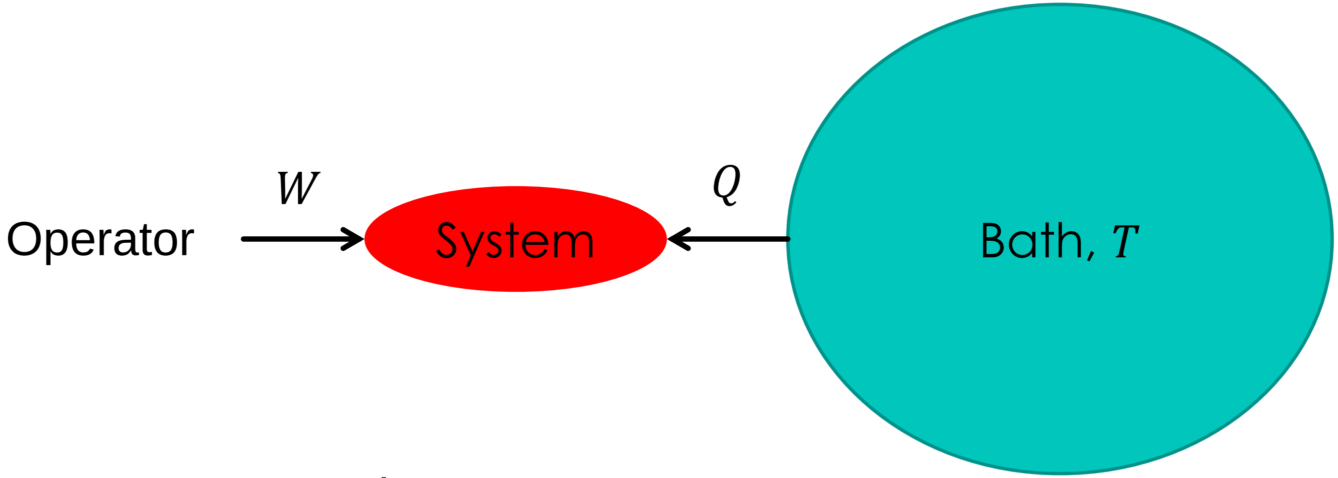
Aalto University
School of Science

Maximizing local violations of the second law with a single-electron transistor

Or: "How much work can we extract from thermal fluctuations at the single thermodynamic trajectory level ?"

O. Maillet, P. A. Erdman, V. Cavina, B. Bhandari, E. Mannila, J. Peltonen, A. Mari, F. Taddei, C. Jarzynski, V. Giovannetti, J. P. Pekola

Context



System energy change $\Delta U = W + Q$

- Jarzynski equality:

$$\left\langle e^{-W/k_B T} \right\rangle = e^{-\Delta F/k_B T}$$

C. Jarzynski, Phys. Rev. Lett. **78,** 2690 (1997)

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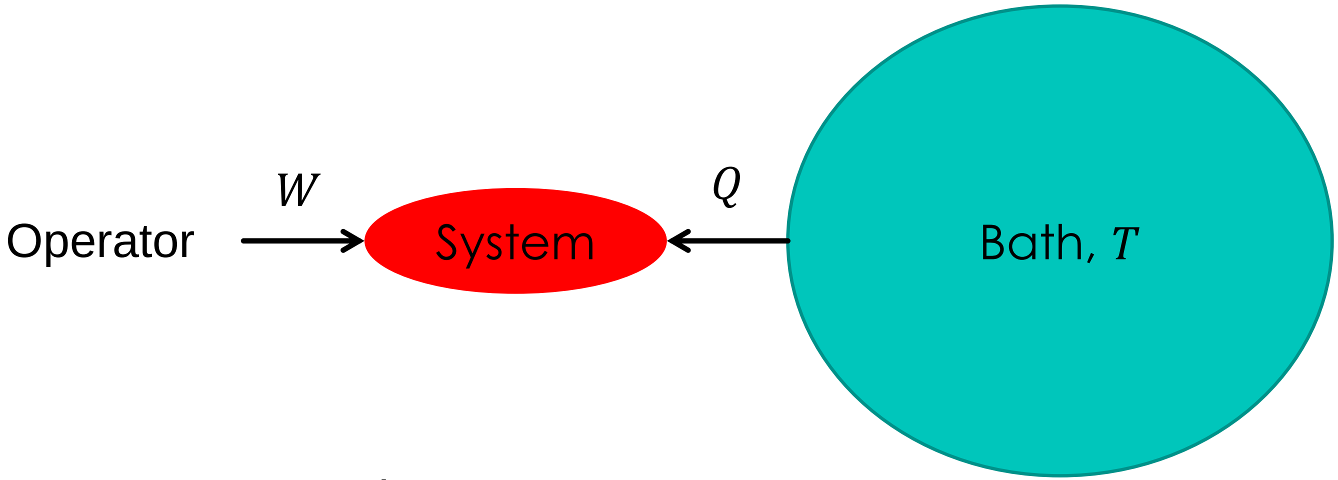

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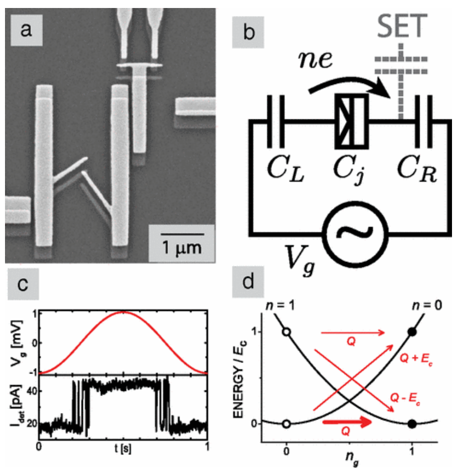
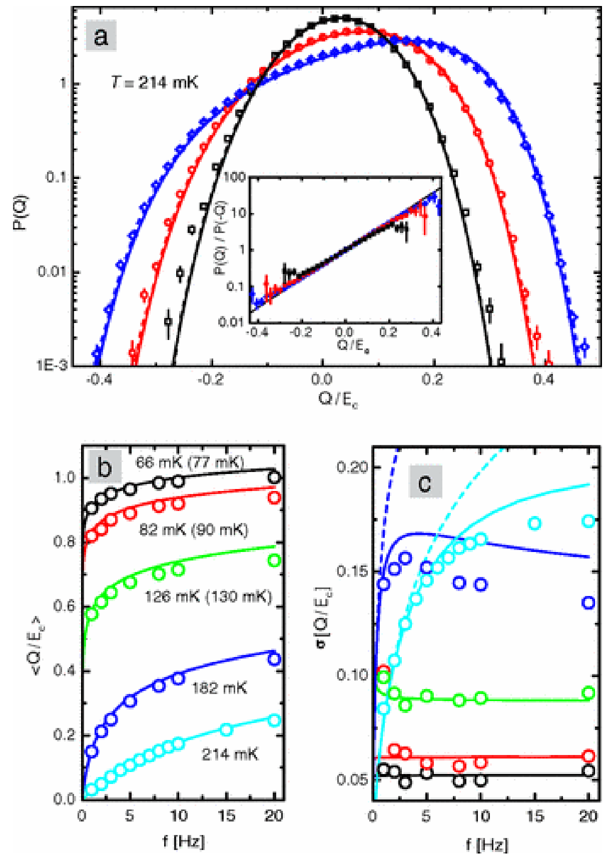
→ Work can be extracted beyond ΔF **at the single trajectory level**

Experiments with mesoscopics

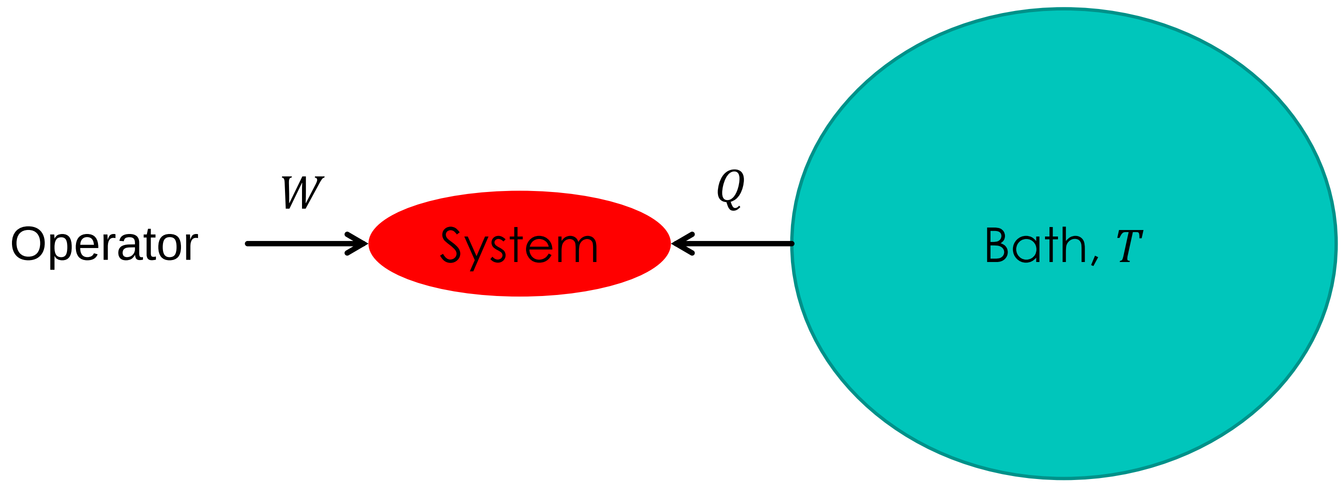
- Test of Jarzynski equality with a single electron box
- Gate driving cycle, measurement of heat exchange (tunneling events) during the cycle





O.-P. Saira *et al.*, Phys. Rev. Lett. **109,** 180601 (2012)

Context



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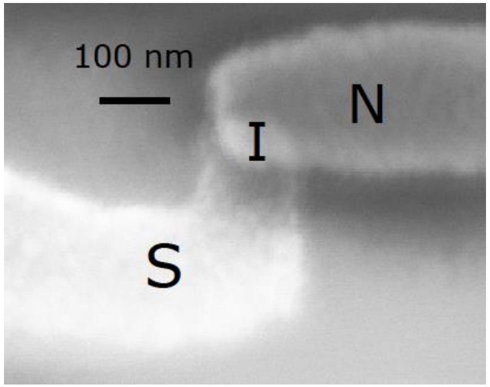
→ **How to optimize (= maximize, make likely…) single-shot work extraction ?**

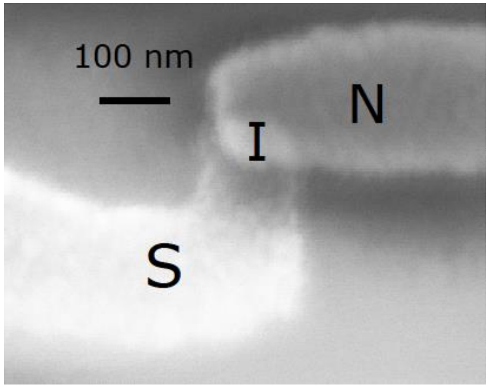
N. Y. Halpern et al., New. J. Phys. **17** 095003 (2015)
V. Cavina et al., Sci. Rep. **6**, 29282 (2016)

