Nondemolition thermometry of a BEC in sub-nK regime

By using Bose-Polaron model

PRL 122 (3), 030403

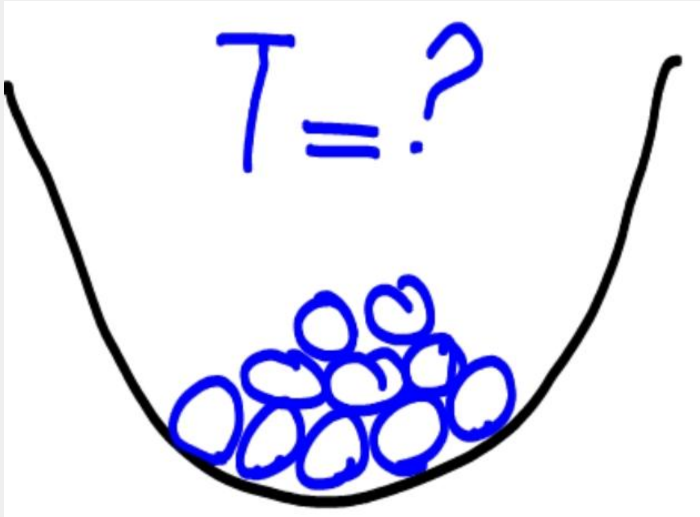
Mohammad Mehboudi
ICFO, Barcelona

QTD2019; Espoo-Finland

This is a theoretical work!

The Problem

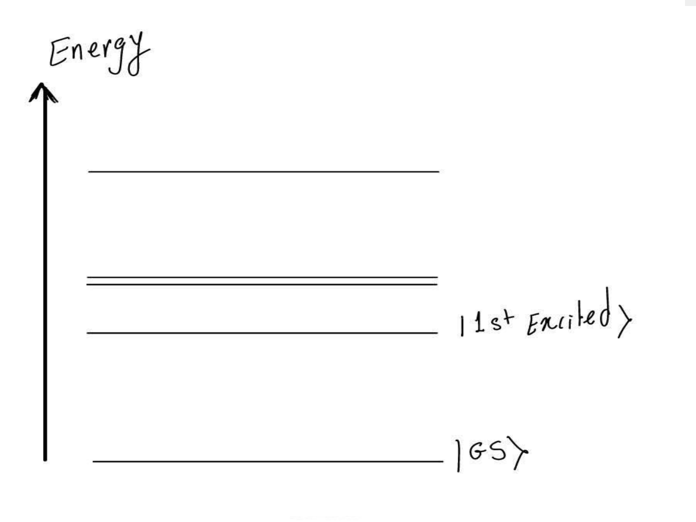
- We have a very cold BEC in a trap (temperature ***T*~nK**)
- We want **not to destroy** the BEC
- We want to **estimate *T*** with minimum **statistical** error



$$\rho = \frac{\exp(-H_{BEC}/T)}{Z}$$

The Problem

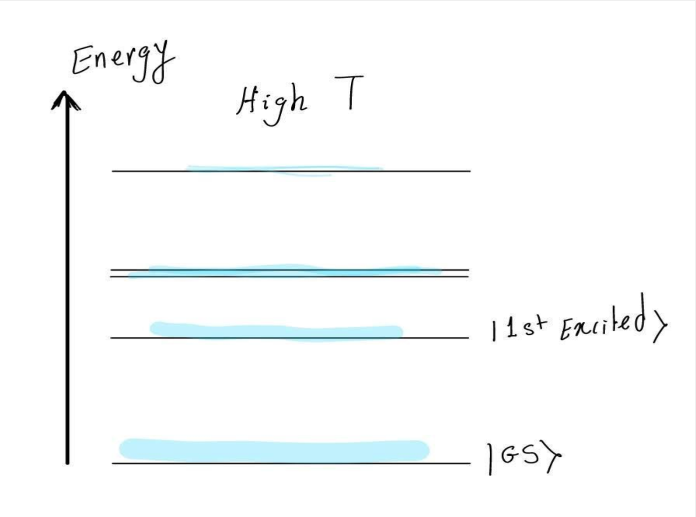
- We have a very cold BEC in a trap (temperature ***T*~nK**)
- We want **not to destroy** the BEC
- We want to **estimate *T*** with minimum **statistical** error



$$\rho = \frac{\exp(-H_{BEC}/T)}{Z}$$

The Problem

- We have a very cold BEC in a trap (temperature ***T*~nK**)
- We want **not to destroy** the BEC
- We want to **estimate *T*** with minimum **statistical** error



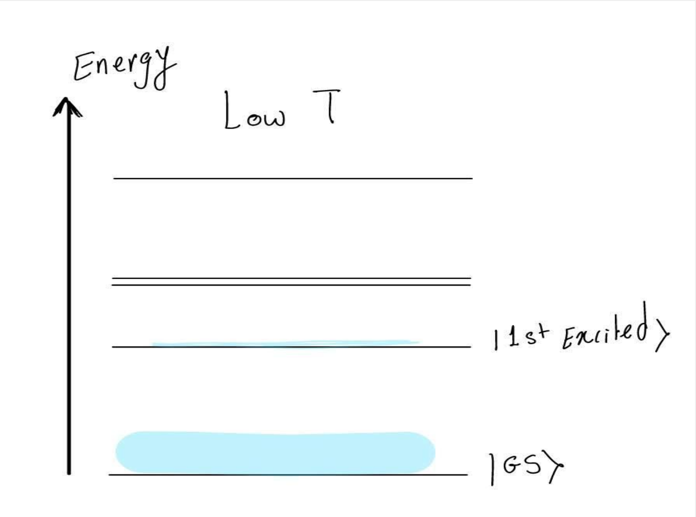
$$\rho = \frac{\exp(-H_{BEC}/T)}{Z}$$

The Problem

- We have a very cold BEC in a trap (temperature ***T*~nK**)

