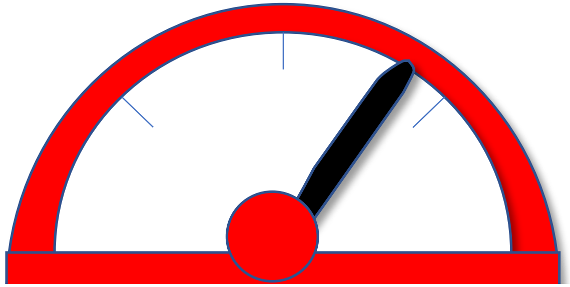
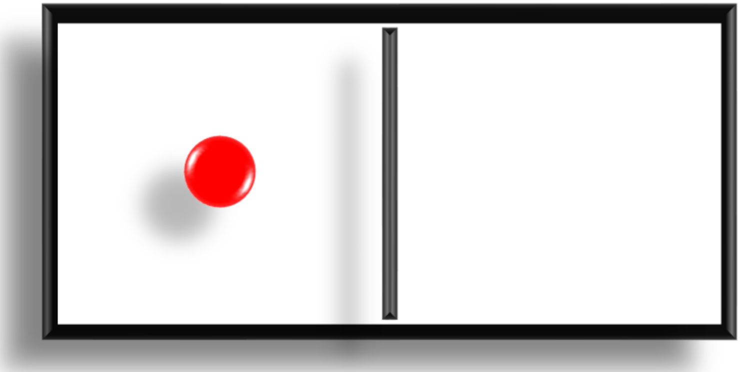
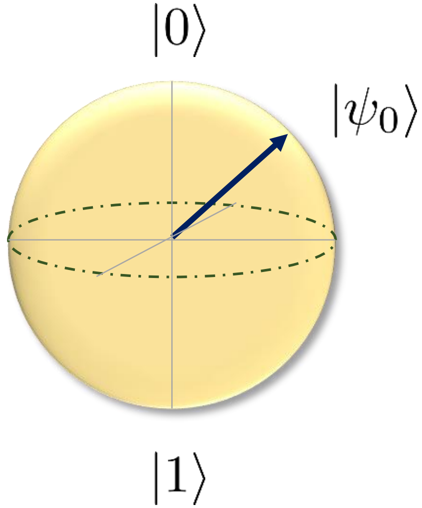
Absolute irreversibility and continuous quantum measurement: a fluctuation theorem perspective

Sreenath K. Manikandan

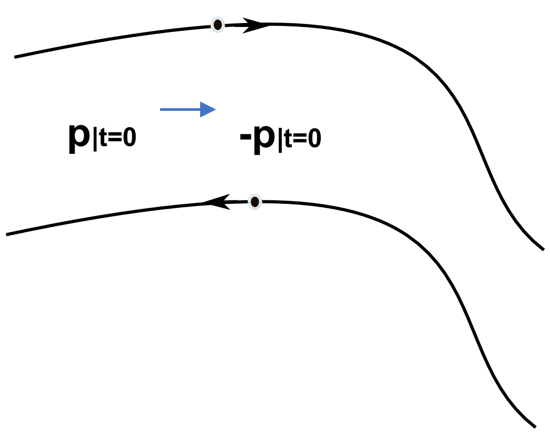






Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Reversal of motion (time reversal)



Classical mechanics:

$$\frac{\partial q_i}{\partial t} = \frac{\partial \mathcal{H}}{\partial p_i}, \qquad \frac{\partial p_i}{\partial t} = -\frac{\partial \mathcal{H}}{\partial q_i},$$

Quantum mechanics:

$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{\mathcal{H}}\Psi,$$

$$\Theta = \hat{U}\mathrm{K} \qquad \Theta\mathbf{x}\Theta^{-1} = \mathbf{x} \qquad \Theta\mathbf{p}\Theta^{-1} = -\mathbf{p} \qquad \Theta\mathbf{J}\Theta^{-1} = -\mathbf{J} \qquad \Theta\mathbf{S}\Theta^{-1} = -\mathbf{S}$$

Quantum measurement:

$$\hat{A} = \sum_n a_n |a_n\rangle\langle a_n|$$

$$|\Psi\rangle = \sum_n \alpha_n |a_n\rangle \longrightarrow$$

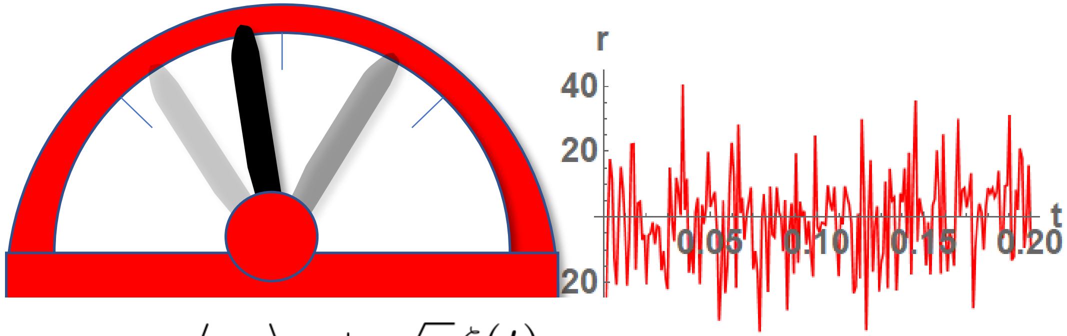
$$\begin{array}{l} \vdots \\ \longrightarrow a_2 \\ \longrightarrow a_3 \\ \longrightarrow a_4 \\ \vdots \end{array} \qquad p_3 = |\alpha_3|^2$$