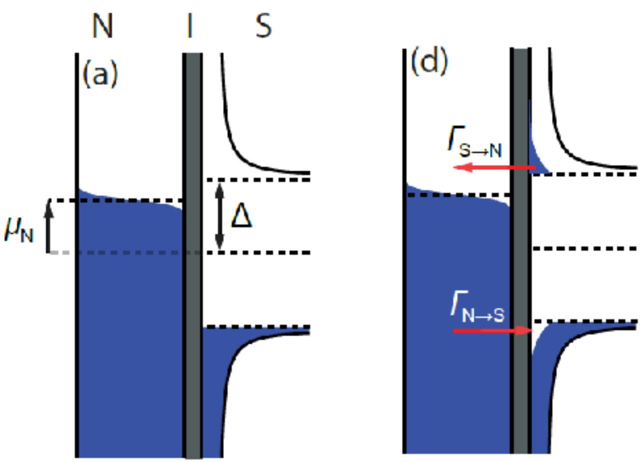
Normal-Insulator-Superconductor tunnel junction



- Fabrication: EBL+ angle evaporation + low pressure oxidation of Aluminum

NIS tunnel junction





$T_s = 0$ $\quad$ $T_s \neq 0$

- Fabrication: EBL+ angle evaporation + low pressure oxidation of Aluminum

- Thermally activated tunneling rate:

$$\Gamma_{S\to N}(\mu_N) = \frac{1}{e^2 R_T}\int dE\,[1-f(E-\mu_N,T_N)]\,f(E,T_s)\,n_s(E)$$

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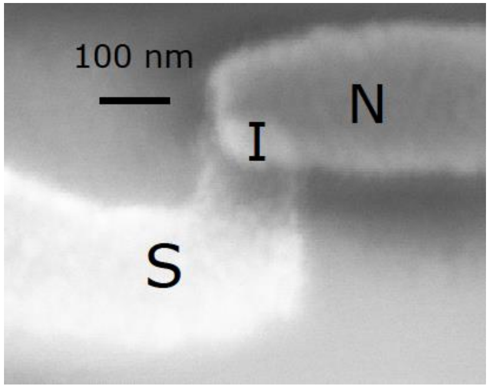

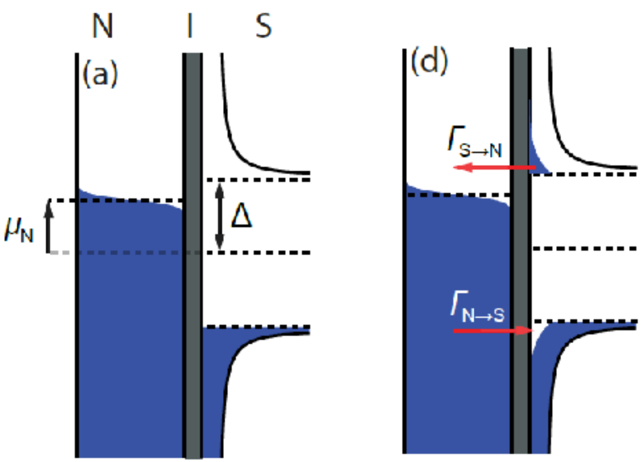

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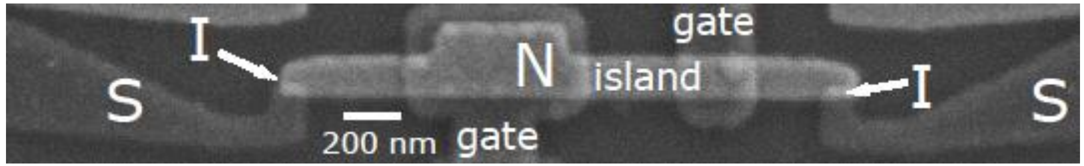
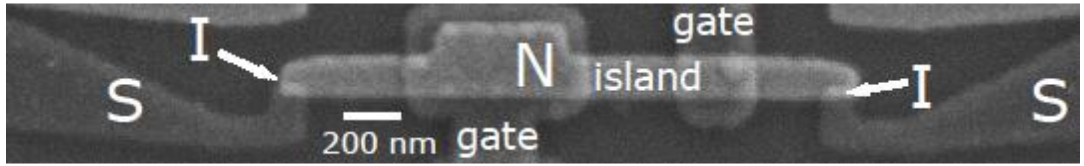
- Use of superconductor: energy gap = low rates

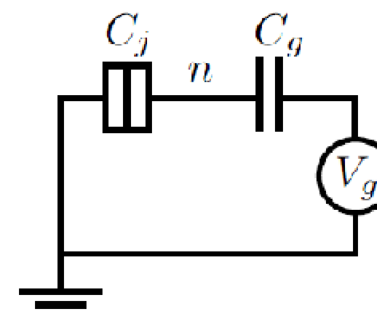
Coulomb blockade



- Small metallic island, with small capacitance C_Σ dominated by tunnel junctions
- Ultrasmall junctions (area < 100 nm x 100 nm): $C_\Sigma \leq 1$ fF
- Energy cost of tunneling: **charging energy** $E_c = \frac{e^2}{2C_\Sigma}$
- Two junctions: SINIS transistor for transport measurements → E_c, R_T can be measured

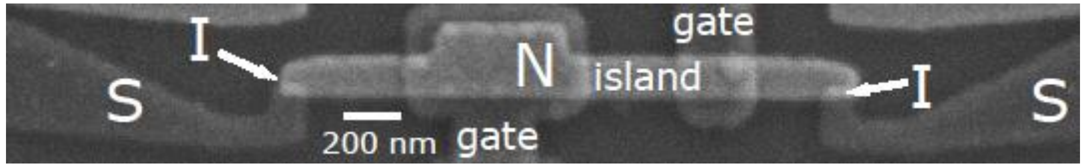
Single electron box





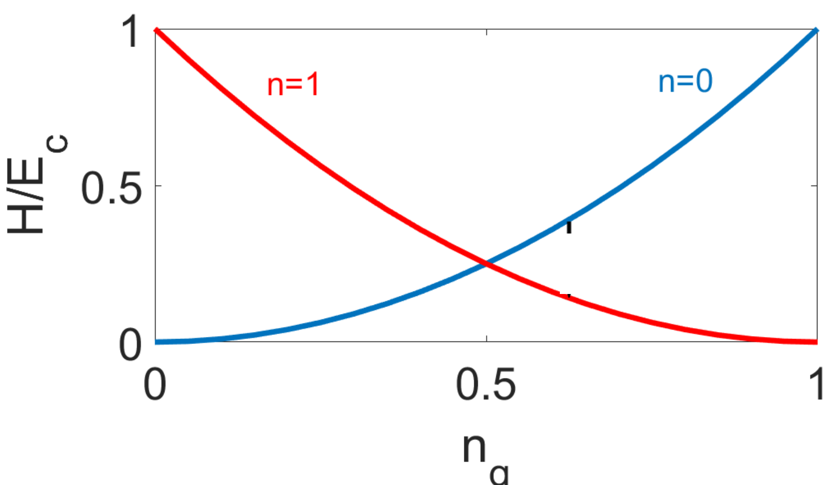
- Hamiltonian for equivalent circuit: $H(n, n_g) = E_C(n - n_g)^2$
- Tunable electrostatic energy with gate voltage $n_g = \frac{C_g V_g}{e}$
- $E_C \sim 1$ K: occupation of two charge states $N_0 + n, n = 0{,}1$ for $n_g \in [0;1]$

Single electron box



Normal-Insulator-Superconductor tunnel junctions + Coulomb blockade

$$H(n, n_g) = E_C(n - n_g)^2$$



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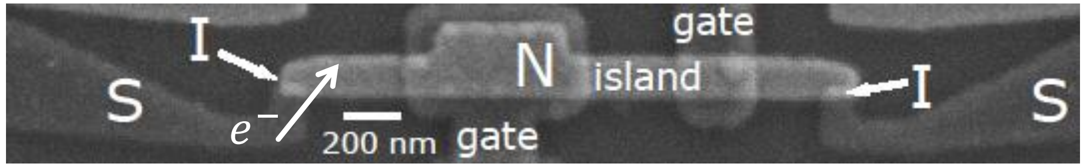

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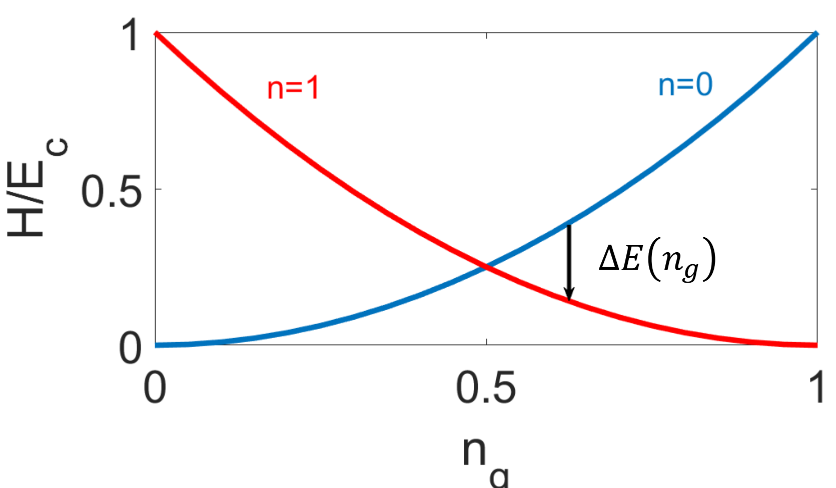
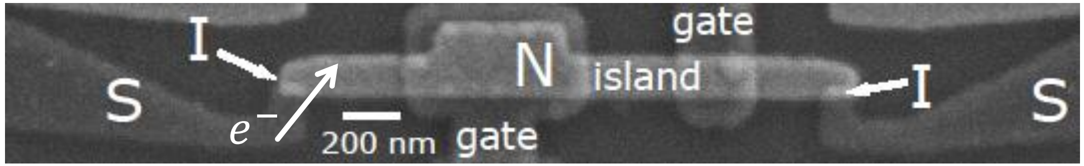


$$H(n, n_g) = E_C(n - n_g)^2$$

- Electron tunneling = heat exchange

$$\Delta E(n_g) = E_C(1 - 2n_g)$$

Single electron box



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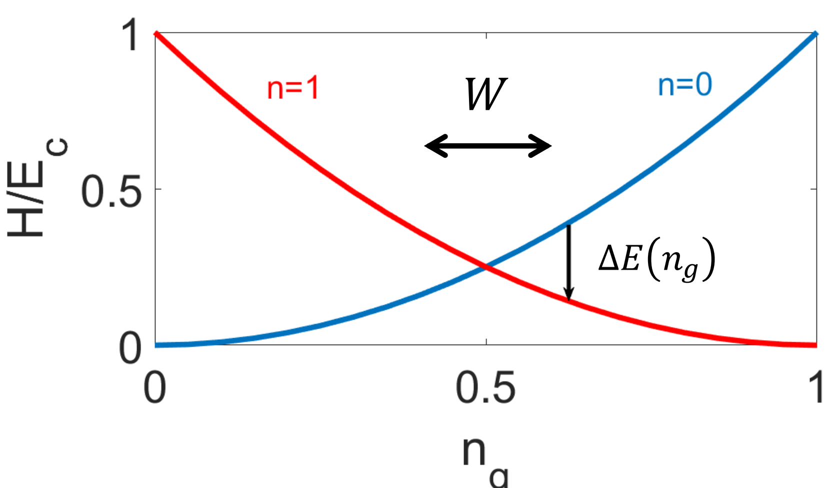