- We want **not to destroy** the BEC

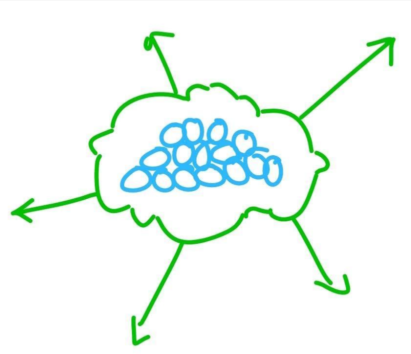
- We want to **estimate *T*** with minimum **statistical** error



$$\rho = \frac{\exp(-H_{BEC}/T)}{Z}$$

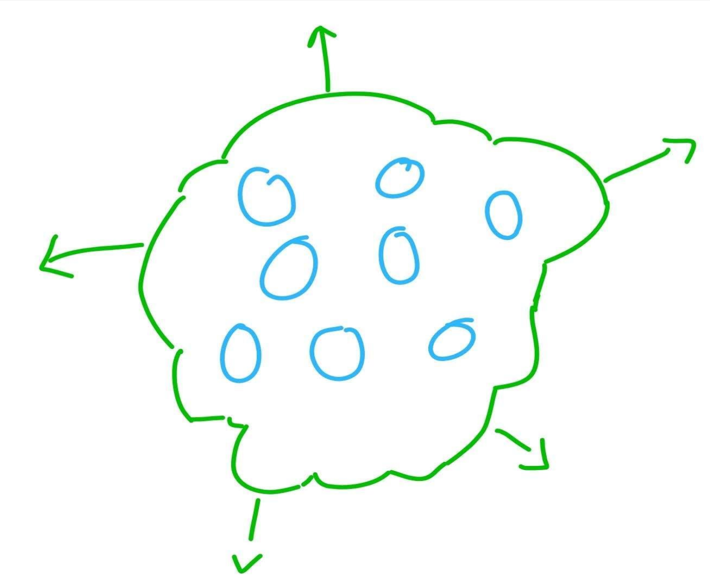
Major thermometry methods in BEC

Time-of-flight measurements



Major thermometry methods in BEC

Time-of-flight measurements



Major thermometry methods in BECs

Time-of-flight measurements

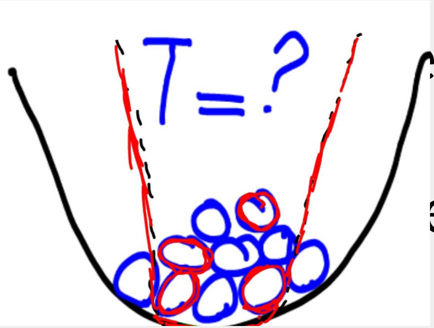
- Performed **directly** on the BEC
  - Good precision even down to **0.5nK**
  - **Demolition**

Leanhardt, *et.al*---Science 301, 1513 (2003)

Major thermometry methods in BECs

Time-of ... ements

- Perf ... on the BEC
  - ... even down to **0.5nK**
  - ...



- Performed on **impurities** embedded inside the BEC
  - **Bad** precision in the **few nK** domain
  - **Nondemolition**

Olf, *et.al*---Nat. Phys. 11, 720 (2015)

Major thermometry methods in BECs

Time-of-flight measurements

- Performed **directly** on the BEC
  - Good precision even down to **0.5nK**
  - **Demolition**
- Performed on **impurities** embedded inside the BEC
  - **Bad** precision in the **few nK** domain
  - **Nondemolition**

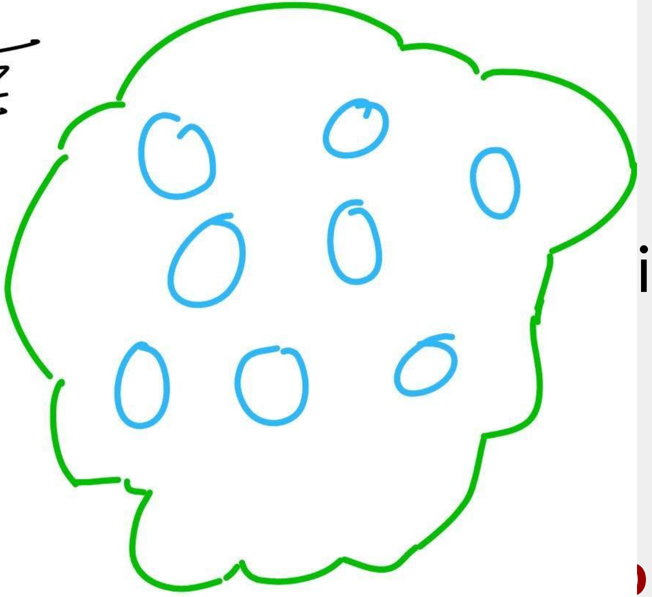
Most works are based on the **thermalization** assumption