$$|\Psi'\rangle = |a_n\rangle\langle a_n||\Psi\rangle \propto |a_n\rangle.$$

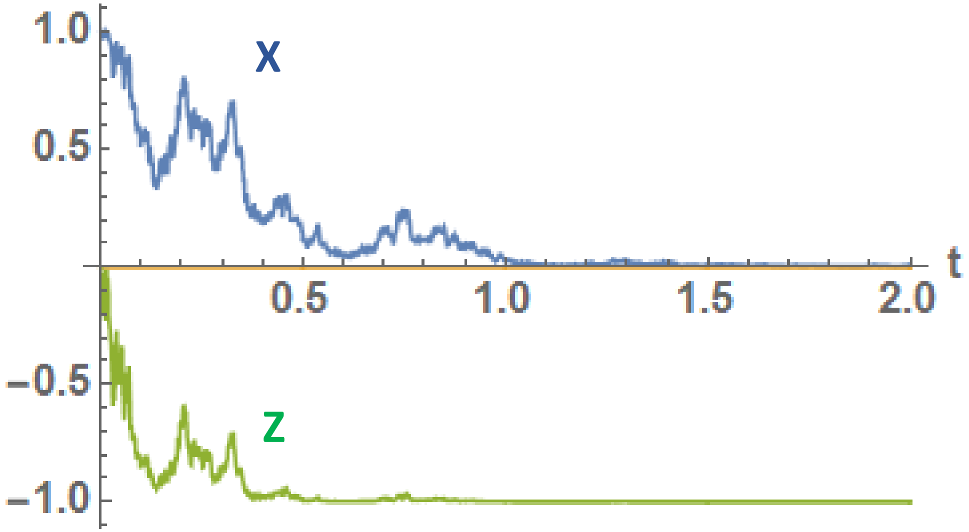
Sakurai, Jun John, and Eugene D. Commins. "Modern quantum mechanics, revised edition." (1995): 93-95.

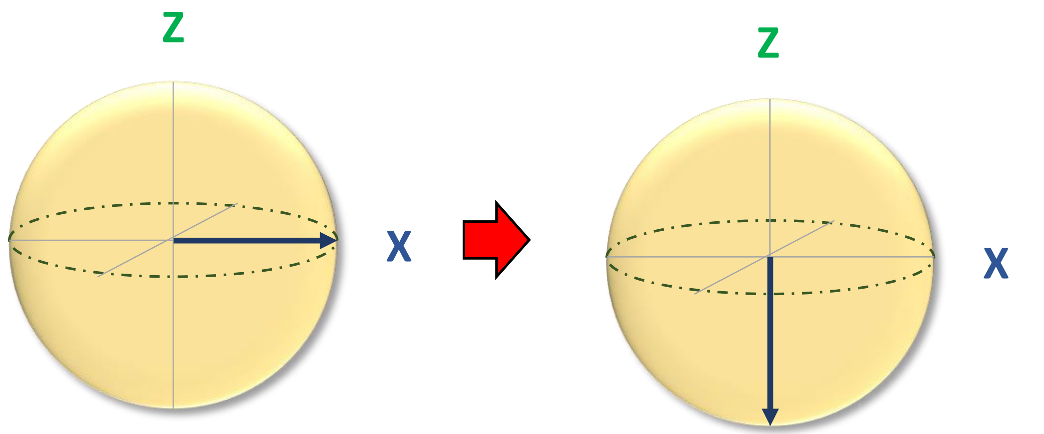
Example: Continuous weak measurement of $\hat{\sigma}_z$

$$M_z(r_n) = \left(\frac{dt}{2\pi\tau}\right)^{\frac{1}{4}} e^{-\frac{dt}{4\tau}(r_n-\sigma_z)^2}$$



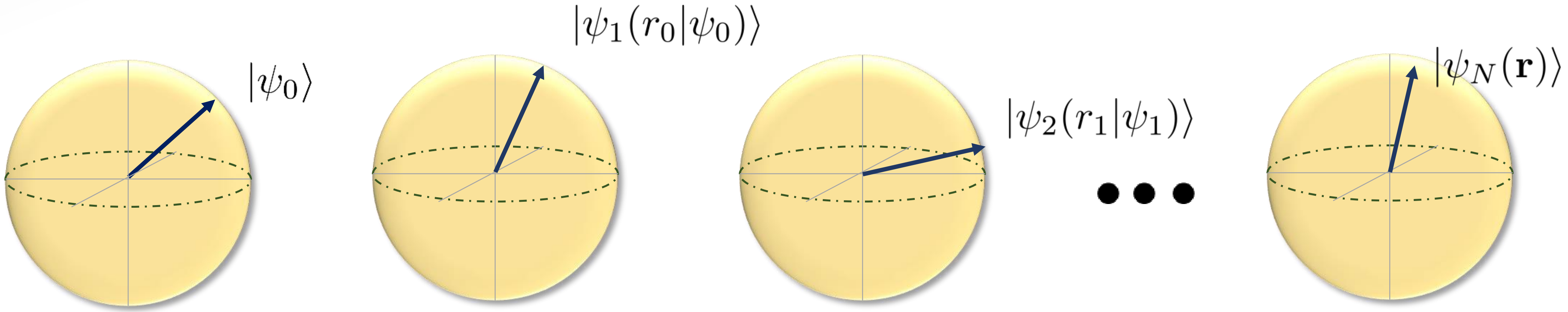
Readout $r_n = \langle\sigma_z\rangle_n + \sqrt{\tau}\zeta(t)$





Reversibility at each step: $M_z(-r_n)M_z(r_n) \propto \hat{1}$

A quantum trajectory



A quantum trajectory is the sequence:

$$\Gamma_{|\psi_0,\mathbf{r}} \equiv \{\psi_0,\ \psi_1(r_0|\psi_0),\ \psi_2(r_1|\psi_1)\ \ldots\ \psi_N(\mathbf{r})\},\ \psi_N(\mathbf{r}) = \psi_N[r_{(N-1)}|\psi_{(N-1)}]$$

The sequence is realized with probability density: $P_F(\Gamma_{|\psi_0,\mathbf{r}}) \equiv P_F(\mathbf{r}|\psi_0) = \|\overleftarrow{\prod_n} M(r_n)|\psi_0\rangle\|^2$

The quantum measurement reversal

We considered general measurements, and obtained the inverse measurement operator:

$$\tilde{M}(r_n) = \Theta^{-1} M(r_n)^\dagger \Theta, \qquad \text{since} \quad \tilde{M}(r_n) M(r_n) \propto \hat{1}. \qquad \Theta = -i\hat{\sigma}_y \mathrm{K}$$

