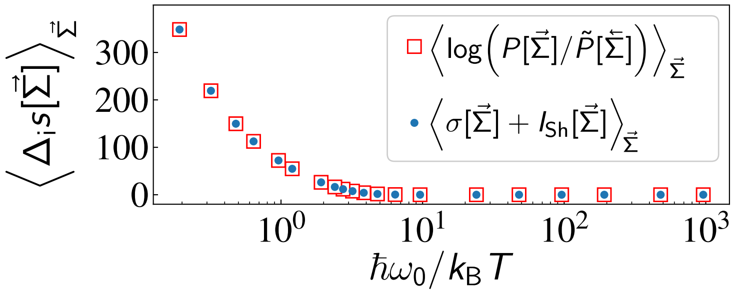
Jarzynski's equality can be probed if $\delta\beta \ll g_m/\Omega \ll |\beta_0|$

Relaxation toward equilibrium



$$\Delta_{\mathrm{i}} s[\vec{\Sigma}] = \log \frac{P[\vec{\Sigma}]}{\tilde{P}[\overleftarrow{\Sigma}]} = \sigma[\vec{\Sigma}] + I_{\mathrm{Sh}}[\vec{\Sigma}]$$

Average entropy production



$$\Delta_i s[\vec{\Sigma}] = \log \frac{P[\vec{\Sigma}]}{\tilde{P}[\overleftarrow{\Sigma}]} = \sigma[\vec{\Sigma}] + I_{Sh}[\vec{\Sigma}]$$

Conclusion

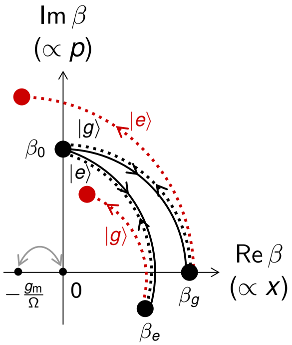
Hybrid optomechanical systems are promising set-ups to explore trajectory thermodynamics:

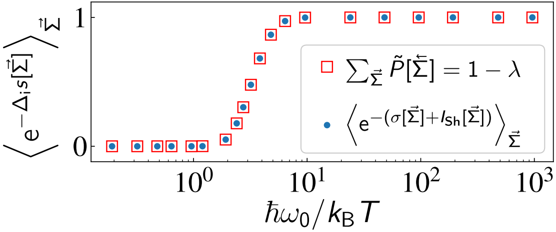
- Battery's energy fluctuations equal work fluctuations
- Jarzynski's equality directly measurable in a open quantum system with a realistic experimental platform

Thank you for your attention!

Further reading: *npj Quantum Information* **4**, 59 (2018)

Absolute irreversibility





Generalized integral fluctuation theorem

$$\left\langle e^{-\Delta_\mathrm{i}s[\vec{\Sigma}]} \right\rangle_{\vec{\Sigma}} = \sum_{\vec{\Sigma}} \tilde{P}[\overleftarrow{\Sigma}] = 1 - \lambda < 1$$

Quantum jump operators

- Master equation: $\dot{\rho}(t) = -\frac{\mathrm{i}}{\hbar}[H, \rho(t)] + \gamma \bar{n}_{\omega(\beta_0(t))} D[\sigma^\dagger \otimes \mathbf{1}_\mathrm{m}]\rho(t) + \gamma\left(\bar{n}_{\omega(\beta_0(t))} + 1\right) D[\sigma \otimes \mathbf{1}_\mathrm{m}]\rho(t).$

$$D[X]\rho = X\rho X^\dagger - \frac{1}{2}\{X^\dagger X, \rho\},\ \bar{n}_\omega = \frac{1}{\exp(\hbar\omega/k_\mathrm{B} T) - 1},$$

$$\omega(\beta) = \omega_0 + 2g_\mathrm{m}\,\mathrm{Re}\,\beta \text{ and } \beta_0(t) = \beta_0 e^{-\mathrm{i}\Omega t}$$

- Unraveling:

Direct protocol

$$J_0(t) = \mathbf{1}_\mathrm{qm} - \frac{\mathrm{i}\Delta t}{\hbar} H_\mathrm{eff}(t)$$

$$J_{-1}(t) = \sqrt{\gamma\Delta t(\bar{n}_{\omega(\beta_0(t))} + 1)}\,\sigma \otimes \mathbf{1}_\mathrm{m}$$

$$J_{+1}(t) = \sqrt{\gamma\Delta t\,\bar{n}_{\omega(\beta_0(t))}}\,\sigma^\dagger \otimes \mathbf{1}_\mathrm{m}$$