Major thermometry methods in BECs

Time-of-f

- Perfo
  - Go
  - **De**
- Perfo ide the BEC
  - Ba
  - **No**

$$\rho_{\text{Global}} = \frac{e^{-H/T}}{Z}$$

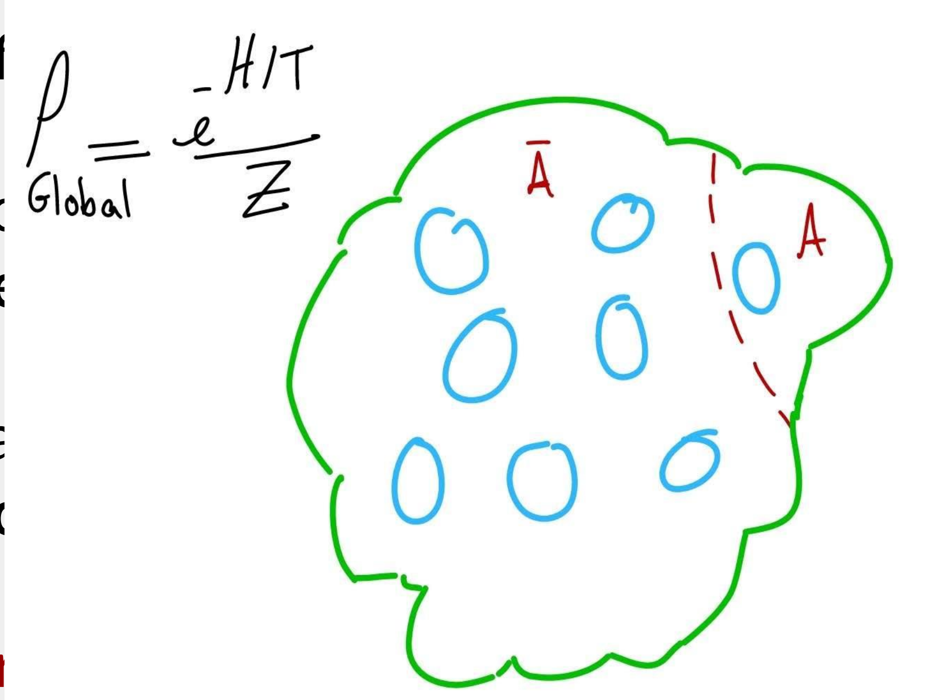


Most wor **on** assumption

Major thermometry methods in BECs

Time-of-f

- Perfo
  - Go
  - **De**
- Perfo … ide the BEC
  - Ba
  - **No**

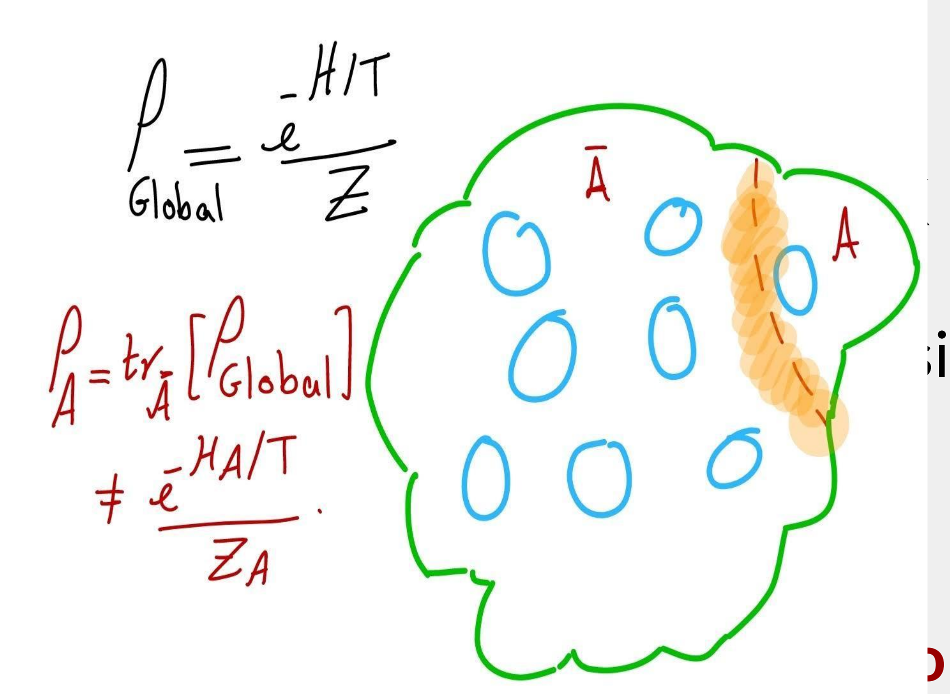


Most wor … **on** assumption

Major thermometry methods in BECs

Time-of-f

- Perfo
  - Go
  - **De**
- Perfo ... side the BEC
  - Ba
  - **No**



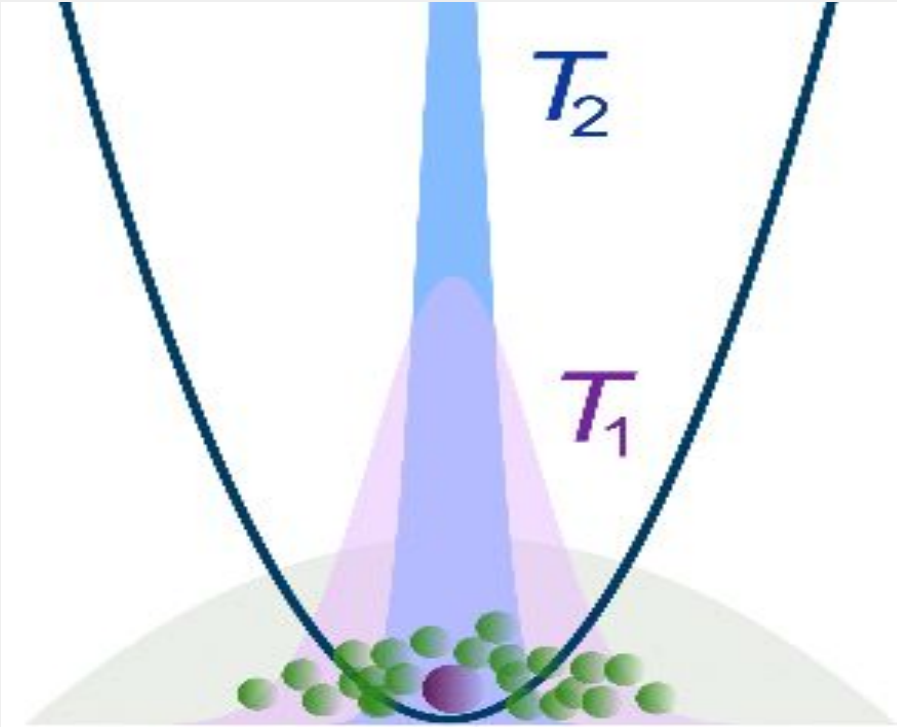
Most wor ... **on** assumption

What do *we* look for?

A thermometry protocol that is:

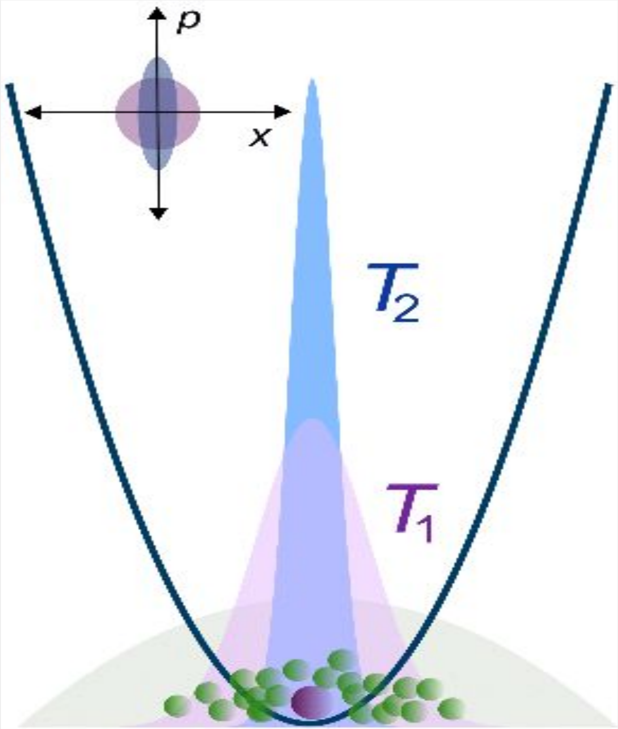
1. **Nondemolition**
2. **Precise** in nK and **sub-nK** regime
3. Built upon a **comprehensive theoretical description**