For the reversed sequence of readouts, $\tilde{\mathbf{r}} = \{r_{N-n}\}_{1\leq n \leq N}$

we have the backward trajectory:

$$\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}} \equiv \{\psi_N(\mathbf{r}) \ \ldots \ \psi_0\}, \quad P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}] \equiv P_B(\tilde{\mathbf{r}}|\psi_N) = \|\overleftarrow{\prod_n} \tilde{M}(\tilde{r}_n)|\psi_N\rangle\|^2.$$

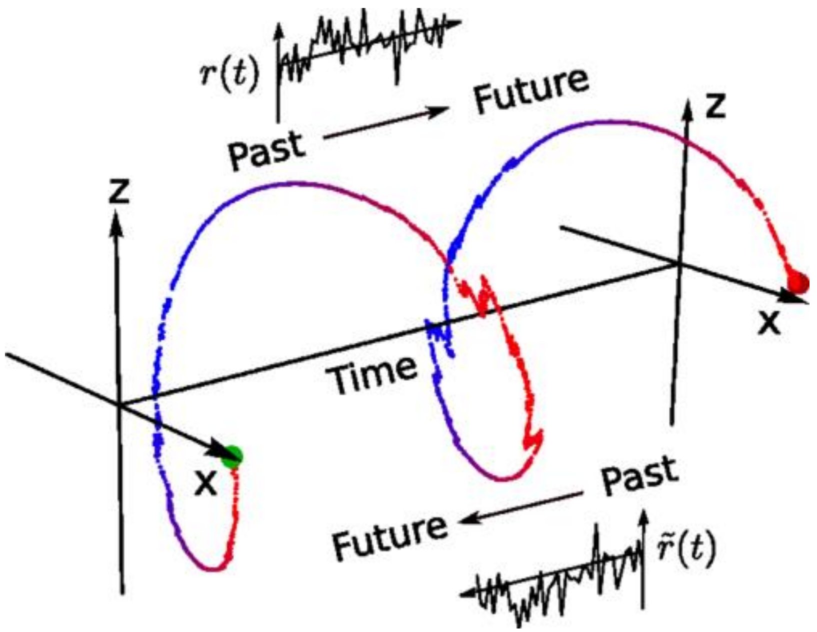
Manikandan, S.K. & Jordan, A.N. Quantum Stud.: Math. Found. (2019). https://doi.org/10.1007/s40509-019-00182-w

Time-symmetry of equations

$$M_z(r_n) = \left(\frac{dt}{2\pi\tau}\right)^{\frac{1}{4}} e^{-\frac{dt}{4\tau}(r_n-\sigma_z)^2}$$

$$r_n = \langle\sigma_z\rangle_n + \sqrt{\tau}\zeta(t)$$

$$\frac{dt}{\tau} \to 0$$



$$\frac{dx}{dt} = \Omega z - \frac{xzr}{\tau}, \qquad \frac{dy}{dt} = -\frac{yzr}{\tau}, \qquad \frac{dz}{dt} = -\Omega x + \frac{(1-z^2)r}{\tau}.$$

- **Dynamical equations time reversal invariant…!!**

Dressel, Justin, et al. "Arrow of time for continuous quantum measurement." *Physical review letters* 119.22 (2017): 220507.

The quantum measurement arrow of time

Using Bayes theorem,

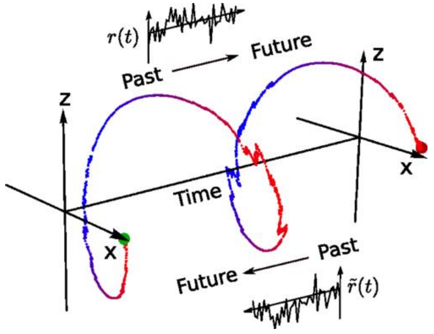
$$P(F|r(t)) = \frac{P_F(\Gamma_{|\psi_0,\mathbf{r}})P(F)}{P_F(\Gamma_{|\psi_0,\mathbf{r}})P(F)+P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}]P(B)}.$$

Assume uniform priors $P(F) = P(B) = 1/2$,

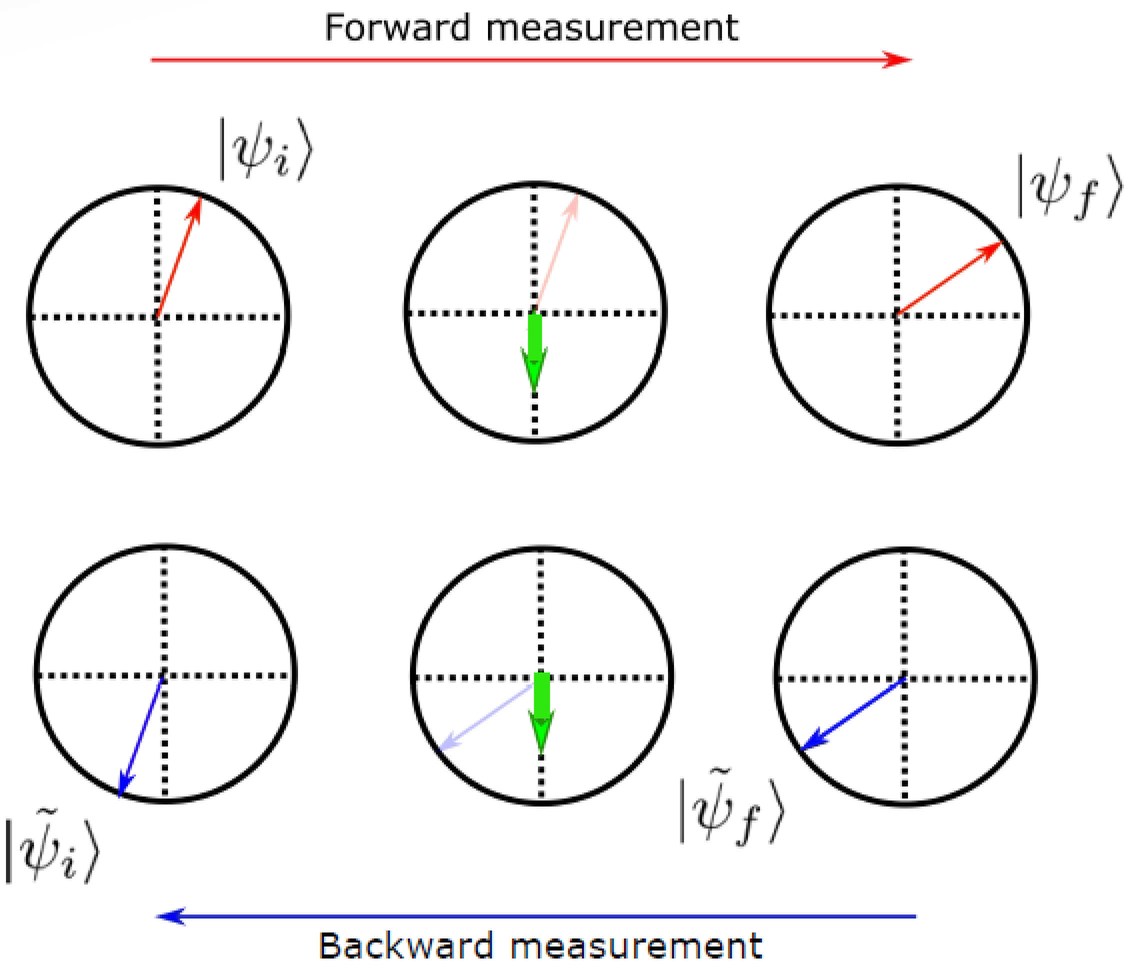
$$P(F|r(t)) = \frac{\mathcal{R}}{1+\mathcal{R}}, \quad \text{where} \quad \mathcal{R} = \frac{P_F(\Gamma_{|\psi_0,\mathbf{r}})}{P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}]}.$$