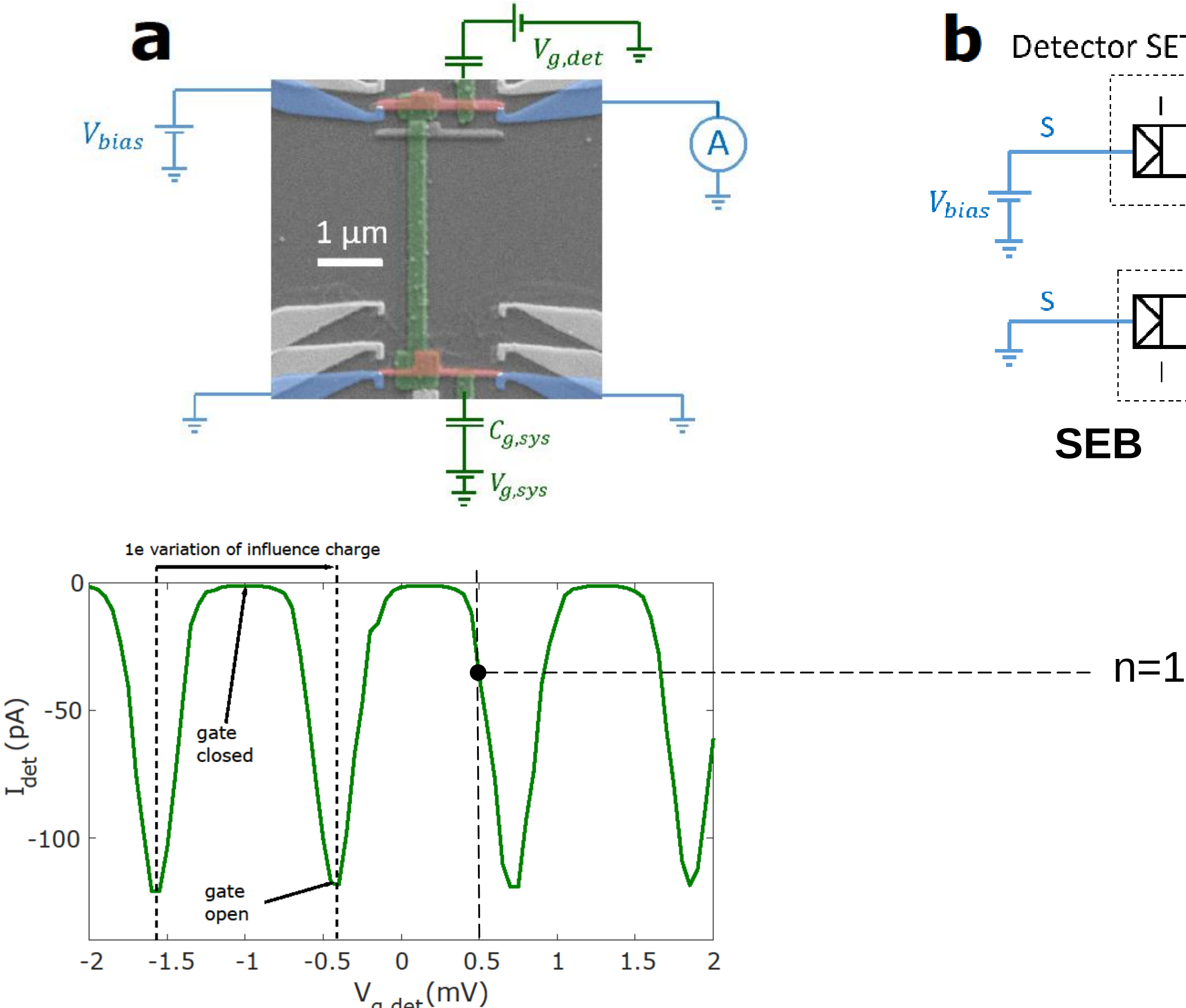
$$H(n, n_g) = E_C(n - n_g)^2$$

- Electron tunneling = heat exchange **(stochastic)**

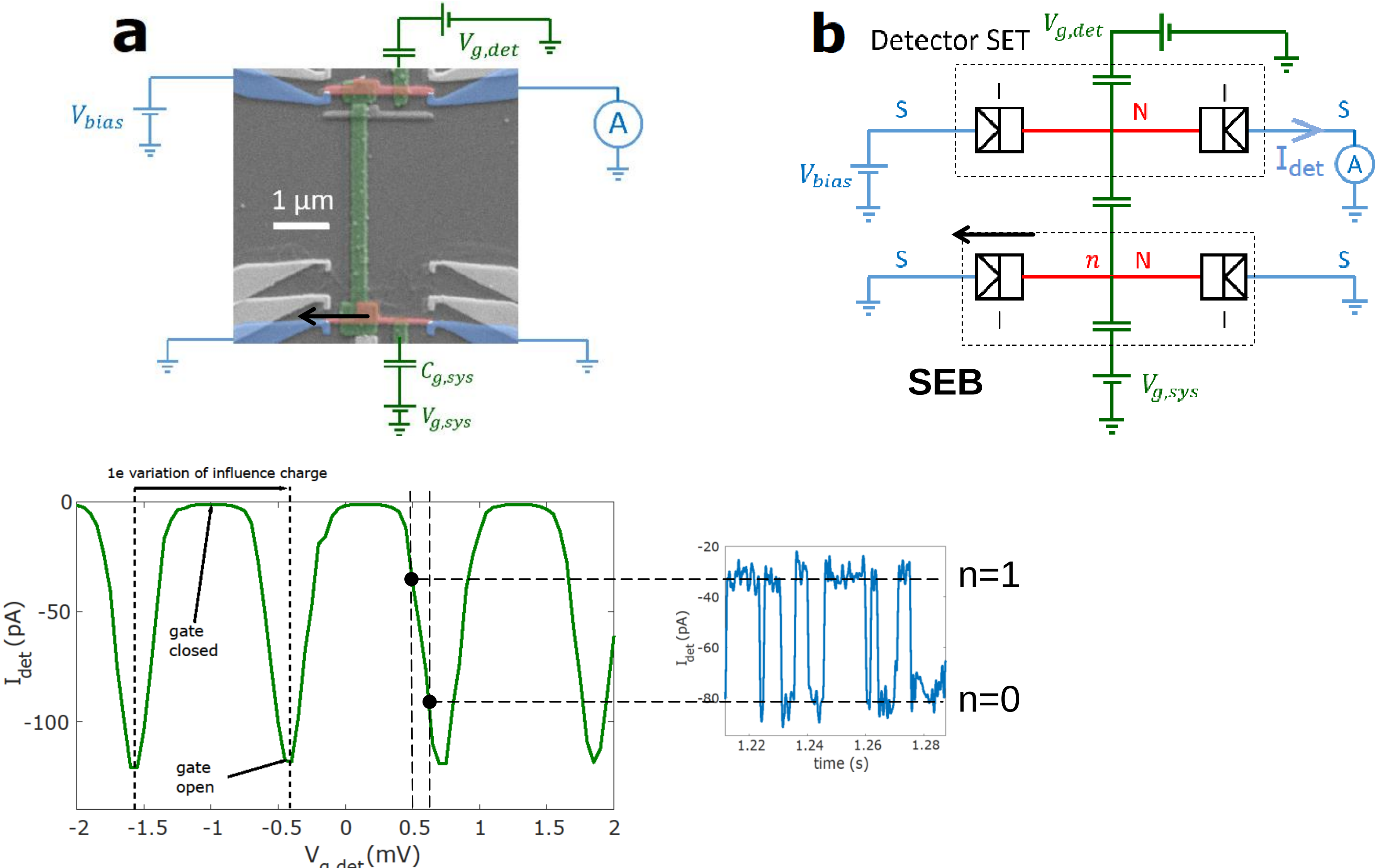
$$\Delta E(n_g) = E_C(1 - 2n_g)$$

- Gate driving $n_g(t)$ = work applied

$$W[n(t), n_g(t)] = \int dt \dot{n}_g(t) \frac{\partial H}{\partial n_g}$$



a
$V_{g,det}$
V_{bias}
A
1 µm
$C_{g,sys}$
$V_{g,sys}$
b
Detector SET
$V_{g,det}$
S
N
S
V_{bias}
I_{det}
A
S
n
N
S
SEB
$V_{g,sys}$
1e variation of influence charge
gate closed
gate open
n=1
I_{det} (pA)
0
-50
-100
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$V_{g,det}$(mV)