Reversed protocol

$$\tilde{J}_0(t) = \mathbf{1}_\mathrm{qm} + \frac{\mathrm{i}\Delta t}{\hbar} H^\dagger_\mathrm{eff}(t)$$

$$\tilde{J}_{-1}(t) = J_{+1}(t)$$

$$\tilde{J}_{+1}(t) = J_{-1}(t)$$

$$H_\mathrm{eff}(t) = H - \mathrm{i}\frac{\hbar}{2}(J^\dagger_{+1}(t)J_{+1}(t) + J^\dagger_{-1}(t)J_{-1}(t))$$

Trajectory probabilities

Trajectory $\vec{\Sigma} = (\Psi_\Sigma(t_0), \ldots, \Psi_\Sigma(t_N))$ with $|\Psi_\Sigma(t_n)\rangle = |\epsilon_\Sigma(t_n)\rangle \otimes |\beta_\Sigma(t_n)\rangle$

$$P[\vec{\Sigma}] = p^\infty_{\beta_0}[\epsilon_\Sigma(t_0)] \prod_{n=1}^{N} P[\Psi_\Sigma(t_n)|\Psi_\Sigma(t_{n-1})]$$

$$\tilde{P}[\overleftarrow{\Sigma}] = p_\mathrm{m}[\beta_\Sigma(t_N)] p^\infty_{\beta_\Sigma(t_N)}[\epsilon_\Sigma(t_N)] \prod_{n=N}^{1} \tilde{P}[\Psi_\Sigma(t_{n-1})|\Psi_\Sigma(t_n)]$$

$$p^\infty_\beta[\epsilon] = \frac{\mathrm{e}^{-\hbar\omega(\beta)/k_\mathrm{B}T}}{1 + \mathrm{e}^{-\hbar\omega(\beta)/k_\mathrm{B}T}}$$

Work and Heat

Average quantities with an external drive:

$$\left\langle \dot{W} \right\rangle = \mathrm{Tr}\left(\rho \dot{H}\right) \text{ and } \left\langle \dot{Q} \right\rangle = \mathrm{Tr}(\dot{\rho} H)$$

Energy variation $\delta\mathcal{E}_{\mathrm{q}}[\Sigma, t_n]$ of the two-level system during the time step $[t_n, t_{n+1}]$:

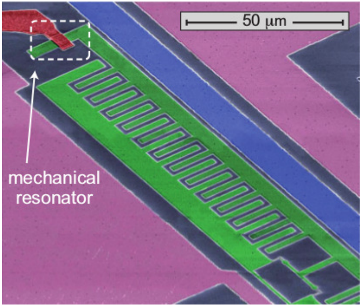
- No jump evolution: work exchange $w[\Sigma, t_n] = \hbar(\omega(\beta_\Sigma(t_n)) - \omega(\beta_\Sigma(t_{n+1}))) = -\delta\mathcal{E}_{\mathrm{m}}[\Sigma, t_n]$
- Quantum jump: heat exchange $q[\Sigma, t_n] = \pm\hbar\omega(\beta_\Sigma(t_n))$

Total work and heat exchanged along the trajectory $\vec{\Sigma}$:

$$W[\vec{\Sigma}] = \sum_{n=0}^{N-1} w[\Sigma, t_n] \text{ and } Q[\vec{\Sigma}] = \sum_{n=0}^{N-1} q[\Sigma, t_n]$$

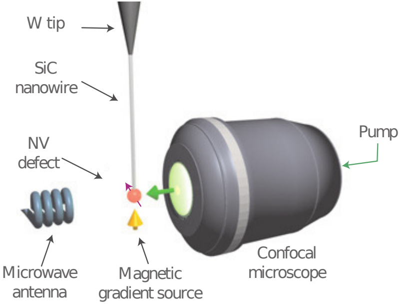
Other hybrid optomechanical systems

Superconducting qubits embedded in oscillating membranes



J.-M. Pirkkalainen et al., *Nature*, **494**, p. 211–215 (2013)

NV defect coupled to a nanowire with a magnetic field gradient



O. Arcizet et al., *Nature Physics*, **7**, p. 879–883 (2011)