The arrow of time measure:

$$\mathcal{Q}(\Gamma_{|\psi_0,\mathbf{r}}) = \log\left\{P_F[\Gamma_{|\psi_0,\mathbf{r}}]/P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}]\right\}.$$



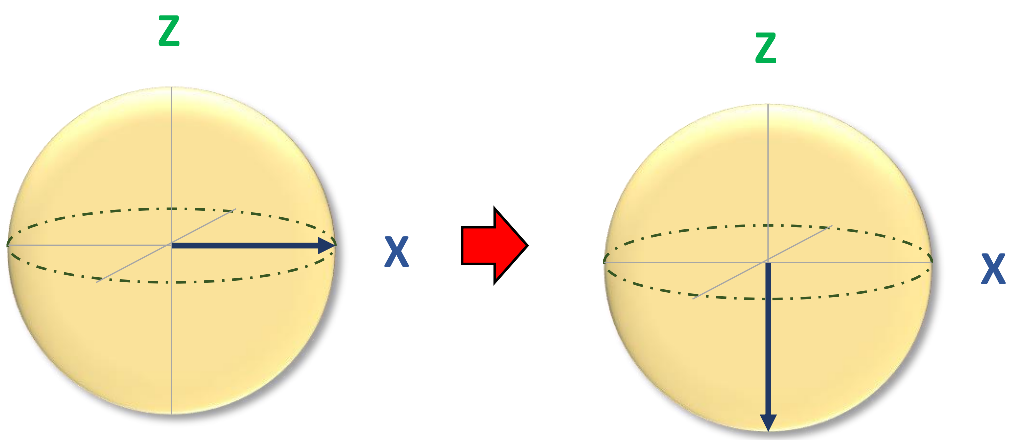
Dressel, Justin, et al. "Arrow of time for continuous quantum measurement." *Physical review letters* 119.22 (2017): 220507.



Manikandan, Sreenath K., and Andrew N. Jordan. "Time reversal symmetry of generalized quantum measurements with past and future boundary conditions." *arXiv preprint arXiv:1801.04364* (2018).

Example: Continuous weak measurement of $\hat{\sigma}_z$

- The arrow of time: $\mathcal{Q}_z(\Gamma_{|z_0,\mathbf{r}}) = 2\log\left[\,\cosh(R) + z_0\sinh(R)\,\right].$
- Here $R = dt\sum_n r_n/\tau.$