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n
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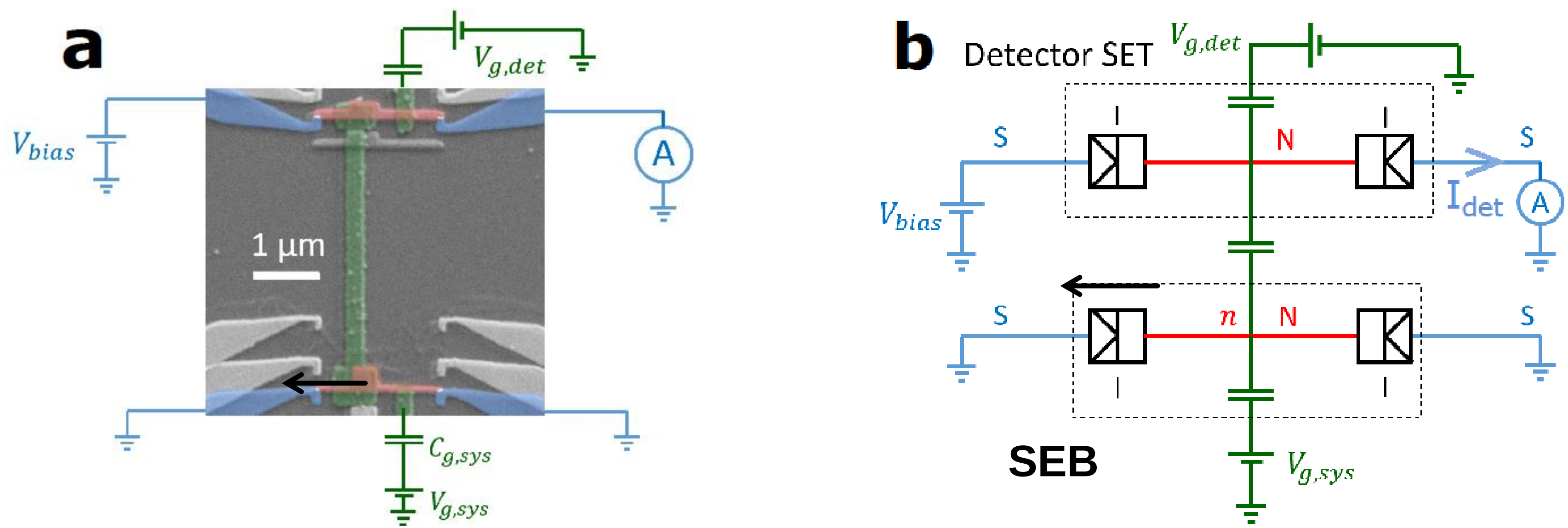
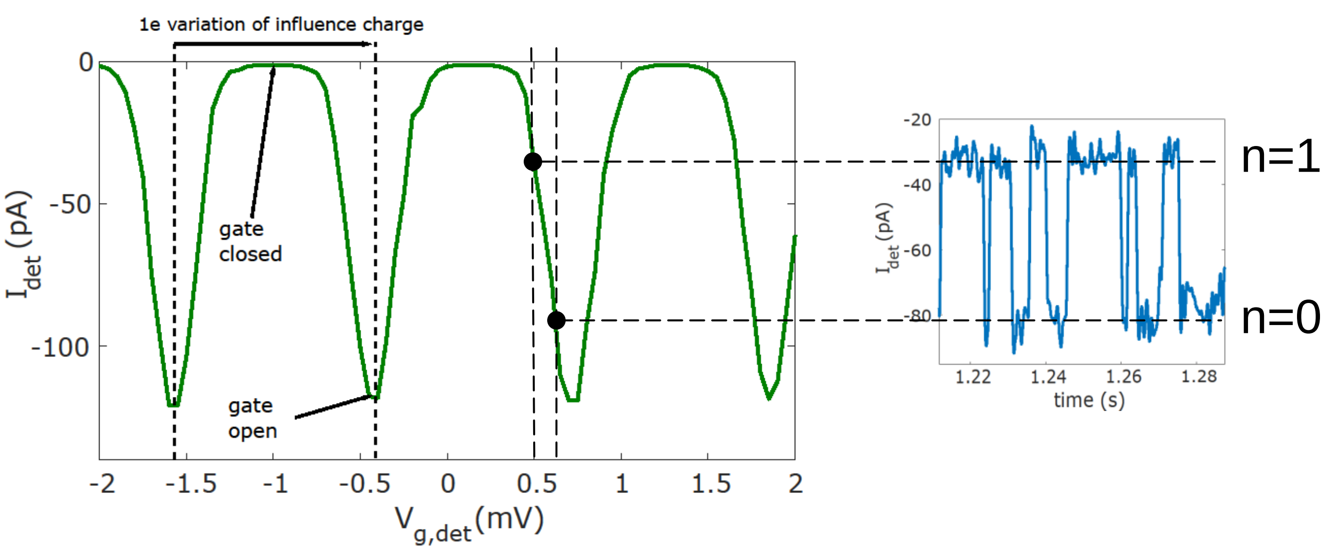
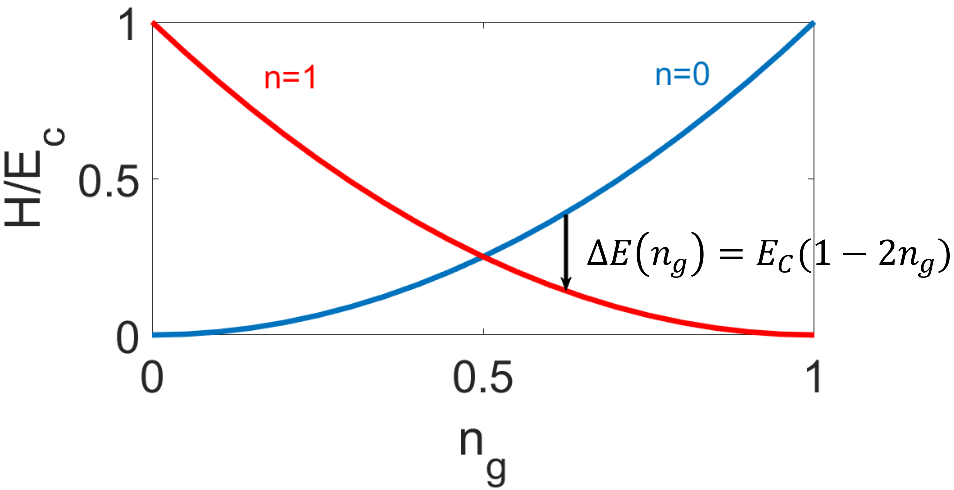
SEB
$V_{g,sys}$
1e variation of influence charge
gate closed
gate open
I_{det} (pA)
$V_{g,det}$(mV)
n=1
n=0
time (s)

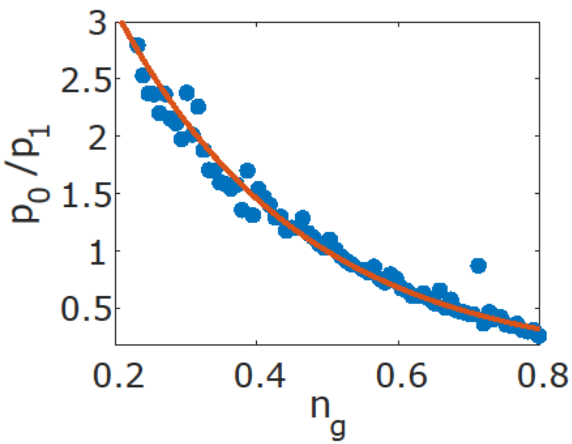




- SET detection: sensitive electrometry
- Real-time monitoring of electron jumps in the SEB (« system »)

Single electron box





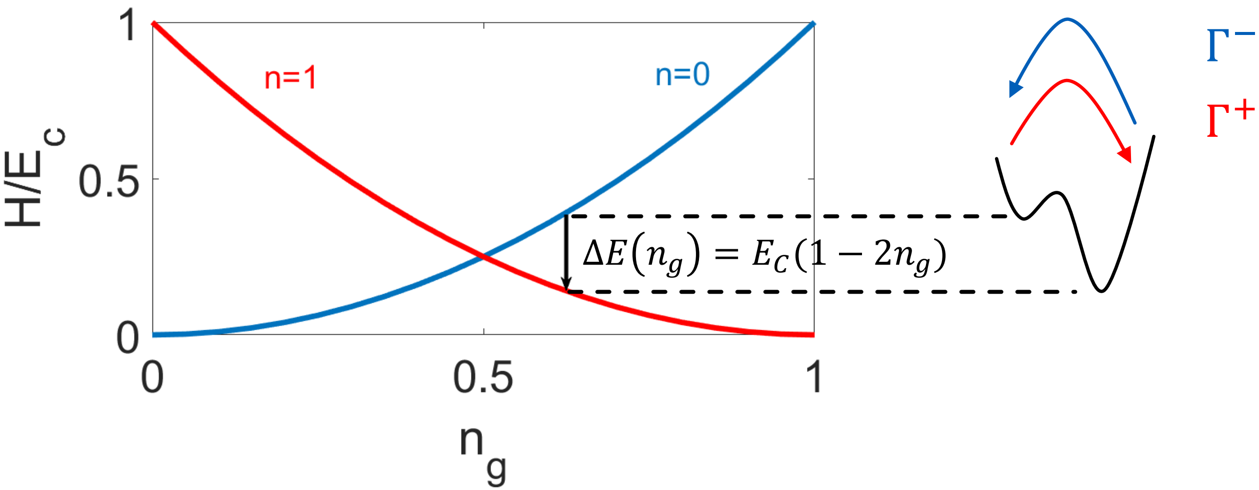
- Rate equations:

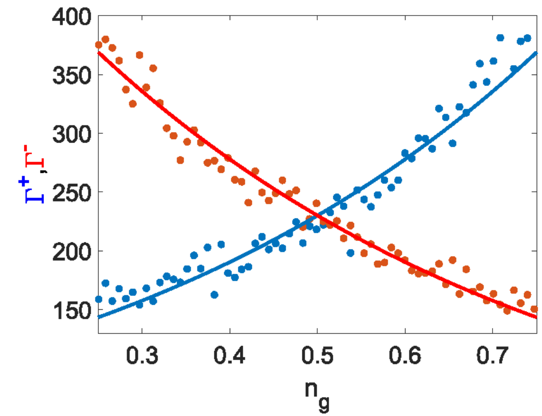
$$\begin{cases} \partial_t p_0 = -\Gamma^+ p_0 + \Gamma^- p_1 \\ \partial_t p_1 = -\Gamma^- p_1 + \Gamma^+ p_0 \end{cases}$$

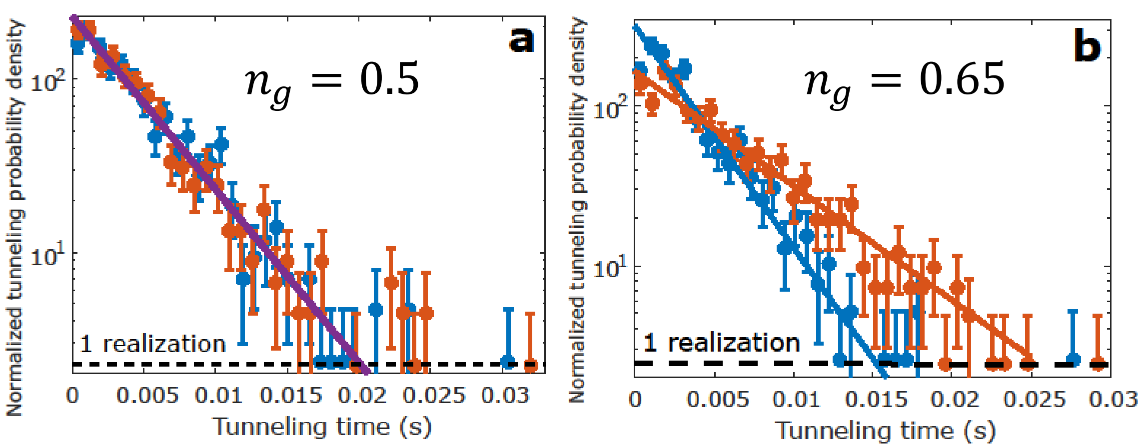
- Detailed balance at equilibrium:

$$\frac{p_0}{p_1} = \frac{\Gamma^-}{\Gamma^+} = e^{\Delta E(n_g)/k_B T}$$

Single electron box

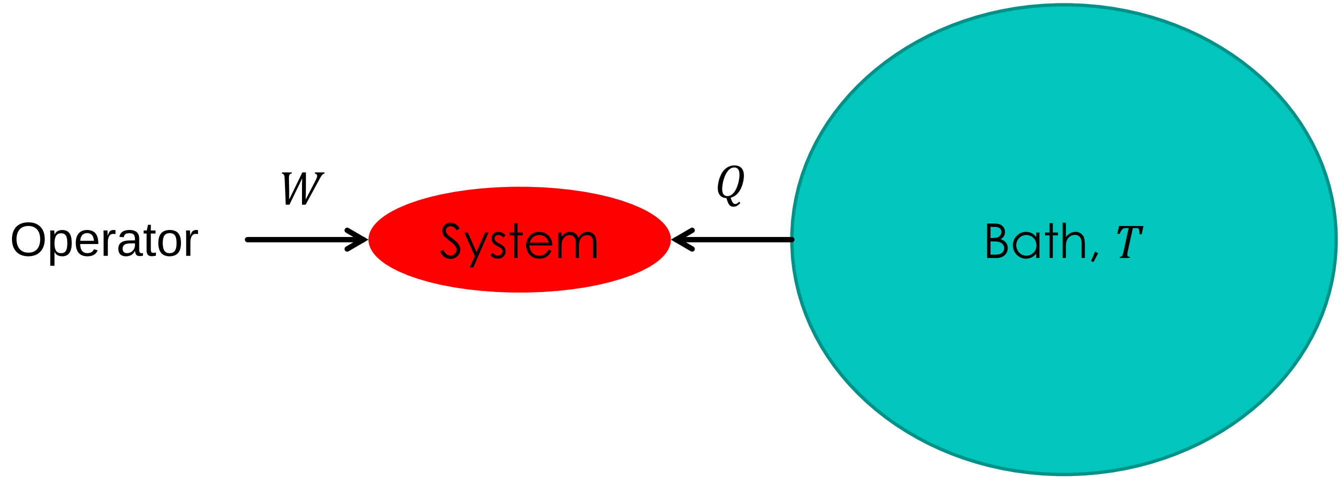






- Typical tunneling rates ~ 100-300 Hz
- Waiting time distribution for a Poisson process