Idea: Bose-Polaron Thermometry



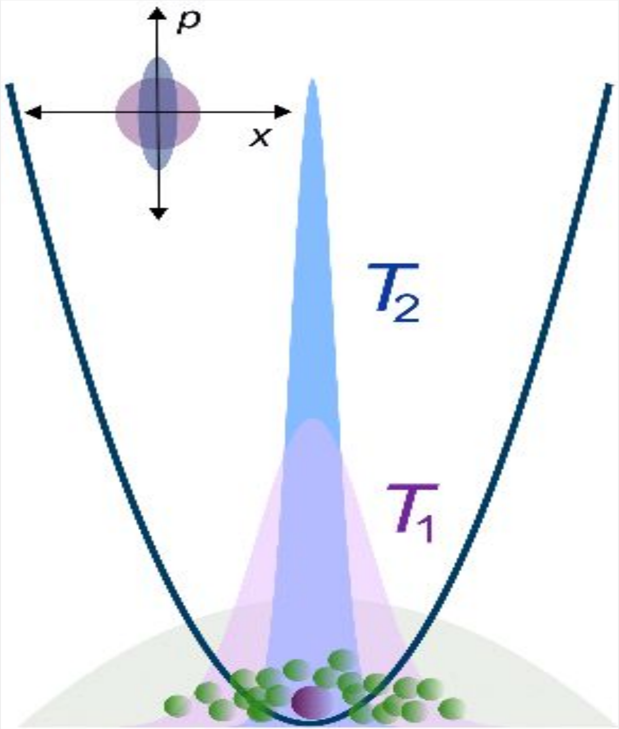
Idea: Bose-Polaron Thermometry

- The 1D BEC:
    - Harmonic trap
    - **Thermal Equilibrium**

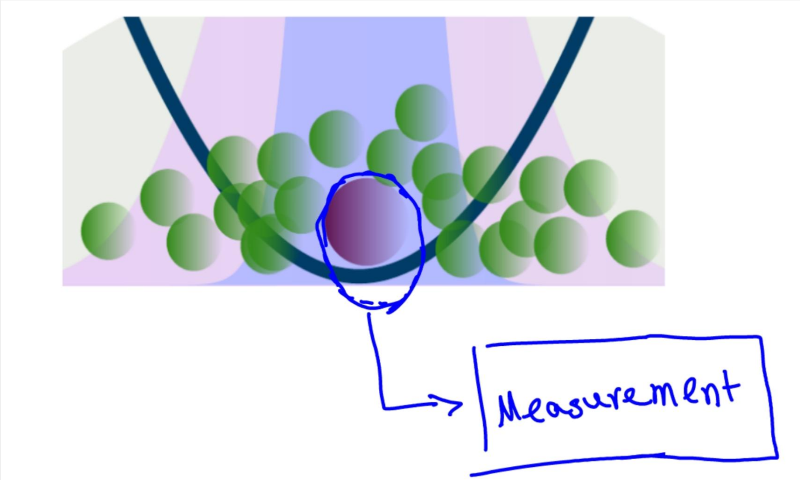


Idea: Bose-Polaron Thermometry

- The 1D BEC:
  - Harmonic trap
  - **Thermal Equilibrium**
- The impurity (thermometer):
  - Interacts with the BEC
  - Relaxes to a **non-thermal** steady state.
  - Its **position** and **momentum** depend on ***T*.**



Idea: Bose-Polaron Thermometry



We have to map the outcomes of the measurement to ***T*.** Thus, ***we need---theoretically---the T-dependence of the statistics of the impurity.***

The Model

$$H = H_{\mathrm{I}} + H_{\mathrm{B}} + H_{\mathrm{BB}} + H_{\mathrm{IB}}$$

Total Hamiltonian

The Model

$$H = \boxed{H_{\mathsf{I}}} + H_{\mathsf{B}} + H_{\mathsf{BB}} + H_{\mathsf{IB}}$$

Total Hamiltonian

$$H_{\mathsf{I}} = \frac{P_{\mathsf{I}}^2}{2m_{\mathsf{I}}} + U(x), U(x) = \frac{m_{\mathsf{I}}\Omega^2 x^2}{2}$$

The impurity

The Model

$$H = H_{\mathsf{I}} + \boxed{H_{\mathsf{B}}} + H_{\mathsf{BB}} + H_{\mathsf{IB}}$$

Total Hamiltonian

$$H_{\mathsf{I}} = \frac{P_{\mathsf{I}}^2}{2m_{\mathsf{I}}} + U(x), U(x) = \frac{m_{\mathsf{I}}\Omega^2 x^2}{2}$$

The impurity

$$H_{\mathsf{B}} = \int dx \Psi^\dagger(x) \left(\frac{P_{\mathsf{B}}^2}{2m_{\mathsf{B}}} + V(x) \right) \Psi(x)$$

The BEC

$$[\Psi(x), \Psi^\dagger(x')] = \delta(x - x'), V(x) = \frac{m_{\mathsf{B}}\omega^2 x^2}{2}$$

The Model

$$H = H_{\mathsf{I}} + H_{\mathsf{B}} + \boxed{H_{\mathsf{BB}}} + H_{\mathsf{IB}}$$

Total Hamiltonian

$$H_{\mathsf{I}} = \frac{P_{\mathsf{I}}^2}{2m_{\mathsf{I}}} + U(x), U(x) = \frac{m_{\mathsf{I}}\Omega^2 x^2}{2}$$

The impurity

$$H_{\mathsf{B}} = \int dx \Psi^\dagger(x)\left(\frac{P_{\mathsf{B}}^2}{2m_{\mathsf{B}}} + V(x)\right)\Psi(x)$$

The BEC

$$[\Psi(x), \Psi^\dagger(x')] = \delta(x - x'), V(x) = \frac{m_{\mathsf{B}}\omega^2 x^2}{2}$$

$$H_{\mathsf{BB}} = g_{\mathsf{BB}} \int dx \Psi^\dagger(x)\Psi^\dagger(x)\Psi(x)\Psi(x)$$

The BEC-BEC

The Model

$$H = H_{\mathsf{I}} + H_{\mathsf{B}} + H_{\mathsf{BB}} + \boxed{H_{\mathsf{IB}}}$$

Total Hamiltonian

$$H_{\mathsf{I}} = \frac{P_{\mathsf{I}}^2}{2m_{\mathsf{I}}} + U(x), U(x) = \frac{m_{\mathsf{I}}\Omega^2 x^2}{2}$$

The impurity

$$H_{\mathsf{B}} = \int dx \Psi^\dagger(x) \left(\frac{P_{\mathsf{B}}^2}{2m_{\mathsf{B}}} + V(x) \right) \Psi(x)$$

$$[\Psi(x), \Psi^\dagger(x')] = \delta(x - x'), V(x) = \frac{m_{\mathsf{B}}\omega^2 x^2}{2}$$

The BEC

$$H_{\mathsf{BB}} = g_{\mathsf{BB}} \int dx \Psi^\dagger(x)\Psi^\dagger(x)\Psi(x)\Psi(x)$$

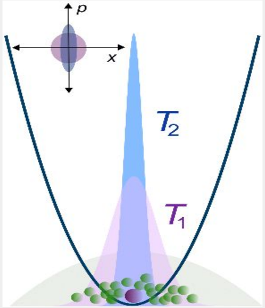
The BEC-BEC

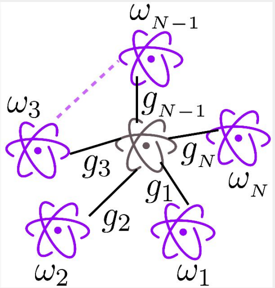
$$H_{\mathsf{IB}} = \eta\, g_{\mathsf{BB}} \Psi^\dagger(x)\Psi(x)$$

The Impurity-BEC

The Bose-Polaron Model ---> The Brownian Motion

➔ In an experimentally **relevant range of parameters**, we can map Bos-Polaron to Brownian motion:



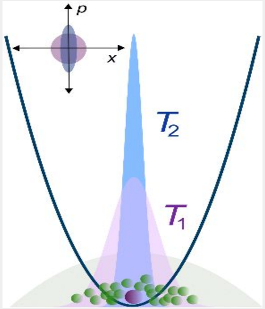


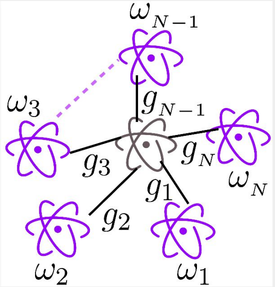
A.Lampo, et.al. Quantum 1, 30
A.Lampo, et.al. PRA 98, 063630
M.Mehboudi, et.al. PRL 122 (3), 030403

$$H = \underbrace{\frac{\hat{p}^2}{2m_{\mathrm{I}}} + \frac{m_{\mathrm{I}}\Omega^2}{2}\hat{x}^2}_{\text{Impurity}} + \underbrace{\sum_k E_k \hat{b}_k^\dagger \hat{b}_k}_{\text{BEC}} + \underbrace{\sum_k \hbar g_k\, \hat{x}\left(\hat{b}_k + \hat{b}_k^\dagger\right)}_{\text{Interaction}}$$

The Bose-Polaron Model ---> The Brownian Motion

➔ In an experimentally **relevant range of parameters**, we can map Bos-Polaron to Brownian motion:





A.Lampo, et.al. Quantum 1, 30
A.Lampo, et.al. PRA 98, 063630
M.Mehboudi, et.al. PRL 122 (3), 030403

$$H = \underbrace{\frac{\hat{p}^2}{2m_\mathrm{I}} + \frac{m_\mathrm{I}\Omega^2}{2}\hat{x}^2}_{\text{Impurity}} + \underbrace{\sum_k E_k \hat{b}_k^\dagger \hat{b}_k}_{\text{BEC}} + \underbrace{\sum_k \hbar g_k\, \hat{x}\left(\hat{b}_k + \hat{b}_k^\dagger\right)}_{\text{Interaction}}$$

- The Hamiltonian is **quadratic**.
- The second moments (**Covariance Matrix**) fully describe the quantum state.

The Covariance Matrix Is All We Need

- In the quantum **Langevin** formaism we can find the covariance matrix

$$\sigma_{ss} = \begin{pmatrix} < x^2 > & < \{x, p\} > \\ < \{x, p\} > & < p^2 > \end{pmatrix}$$

Thermometry Precision

- The **relative error** if we measure observable O:

$$\frac{\delta T(\hat{O})}{T} \equiv \frac{\Delta \hat{O}}{\sqrt{\nu} T |\chi_T(\hat{O})|}$$

Uncertainty

Temperature susceptibility

# of measurements

$$\chi_T(\hat{O}) = \partial_T < \hat{O} >$$

Thermometry Precision

- The **quantum Cramer-Rao** bound:

$$\frac{\delta T(\hat{O})}{T} \equiv \frac{\Delta \hat{O}}{\sqrt{\nu} T |\partial_T < \hat{O} > |} \geq \frac{1}{\sqrt{\nu} T \sqrt{\mathcal{F}(T)}}$$

$$\mathcal{F}(T) = \mathrm{tr}[\Lambda_T^2 \rho_{ss}]; \Lambda_T = \arg[\min_{\hat{O}} \frac{\delta T(\hat{O})}{T}]$$

Thermometry Precision

- The **quantum Cramer-Rao** bound:

$$\frac{\delta T(\hat{O})}{T} \equiv \frac{\Delta \hat{O}}{\sqrt{\nu} T |\partial_T < \hat{O} >|} \geq \frac{1}{\sqrt{\nu} T \sqrt{\mathcal{F}(T)}}$$

$$\mathcal{F}(T) = \mathrm{tr}[\Lambda_T^2 \rho_{ss}]; \Lambda_T = \arg[\min_{\hat{O}} \frac{\delta T(\hat{O})}{T}]$$

- The **optimal** measurement in Gaussian formalism

A.Monras, arXiv:1303.3682
Z.Jiang, Phys. Rev. A 89, 032128 (2014)
R.Nichols, et.al. Phys. Rev. A 98, 012114 (2018)
M.Mehboudi, et.al. arXiv:1901.05709

Thermometry Precision

- The **quantum Cramer-Rao** bound:

$$\frac{\delta T(\hat{O})}{T} \equiv \frac{\Delta \hat{O}}{\sqrt{\nu}T|\partial_T <\hat{O}>|} \geq \frac{1}{\sqrt{\nu}T\sqrt{\mathcal{F}(T)}}$$

$$\mathcal{F}(T) = \mathrm{tr}[\Lambda_T^2 \rho_{ss}]; \Lambda_T = \arg[\min_{\hat{O}} \frac{\delta T(\hat{O})}{T}]$$

- The **optimal** measurement?

$$\Lambda_T = C_x \left(x^2 - < x^2 >\right) + C_p \left(p^2 - < p^2 >\right)$$

$$C_x = \frac{4<x^2>^2\chi_T(x^2)+\hbar^2\chi_T(p^2)}{8<x^2>^2<p^2>^2-\hbar^4/2}$$