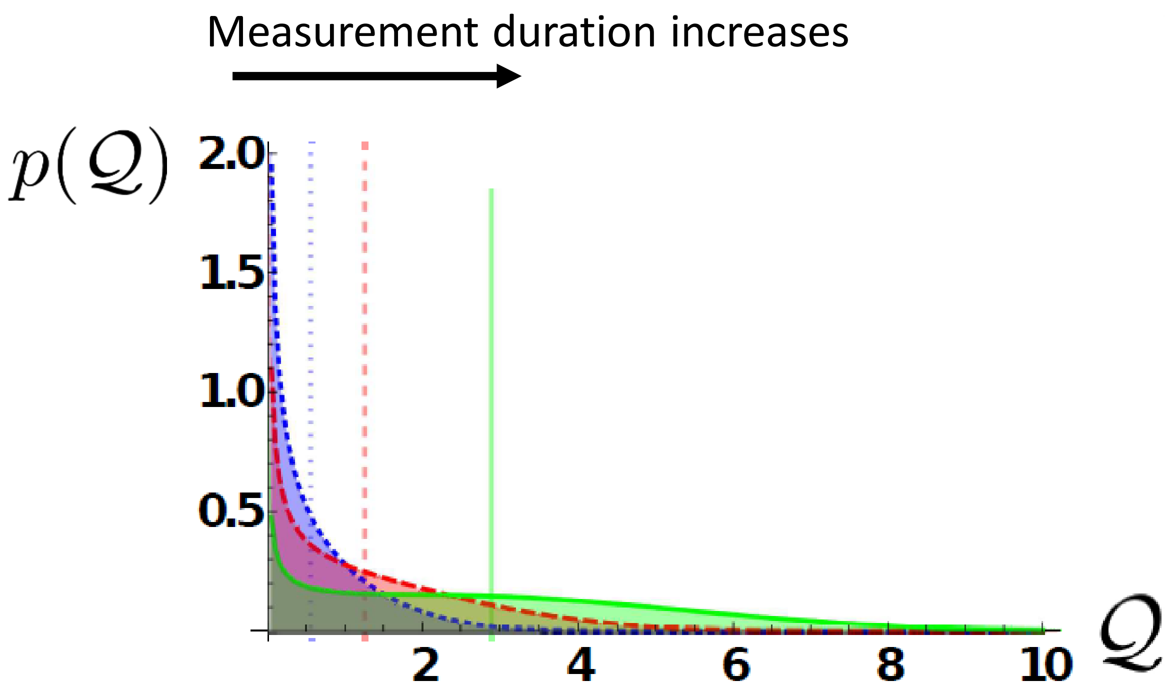
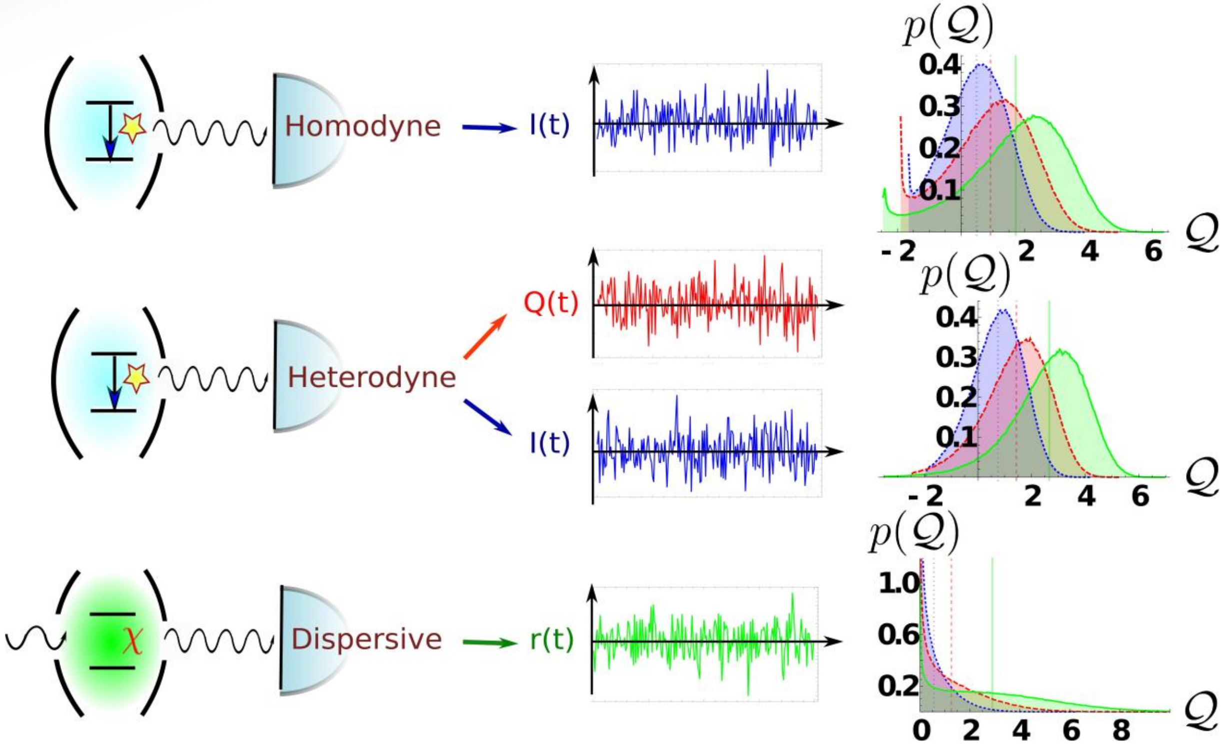


Image from: Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Probability distribution of the arrow of time



$$M_{\mathrm{Ho}}(r_n) = \frac{e^{-r_n^2/2}}{\pi^{1/4}} \begin{pmatrix} \sqrt{1-\epsilon/2} & 0 \\ \sqrt{\epsilon} r_n & 1 \end{pmatrix}$$

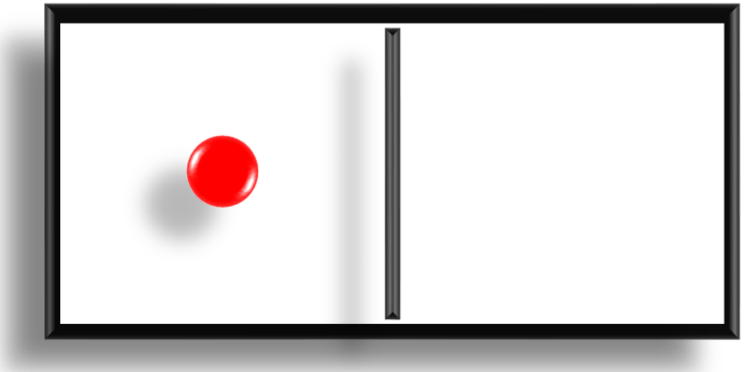
$$M_{\mathrm{He}}(r_n) = \frac{e^{-|r_n|^2/2}}{\sqrt{\pi}} \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ \sqrt{\epsilon} r_n^* & 1 \end{pmatrix}$$

Here $\epsilon = \gamma dt$. We set $\gamma^{-1} = \tau$. $T = 0.5\tau,\ \tau,\ 2\tau$. All starting from $x = 1$.

Image from: Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Thermodynamics of small systems (free expansion)

Canonical distribution with inverse temperature $\beta = \frac{1}{k_B \mathrm{T}}$.