Context



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N. Y. Halpern et al., New. J. Phys. **17** 095003 (2015)
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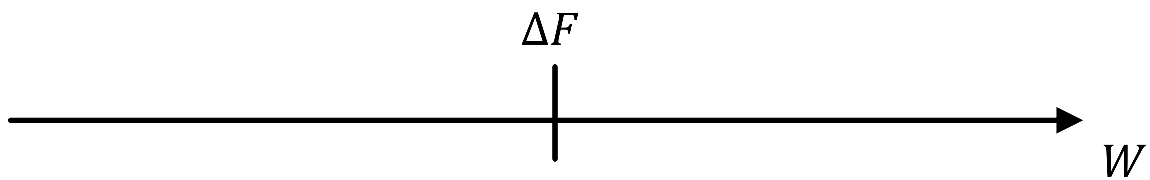
Probabilistic work extraction

Work applied $W[n(t), n_g(t)] = \int_0^T dt \dot{n}_g(t) \frac{\partial H}{\partial n_g}$



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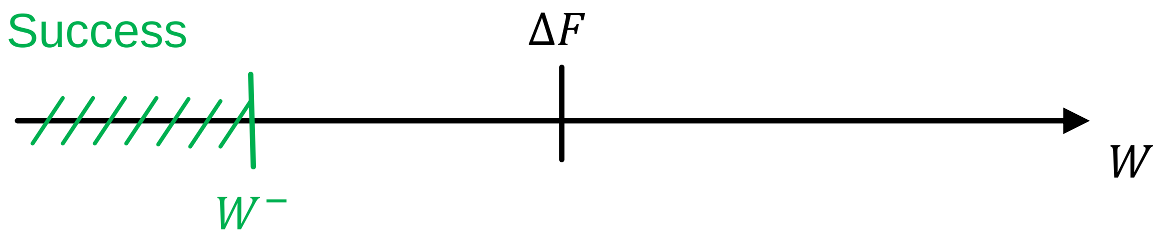

<u>Requirements for the driving protocol over a cycle</u>:

- Bound on minimum work extracted $W^- - \Delta F$ if successful attempt

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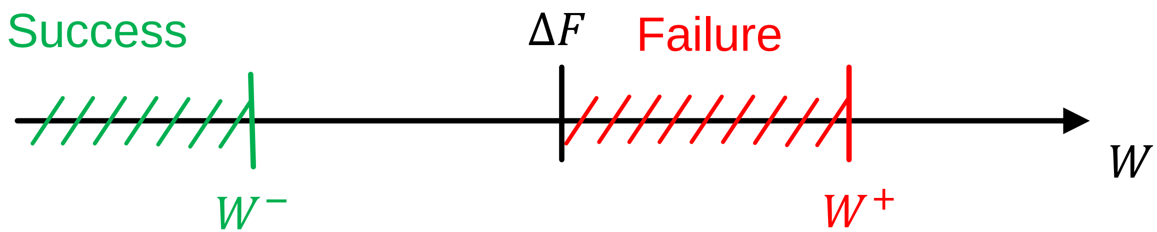

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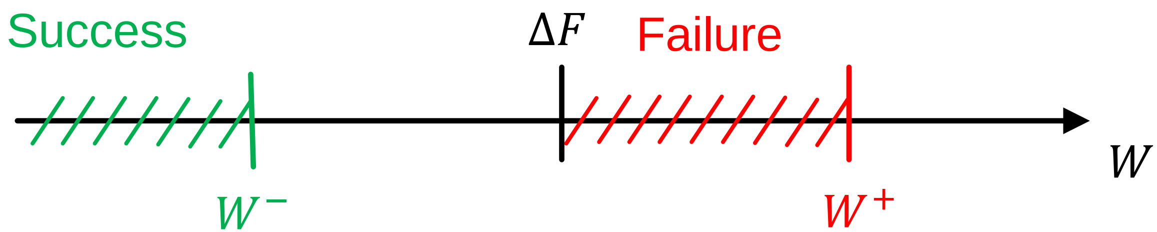
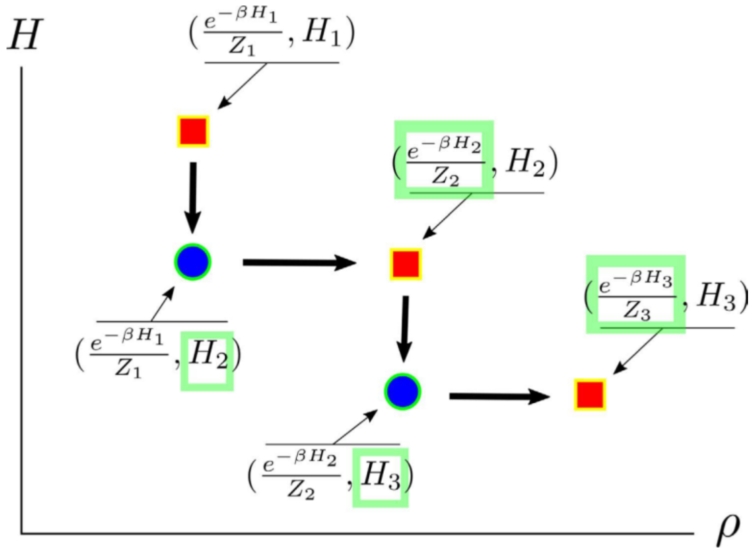


<u>Requirements for the driving protocol over a cycle</u>:

- Bound on minimum work extracted $W^- - \Delta F$ if successful attempt
- Bound on maximum work paid $W^+ - \Delta F$ if failed attempt
- Maximum probability $p_{W^+}(W \leq W^-)$ allowed by Jarzynski's equality

$$p_{W^+}(W \leq W^-) \leq \frac{e^{\Delta F/k_B T} - e^{-W^+/k_B T}}{e^{-W^-/k_B T} - e^{-W^+/k_B T}}$$

V. Cavina et al., Sci. Rep. **6**, 29282 (2016)



V. Cavina et al., Sci. Rep. **6**, 29282 (2016): sequence of « discrete » steps describing a transformation:

- **Quench**: fast = no heat exchanged, only work

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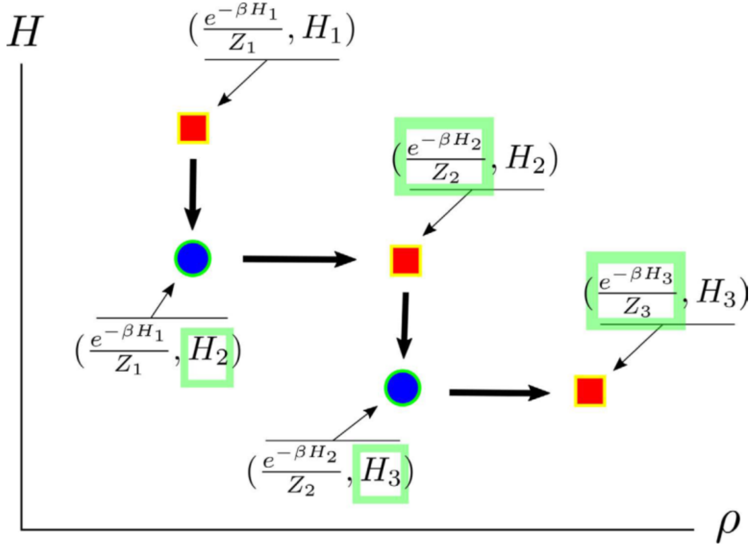

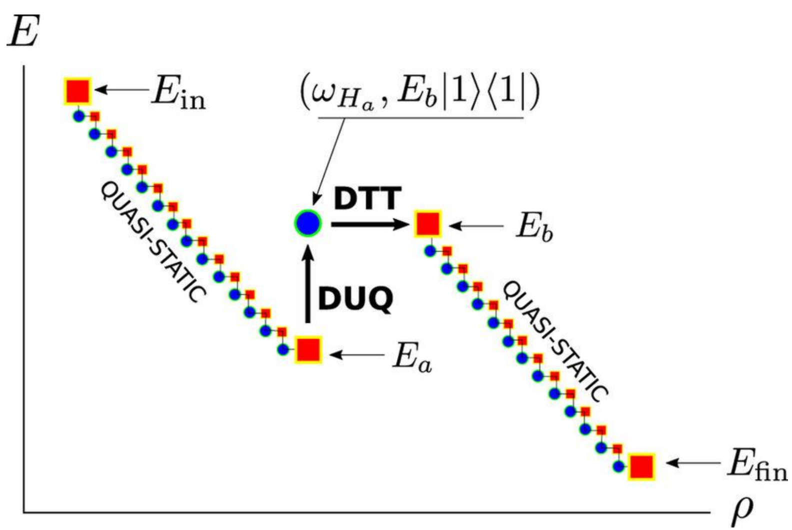

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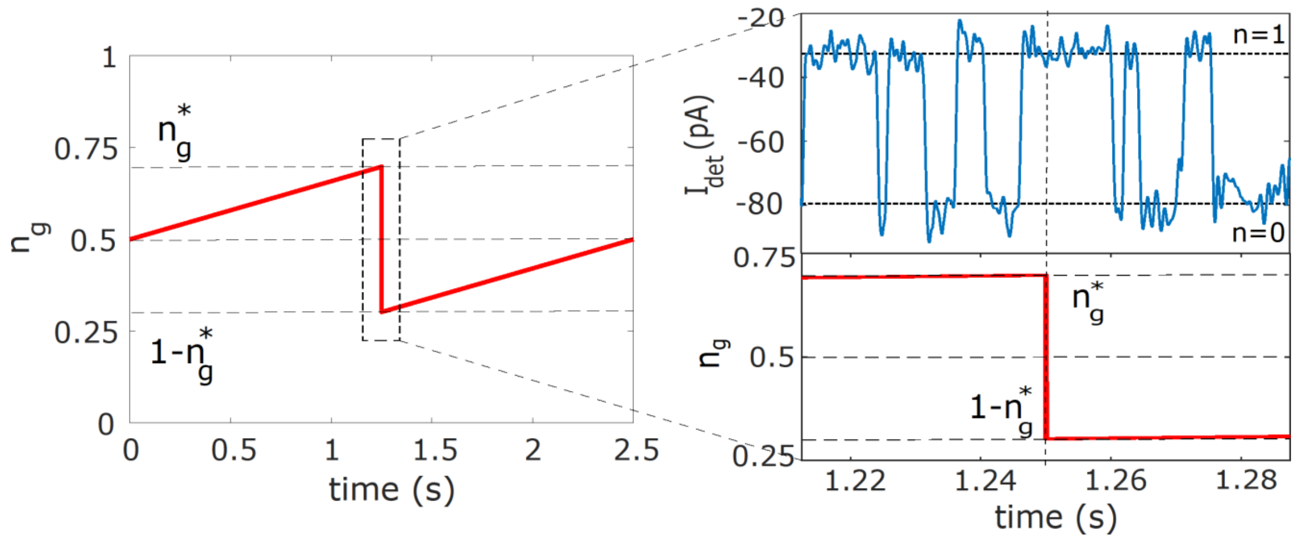
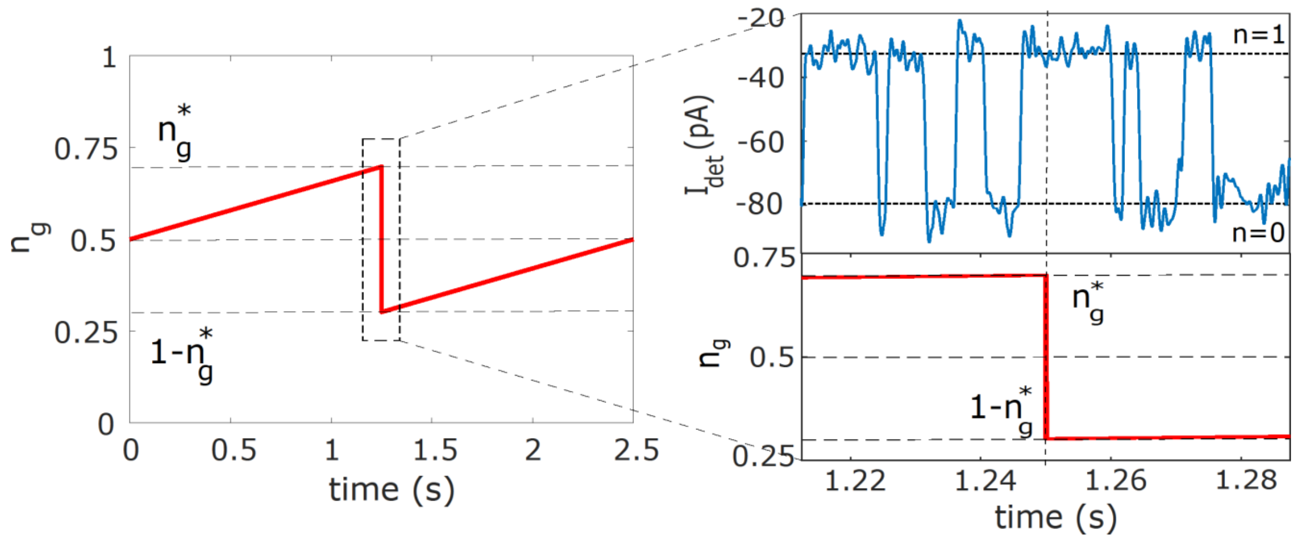
- **Quench**: fast = no heat exchanged, only work
- **Thermalization**: no work done, heat removed from/released to the bath
- Large quench = splits in two the work distribution