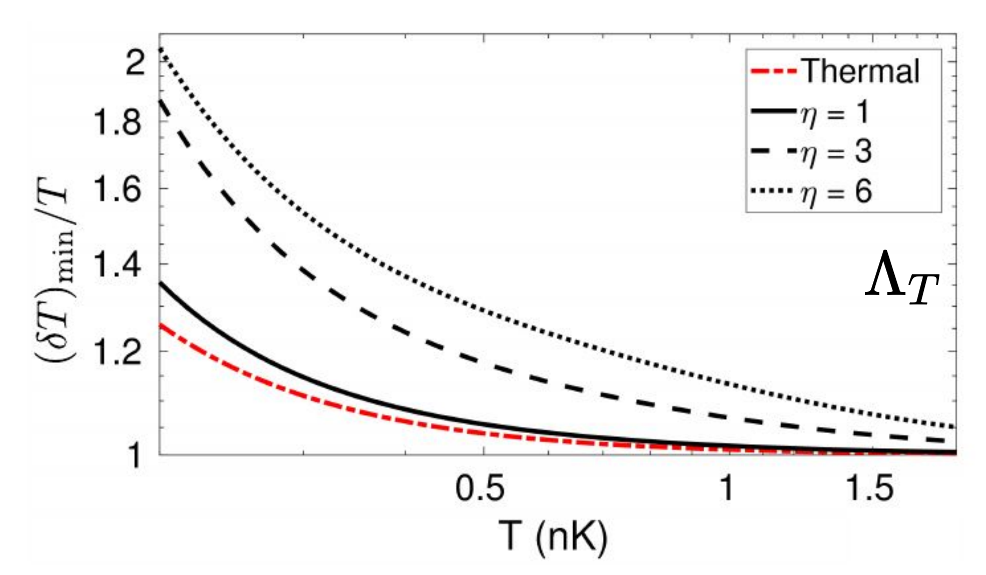
Ultimate bounds of thermometry

See PRL **122**, 030403 (2019) for parameters

BEC: **K**
Impurity: **Yb**

η: The strength of the BEC-impurity interaction

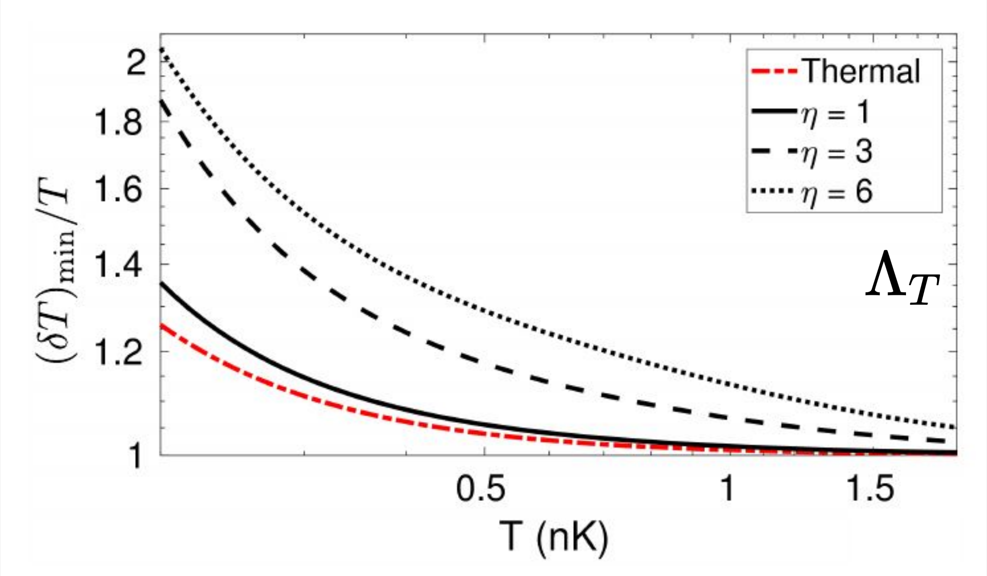
T: From 0.2nK to 2nK



Ultimate bounds of thermometry

See PRL **122**, 030403 (2019) for parameters

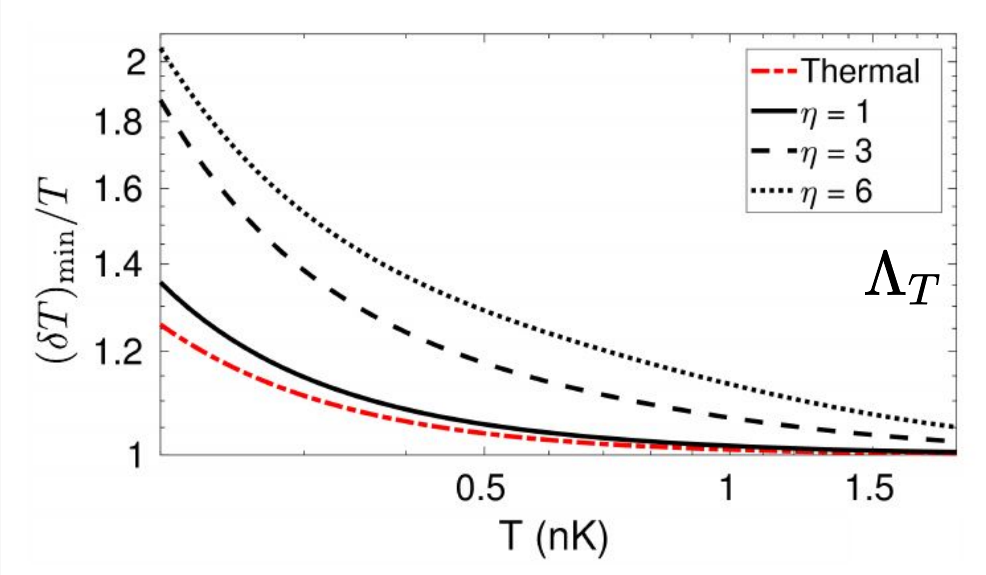
1. **Stronger** couplings *worsen* the precision
2. Relative error **below** 14% with 100 measurements
3. **Thermalization** assumption is *wrong*



Ultimate bounds of thermometry

See PRL **122**, 030403 (2019) for parameters

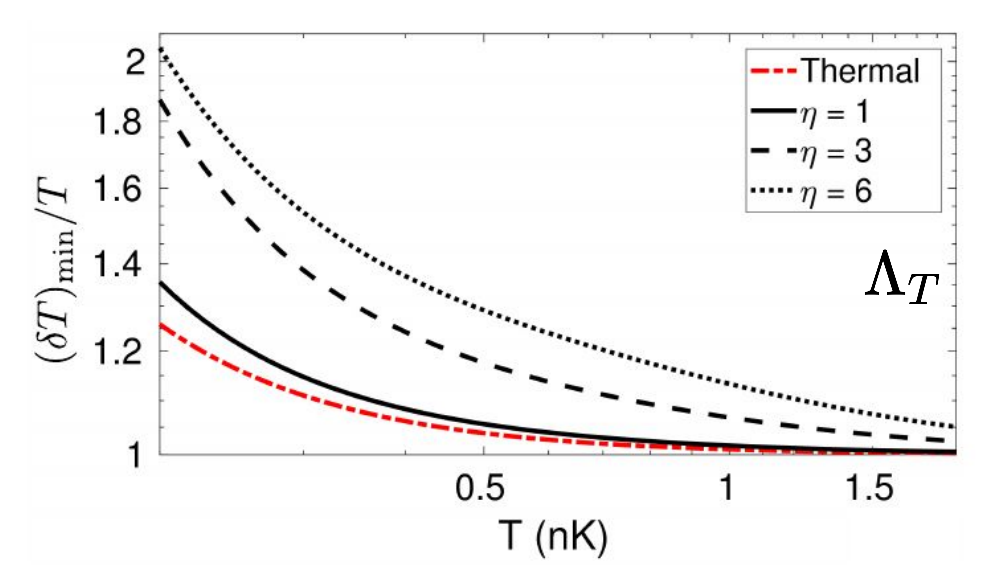
1. **Stronger** couplings *worsen* the precision
2. Relative error **below** 14% with 100 measurements
3. **Thermalization** assumption is *wrong*