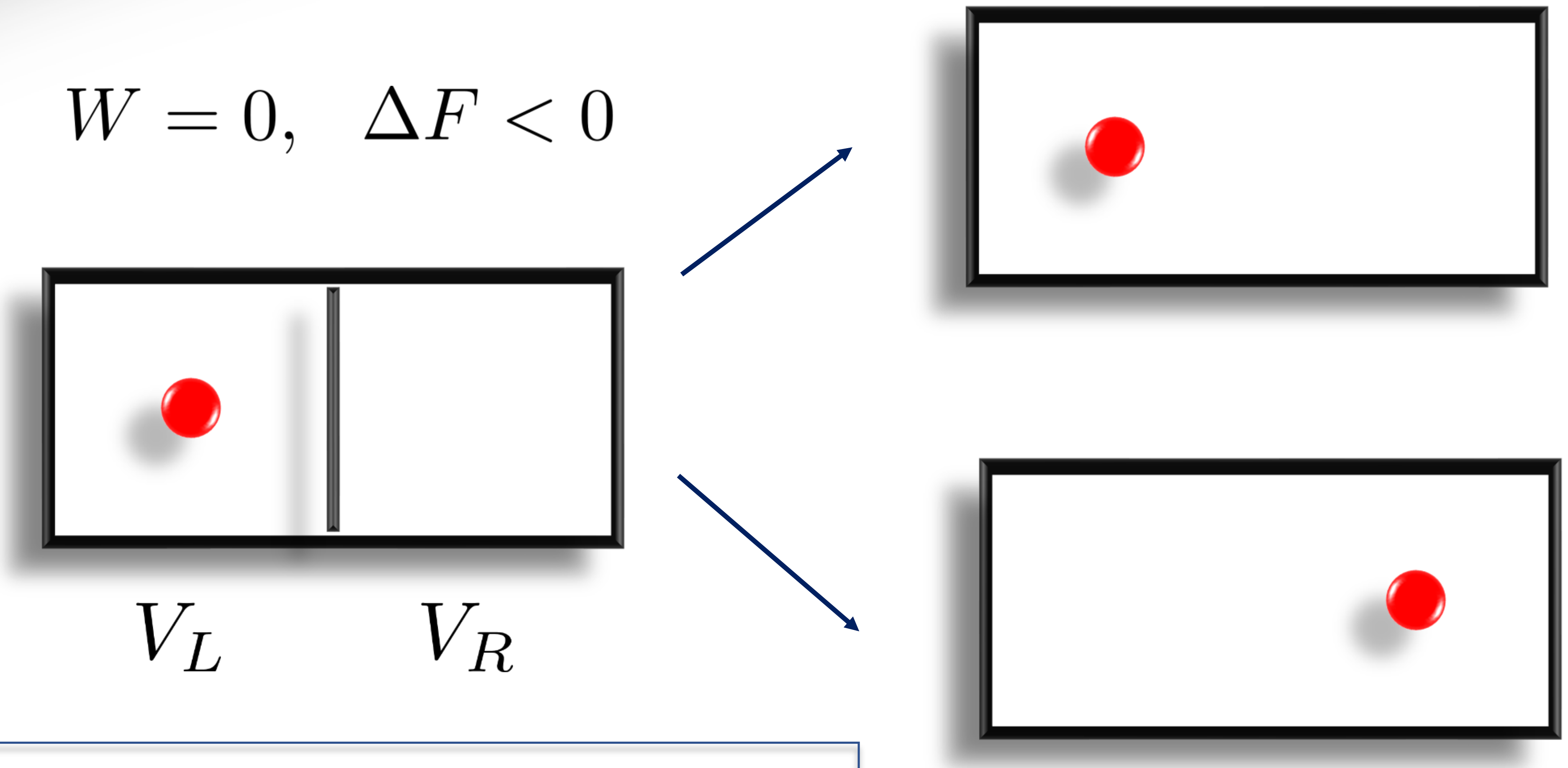
V_L $\quad$ V_R

$$F_i = -k_B \mathrm{T} \log \frac{V_L}{\lambda_{\mathrm{T}}^3}, \quad \lambda_{\mathrm{T}} = \frac{h}{\sqrt{2\pi m k_B \mathrm{T}}}.$$

Murashita, Yûto, Ken Funo, and Masahito Ueda. "Nonequilibrium equalities in absolutely irreversible processes." *Physical Review E* 90.4 (2014): 042110.

Thermodynamic arrow of time

$$W = 0, \quad \Delta F < 0$$



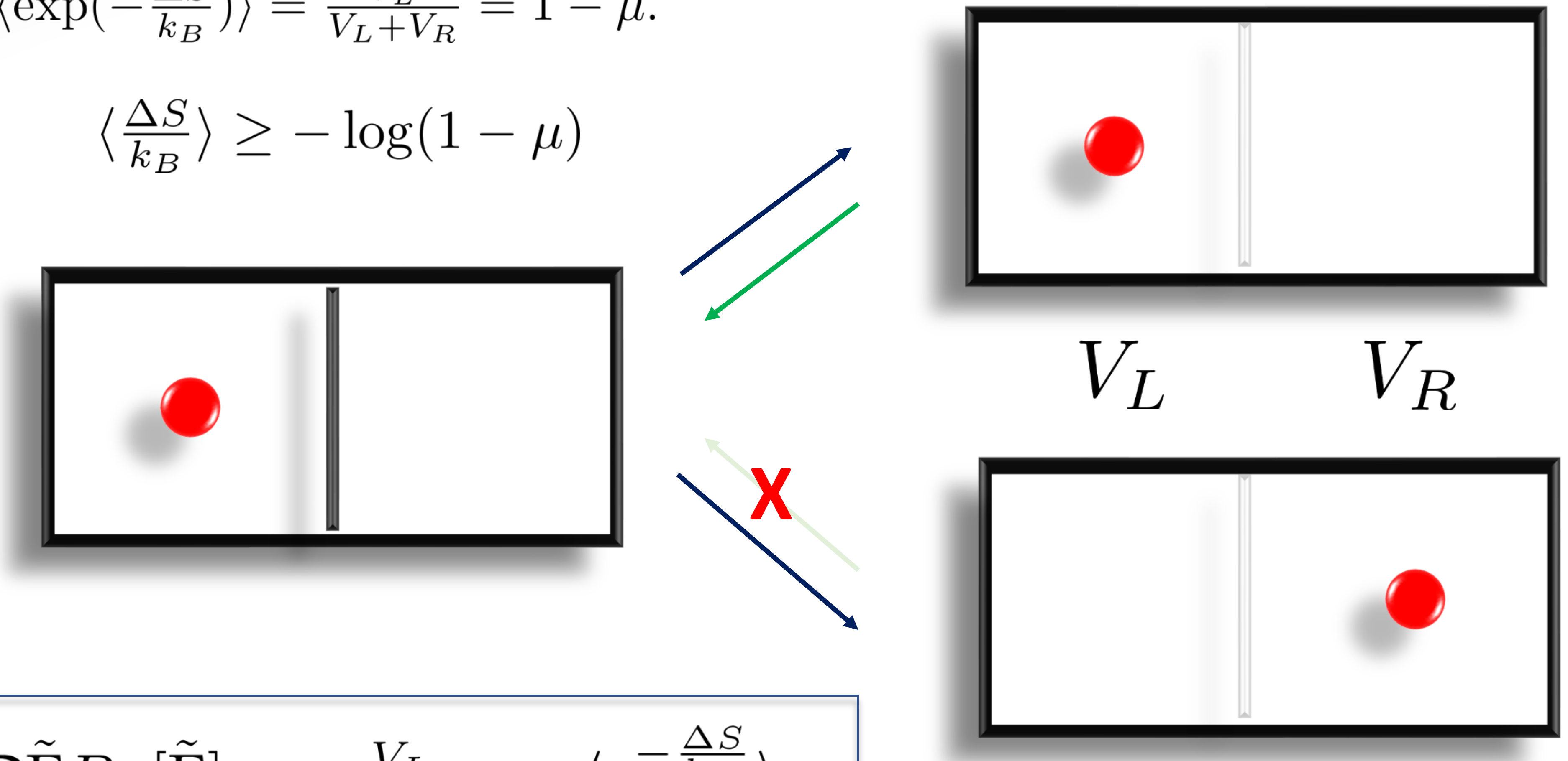
$$\boxed{\frac{\Delta S}{k_B} = \beta(W - \Delta F) > 0, \quad \beta = \frac{1}{k_B \mathrm{T}}.}$$