$$p(W) = p_0\delta(W - W^+) + (\mathbf{1} - \boldsymbol{p_0})\boldsymbol{\delta}(\boldsymbol{W} - \boldsymbol{W}^-)$$

- $p_0 = \frac{1}{e^{-(E_b - E_a)/k_B T} + 1}$ (Gibbs)



- Two « reversible » ramps, time interval ≫ characteristic tunneling time
- Quench time ≪ characteristic tunneling time: **no heat exchange**



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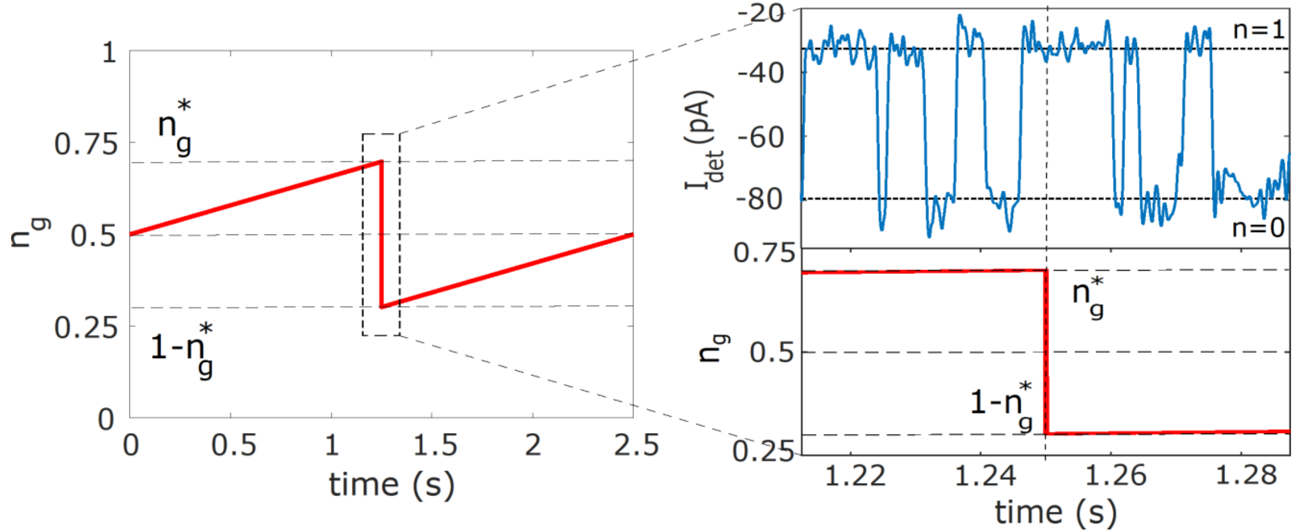
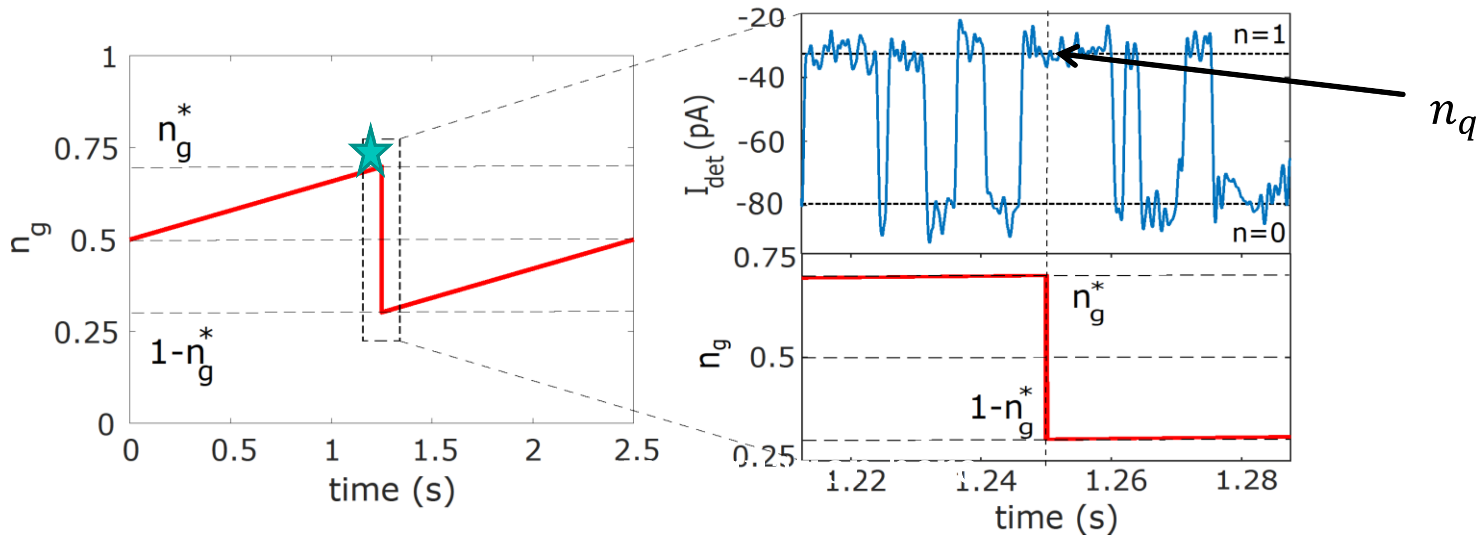
- Quench time ≪ characteristic tunneling time: **no heat exchange**

- Driving cycle: $W = -Q = -\sum_i \Delta E\left[n_g(t_i)\right]\Delta n(t_i)$
  → work over one trajectory can be inferred

- ca. 1000 repetitions = distribution of work fluctuations



$$W^{\pm} = \mp \Delta E\left(n_g^*\right)$$



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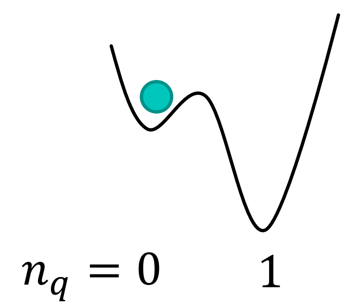
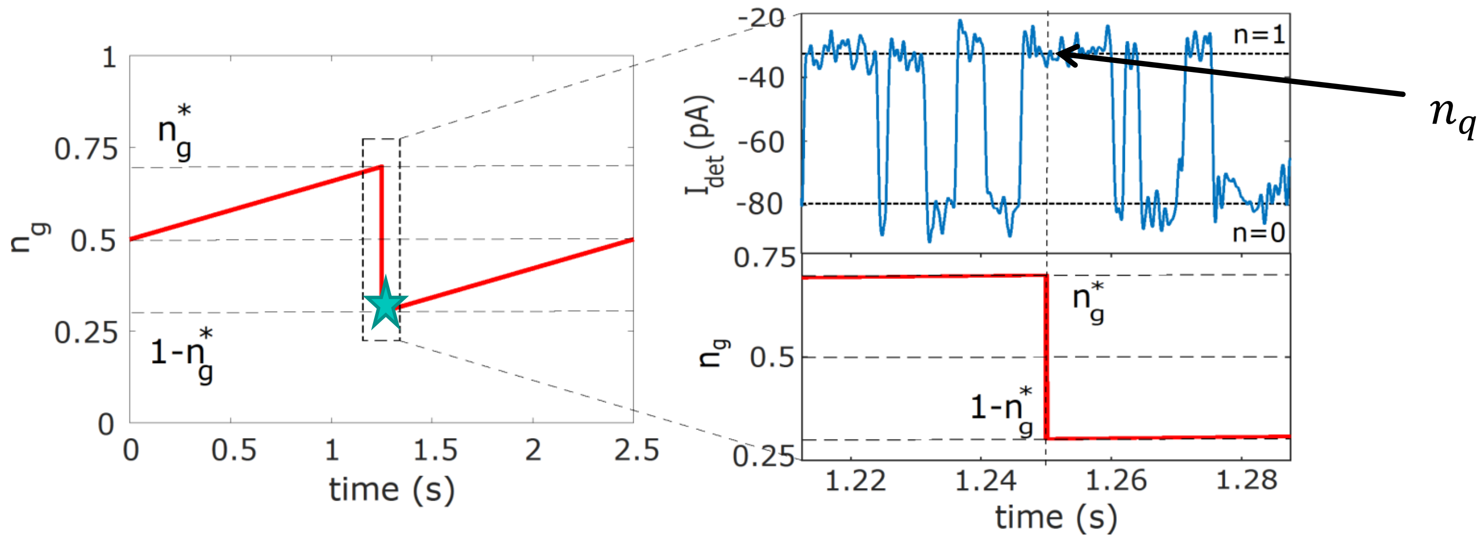
➢ Depends only on state n_q at quench onset:





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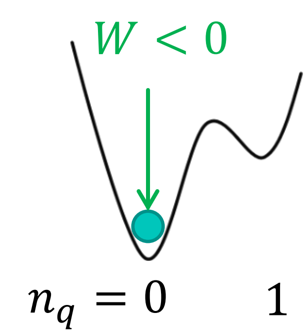
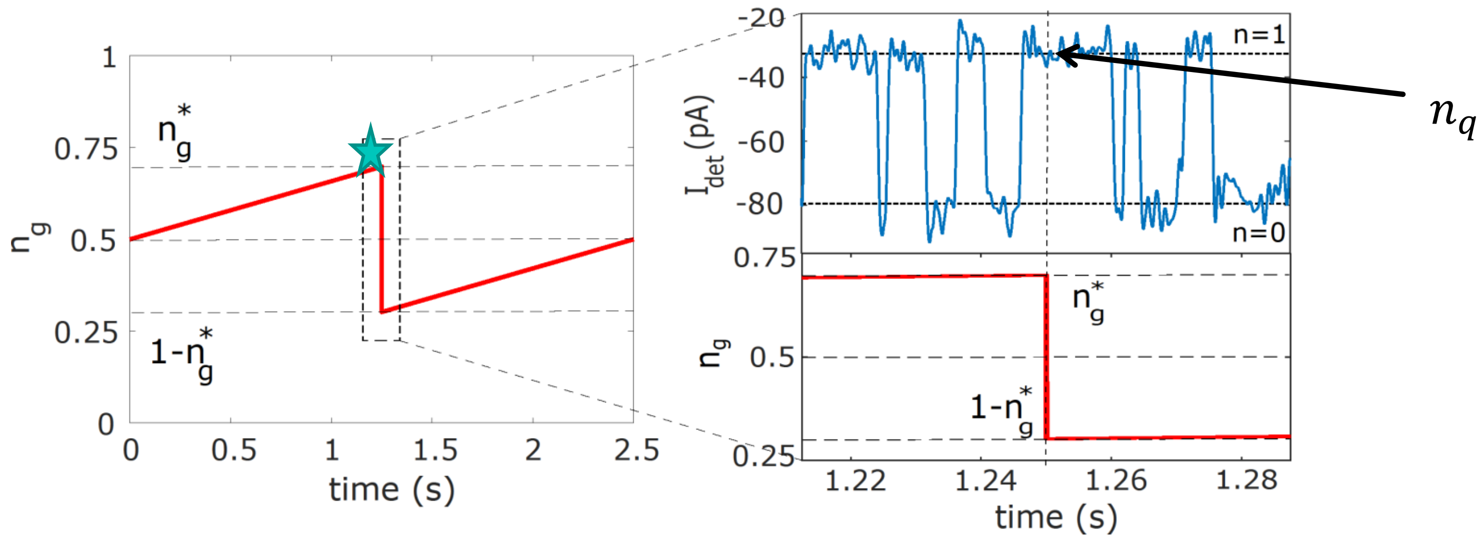
- Depends only on state n_q at quench onset: win if excited state





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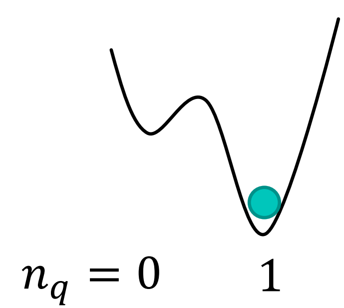

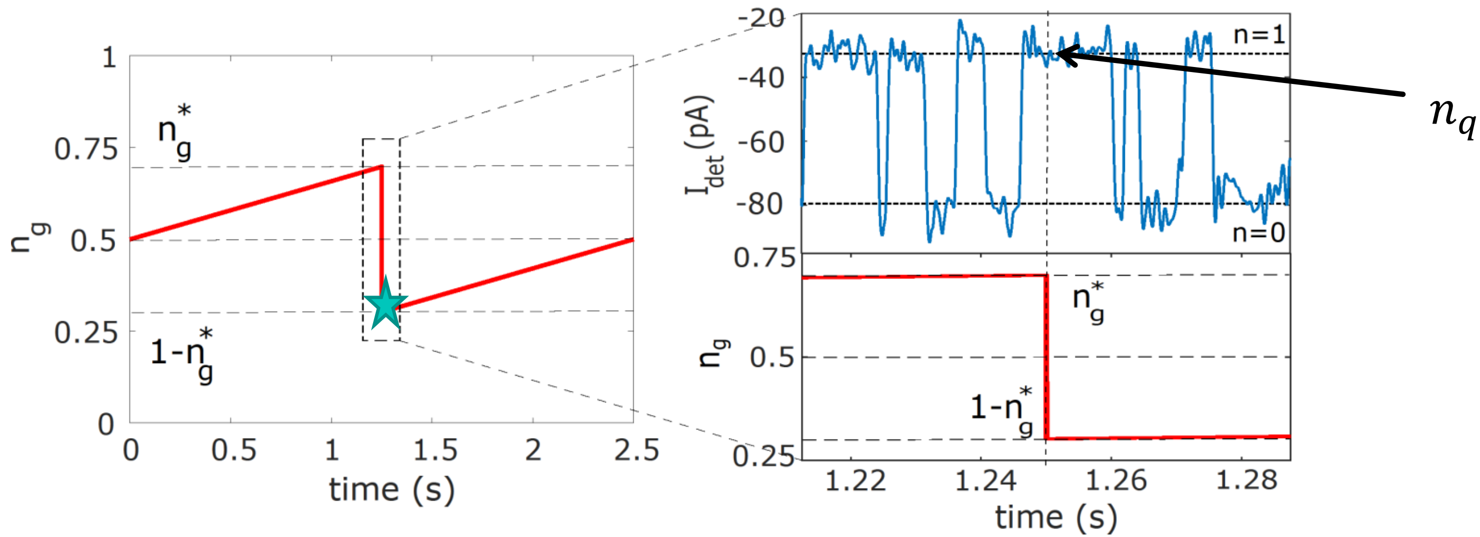

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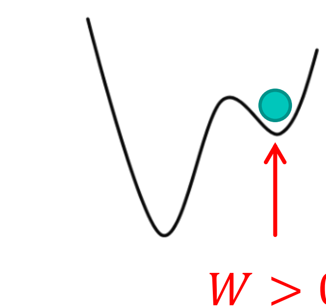
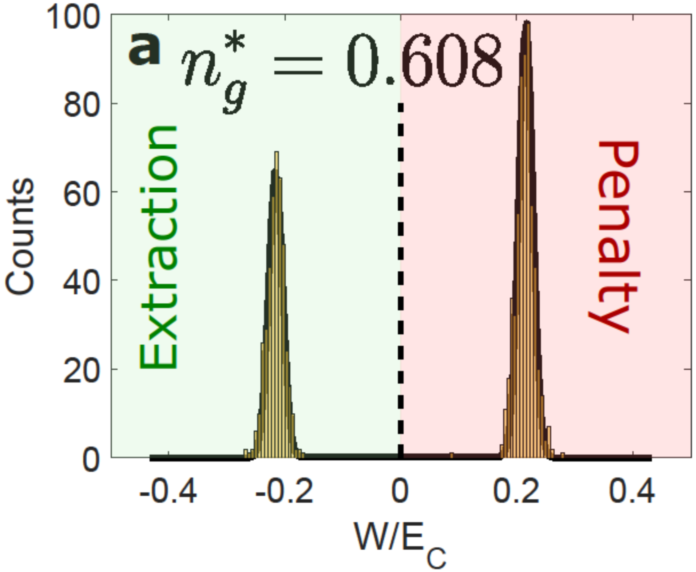


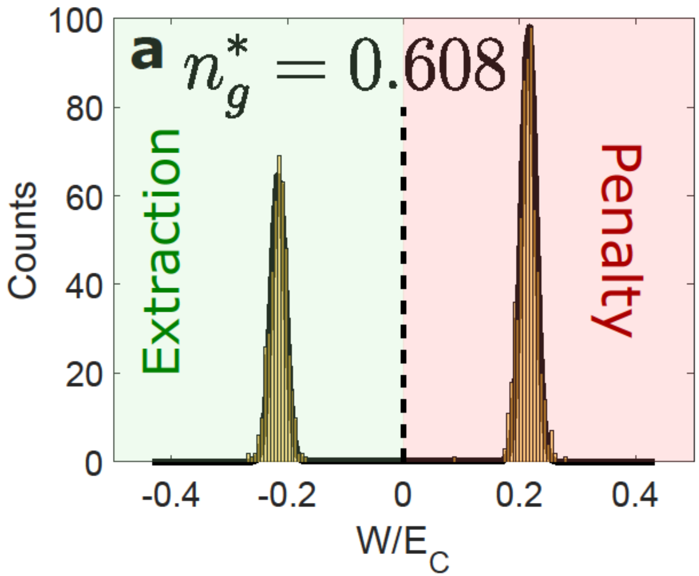
- Depends only on state n_q at quench onset: lose if ground state



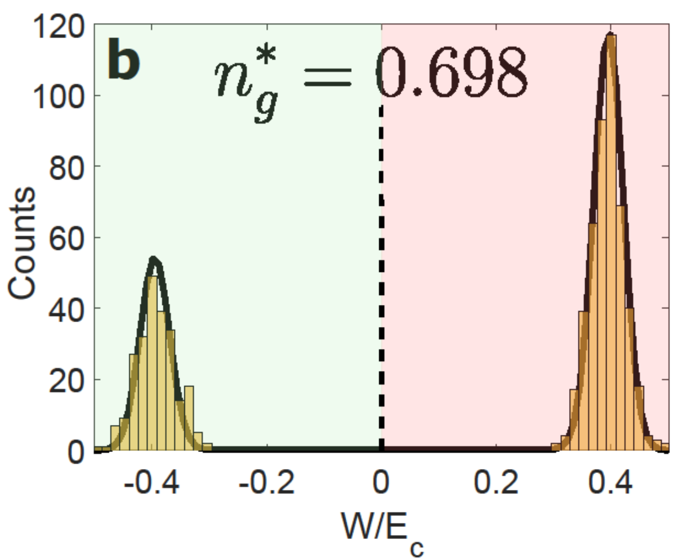
- $\Delta n_g = n_g^* - 1/2 = 0.11 \rightarrow$ small quench amplitude

- Finite peak width: imperfect quasistatic ramp

O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)

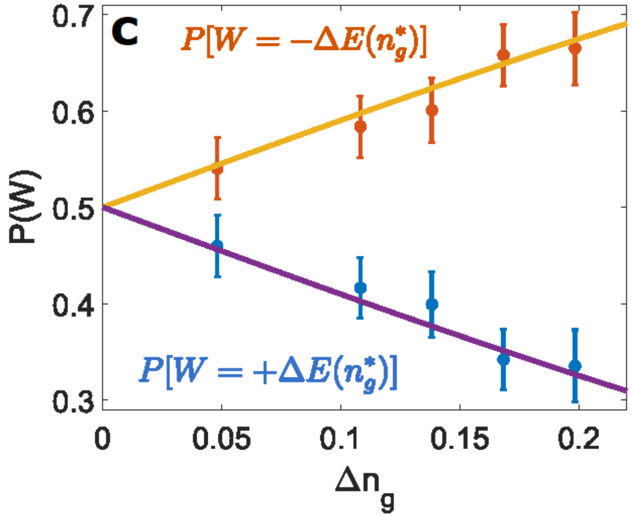


- $\Delta n_g = n_g^* - 1/2 = 0.11 \rightarrow$ small quench amplitude
- Finite peak width: imperfect quasistatic ramp

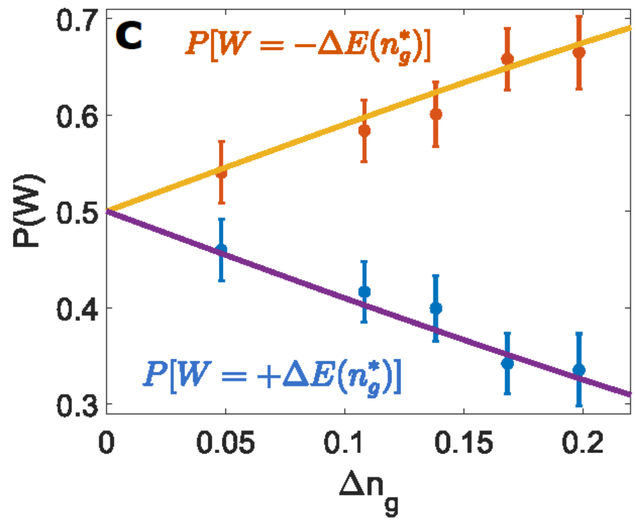


- $\Delta n_g = 0.17 \rightarrow$ large quench amplitude

O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)