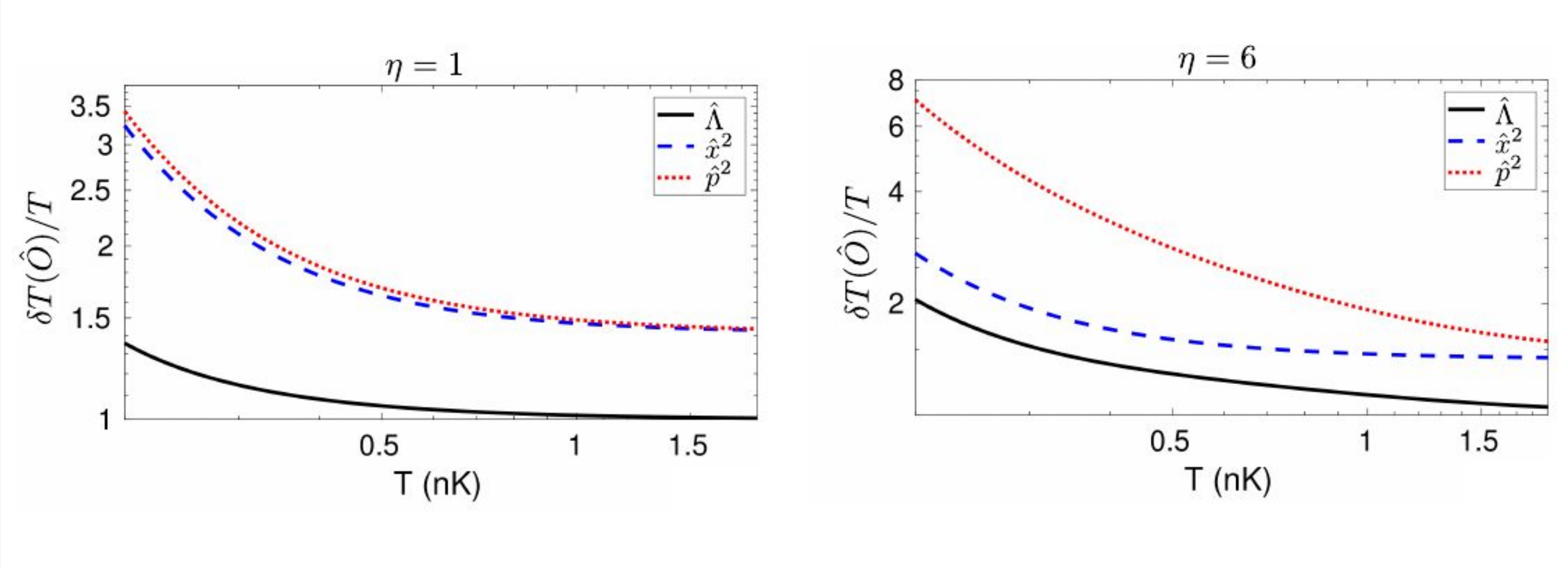
Ultimate bounds of thermometry

See PRL **122**, 030403 (2019) for parameters

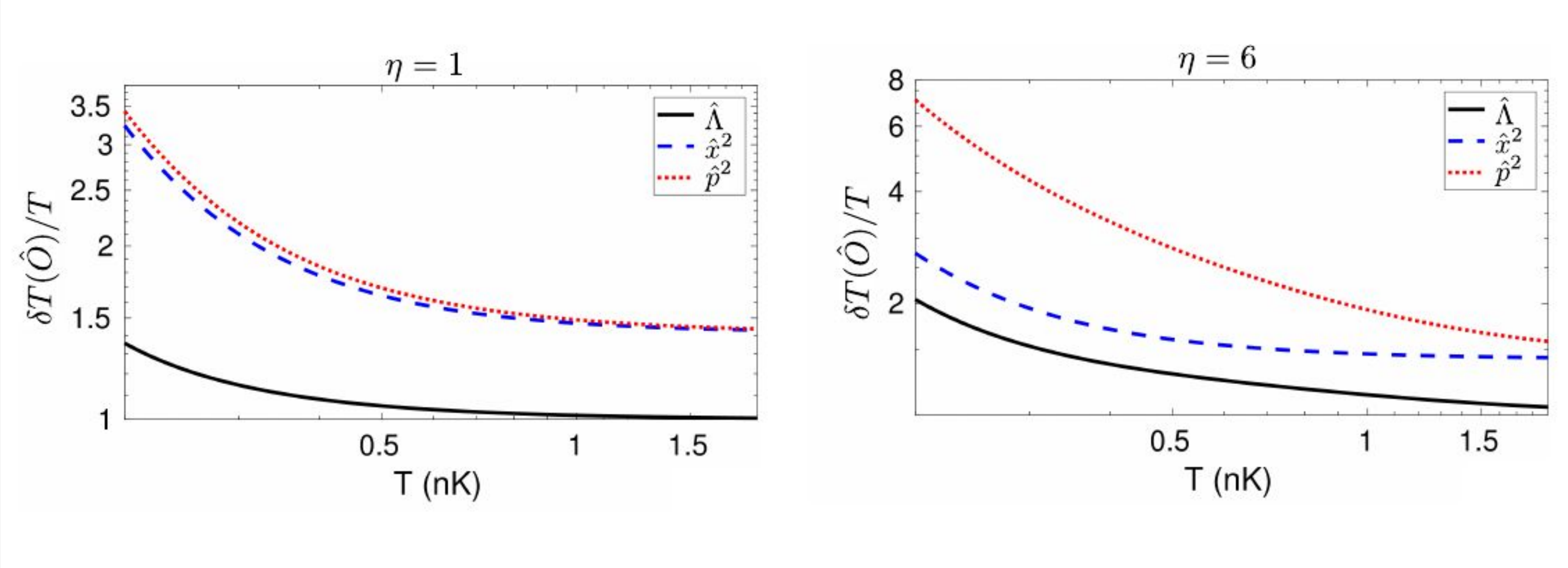
1. **Stronger** couplings *worsen* the precision
2. Relative error **below** 14% with 100 measurements
3. **Thermalization** assumption is *wrong*



Suboptimal measurements (Position and Momentum)

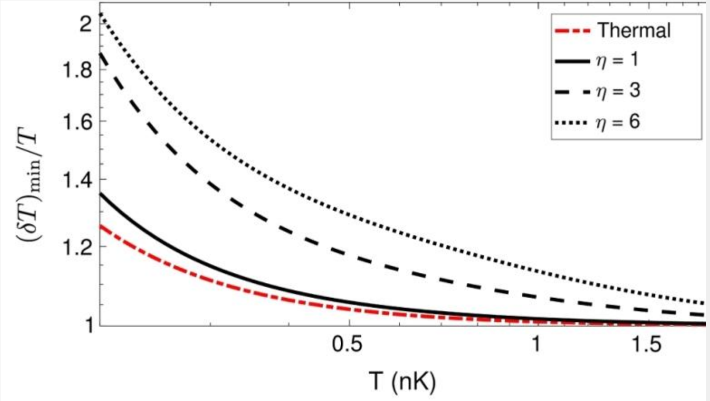


Suboptimal measurements (Position and Momentum)