Murashita, Yûto, Ken Funo, and Masahito Ueda. "Nonequilibrium equalities in absolutely irreversible processes." *Physical Review E* 90.4 (2014): 042110.

Backward process

$$\langle \exp(-\frac{\Delta S}{k_B}) \rangle = \frac{V_L}{V_L + V_R} = 1 - \mu.$$

$$\langle \frac{\Delta S}{k_B} \rangle \geq -\log(1-\mu)$$



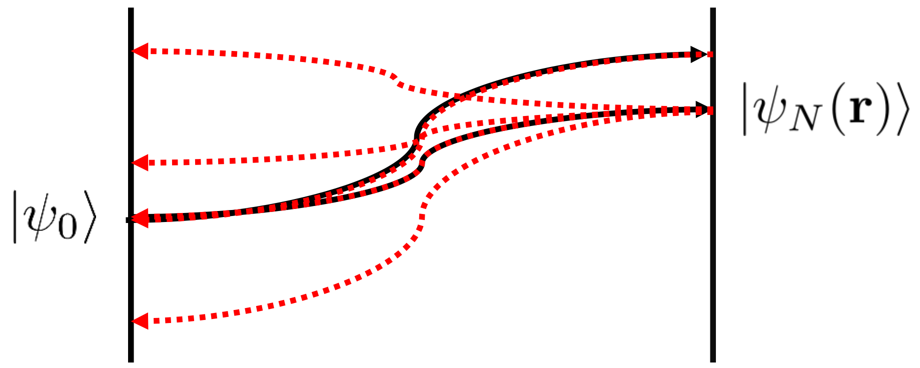
$$\boxed{\int_{V_L} \mathcal{D}\tilde{\Gamma} P_B[\tilde{\Gamma}] = \frac{V_L}{V_L + V_R} \equiv \langle e^{-\frac{\Delta S}{k_B}} \rangle}$$

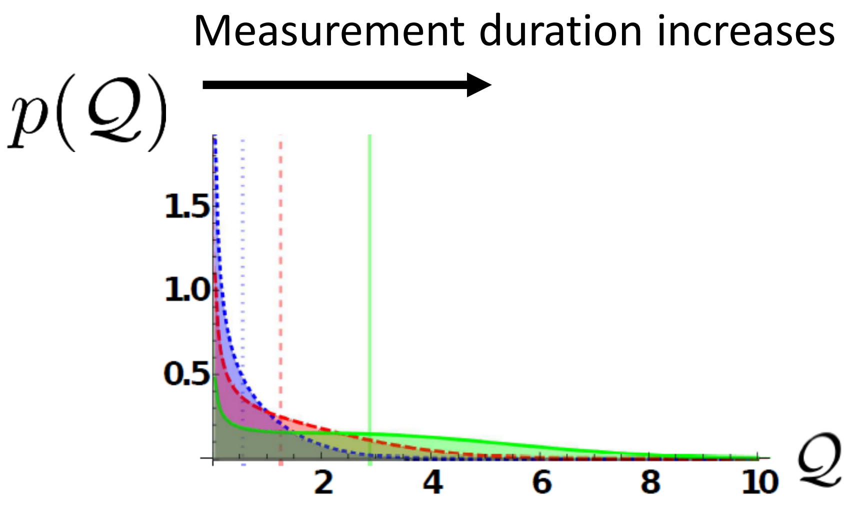
Use this approach in the quantum measurement problem!

Absolute irreversibility

$$\langle e^{-\mathcal{Q}(\Gamma)}\rangle = 1-\mu, \qquad \mu = 1 - \int D\tilde{\Gamma} P_B(\tilde{\Gamma}).$$

$$\text{Using} \quad e^{-\langle \mathcal{Q}\rangle} \leq \langle e^{-\mathcal{Q}}\rangle, \quad \text{we find} \quad \langle \mathcal{Q}\rangle \geq -\log(1-\mu),$$





The average in three distributions can be explained using trajectories which do not go back. We find that is not the complete story.

Strictly positive average arrow of time!

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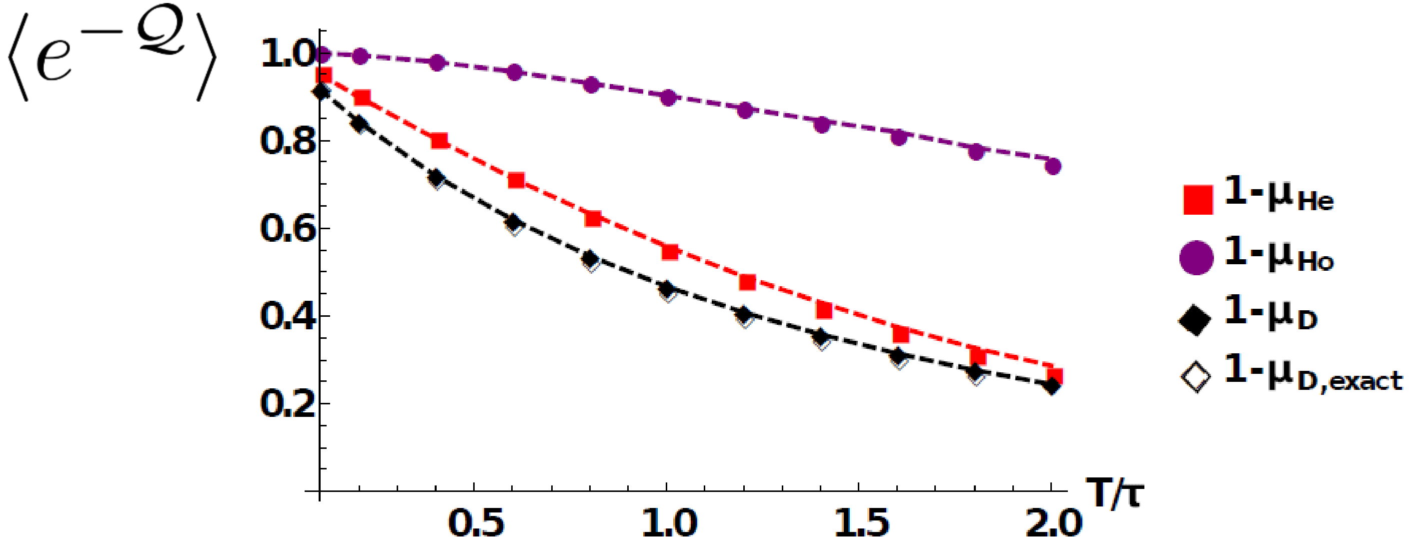
$$\langle e^{-\mathcal{Q}(\Gamma)}\rangle = 1-\mu, \qquad \mu = 1-\int D\tilde{\Gamma} P_B(\tilde{\Gamma}) = \int D\mathbf{r} \frac{|\langle\bar{\psi}_0|\mathcal{M}^\dagger(\mathbf{r})\mathcal{M}(\mathbf{r})|\psi_0\rangle|^2}{\langle\psi_0|\mathcal{M}^\dagger(\mathbf{r})\mathcal{M}(\mathbf{r})|\psi_0\rangle}.$$

- **A complementary description is contained in μ**
- **Reflects the many to one mapping aspect of the quantum measurement problem.**

Results from: Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Simulated experiments

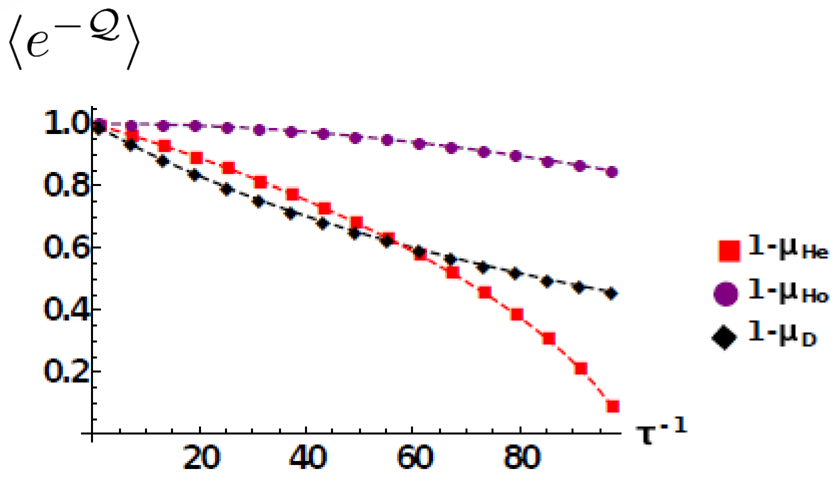


We set $\gamma^{-1} = \tau$. All starting from $x = 1$.

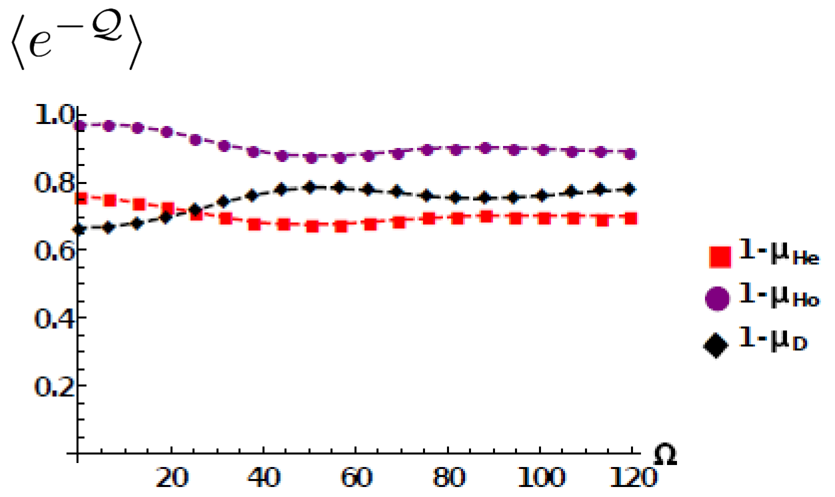
The quantum state collapse becomes more and more absolutely irreversible with the measurement duration

Results from: Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Simulated experiments



(a)



(b)

(a) Single step measurements where we set $\gamma = \tau^{-1}$, and (b) with Rabi drive, at $T = 0.5\tau$. We used $\hat{U}(\delta t) = e^{-i\frac{\Omega \hat{\sigma}_y}{2}\delta t}$.

The quantum state collapse becomes more and more absolutely irreversible with increasing measurement strength.

Results from: Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Conclusions & forward

- We discussed the idea of measurement reversal and introduced the notion of a statistical arrow of time for arbitrary weak measurements on a qubit.
- We found that the quantum measurement dynamics is absolutely irreversible, similar to the free expansion of a single particle gas.
- This absolute irreversibility is deeply connected to information acquisition in quantum measurements, and to the many-to-one mapping aspect of the quantum measurement problem.
- Experimental verification on superconducting qubits: Harrington, P. M., et al. "Characterizing a statistical arrow of time in quantum measurement dynamics." *arXiv preprint arXiv:1811.07708* (2018).
- Quantum measurement process is inherently probabilistic, which disturbs the system being measured similar to the uncontrollable fluctuations created by a thermal bath. Our microscopic reversibility approach to the measurement problem open new directions for probing the resource nature of quantum coherence and correlations, similar to energy in the thermodynamic sense.

Manikandan, Sreenath K., Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Thank you for your time.