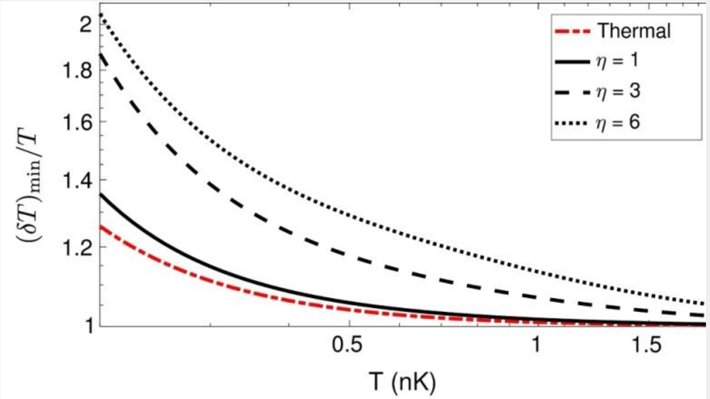


Relative error below 18%, by measuring **x^2**; number of measurements 400 times.

Strong Coupling Enhances Thermometry Precision?



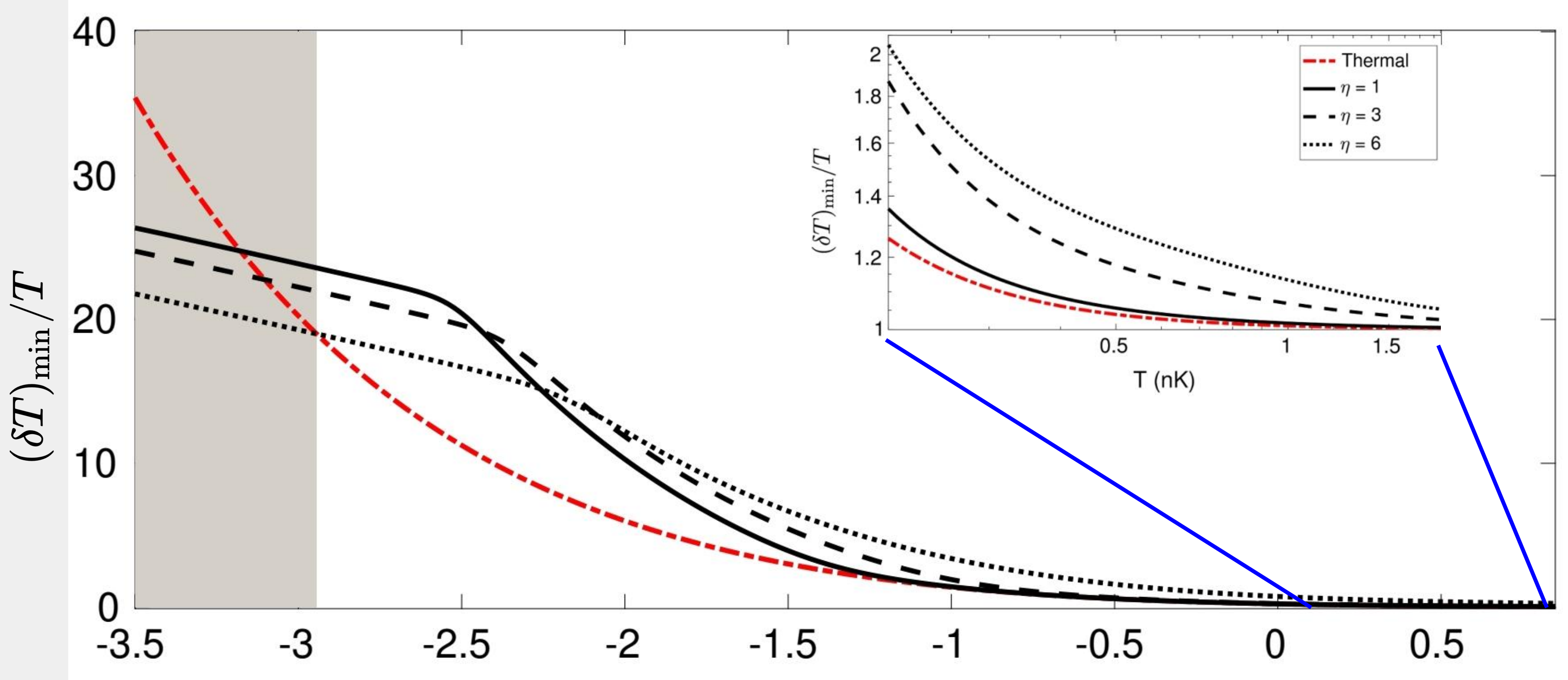
Strong Coupling Enhances Thermometry Precision?



Enhancement of low-temperature thermometry by strong coupling

Luis A. Correa, Martí Perarnau-Llobet, Karen V. Hovhannisyan, Senaida Hernández-Santana, Mohammad Mehboudi, and Anna Sanpera
Phys. Rev. A **96**, 062103 – Published 4 December 2017

Strong Coupling Enhances Thermometry Precision?



T (nK)

Quantum optics theory @ ICFO

Univ. of Exeter

A.Lampo
ICFO

C.Charalambos
ICFO

M.A.Garcia-March
ICFO

M.Lewenstein
ICFO

L.A.Correa

ICFO

Fundació Privada
MIR-PUIG

Generalitat
de Catalunya

MINISTERIO
DE ECONOMÍA
Y COMPETITIVIDAD

Fundació Privada
CELLEX

Summary and **Outlook**

- ➔ **Optimal:** relative error 14%; # of measure. 100
- ➔ **Suboptimal**: X^2; relative error 18%; # of measure. 400
- ❑ Extension to **2D** and **3D** BEC?
- ❑ Exploring **double-impurities**, and multiple impurities?
  - ❑ The role of quantum **correlations** and **interaction**?

1. PRL **122**, 030403 (2019)
2. arXiv:1811.03988 *(J.Phys.A)*

Summary and **Outlook**

- ➔ **Optimal:** relative error 14%; # of measure. 100
- ➔ **Suboptimal:** X^2; relative error 18%; # of measure. 400
- ❑ Extension to **2D** and **3D** BEC?
- ❑ Exploring **double-impurities**, and multiple impurities?
    - ❑ The role of quantum **correlations** and **interaction**?

1. PRL **122**, 030403 (2019)
2. arXiv:1811.03988 (*J.Phys.A*)

Thanks for your attention

Which Regime Is Exactly Solvable?

Treating the model is not easy in general!

- The BEC approximation: Almost all atoms are in the ground state $N - N_0 \ll N$
- Thomas Fermi density profile: $n(x) = \frac{\mu}{g_B B}(1 - x^2/R^2)$

The Hamiltonian of the BEC can be mapped into **non-interacting Bogoliubov modes**!

- Linear interaction: $\langle x \rangle = 0, \delta x \ll R$