- Probability of violation decreases with quench amplitude $\Delta n_g = n_g^* - 1/2$
- Weights: Gibbs functions for favorable/unfavorable state just before the quench



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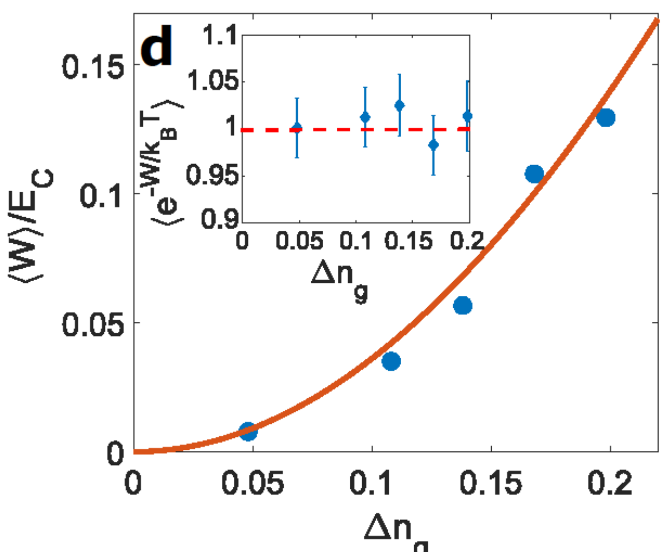
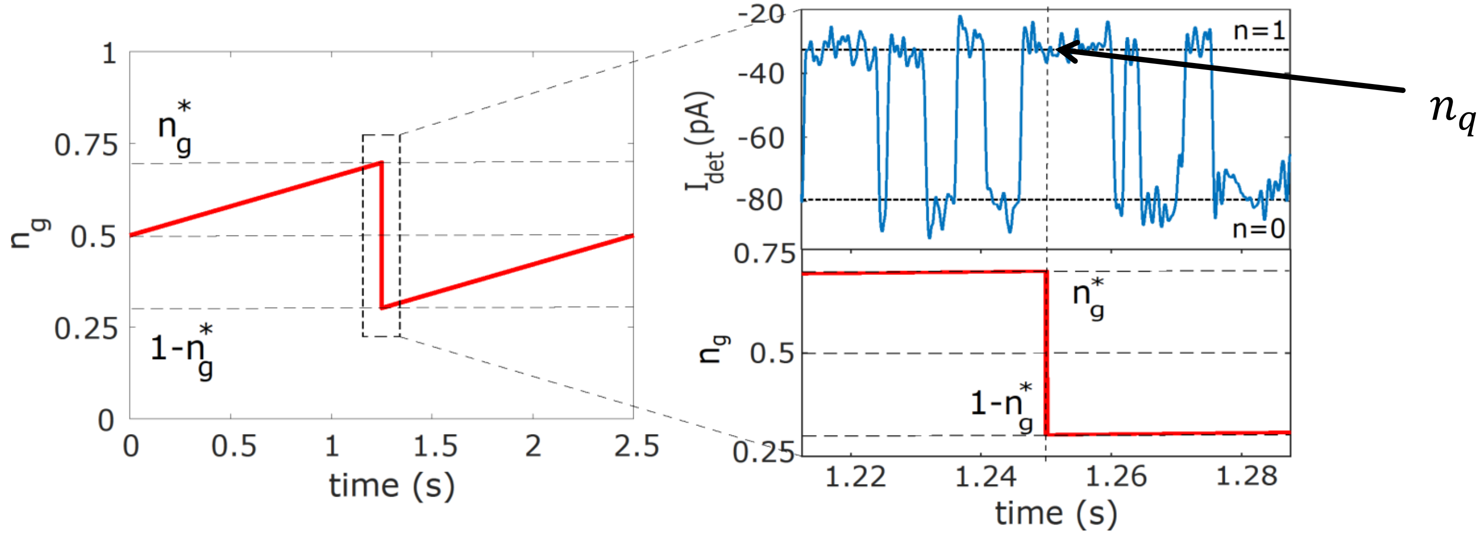


- Average work performed on system positive: in agreement with second law ($\Delta F = 0$ for our closed cycle)

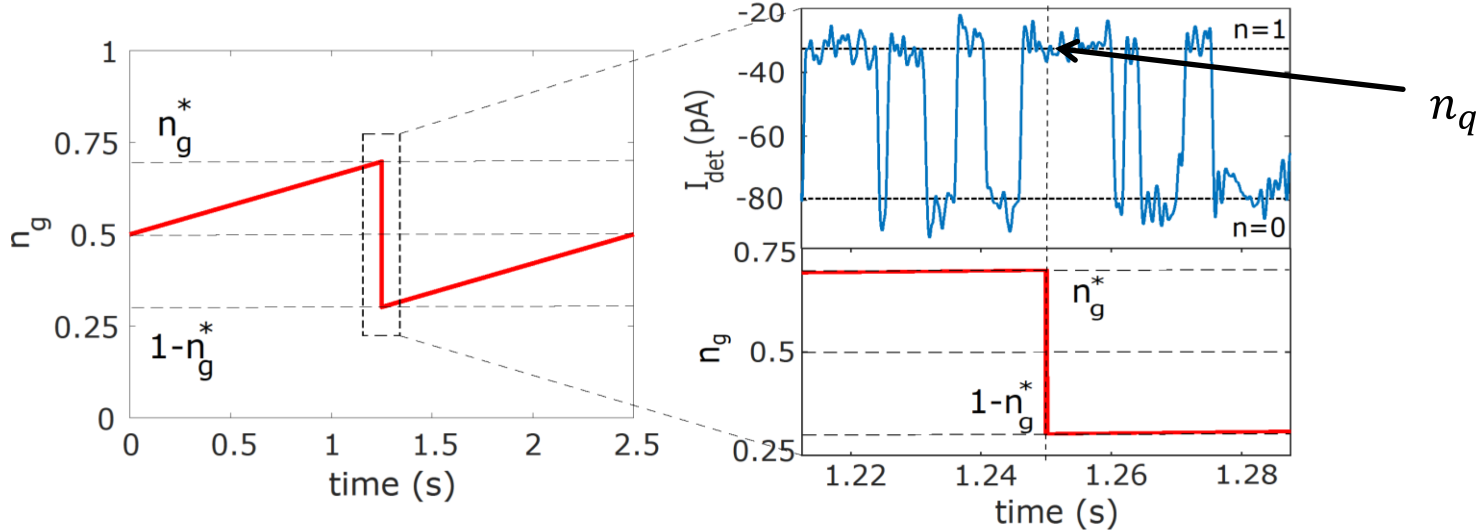
- Increases with quench amplitude: more irreversibility introduced

O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)



$$W^{\pm} = \mp \Delta E\left(n_g^*\right)$$

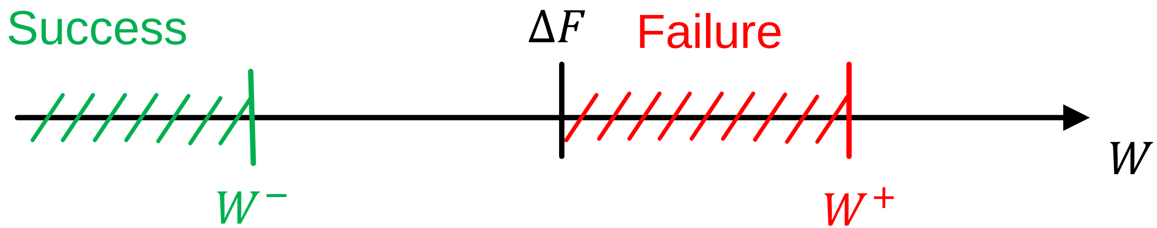
- Depends only on state at quench onset: win if excited state = less likely
- The larger, the less probable



$$W^{\pm} = \mp \Delta E\left(n_g^*\right)$$

- ➢ Depends only on state at quench onset: win if excited state = less likely
- ➢ **The larger, the less probable = how to make extraction more probable ?**

A neat limit



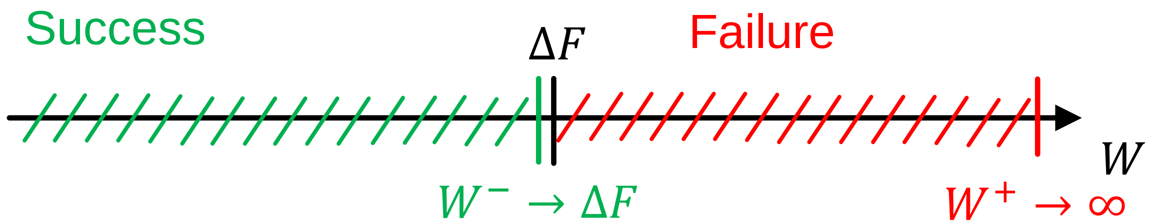
Requirements:

- Bound on minimum work extracted $W^- - \Delta F$ if successful attempt
- Bound on maximum work paid $W^+ - \Delta F$ if failed attempt
- Maximum probability $p_{W^+}(W \leq W^-)$ allowed by Jarzynski's equality

$$p_{W^+}(W \leq W^-) \leq \frac{e^{\Delta F/k_B T} - e^{-W^+/k_B T}}{e^{-W^-/k_B T} - e^{-W^+/k_B T}}$$

V. Cavina et al., Sci. Rep. (2016)

A neat limit

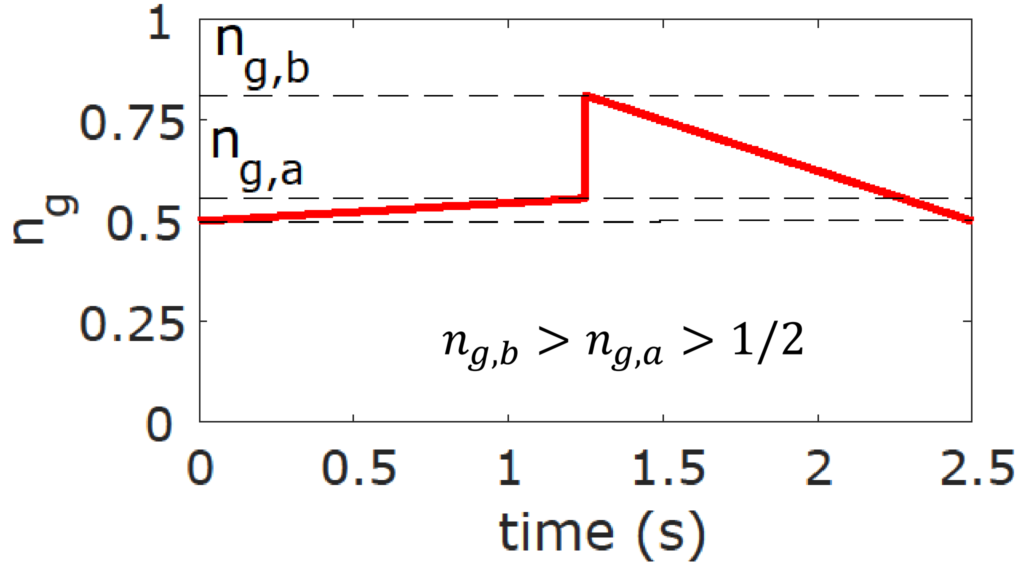


If we loosen requirements: $W^- \to \Delta F$, $W^+ \to \infty$

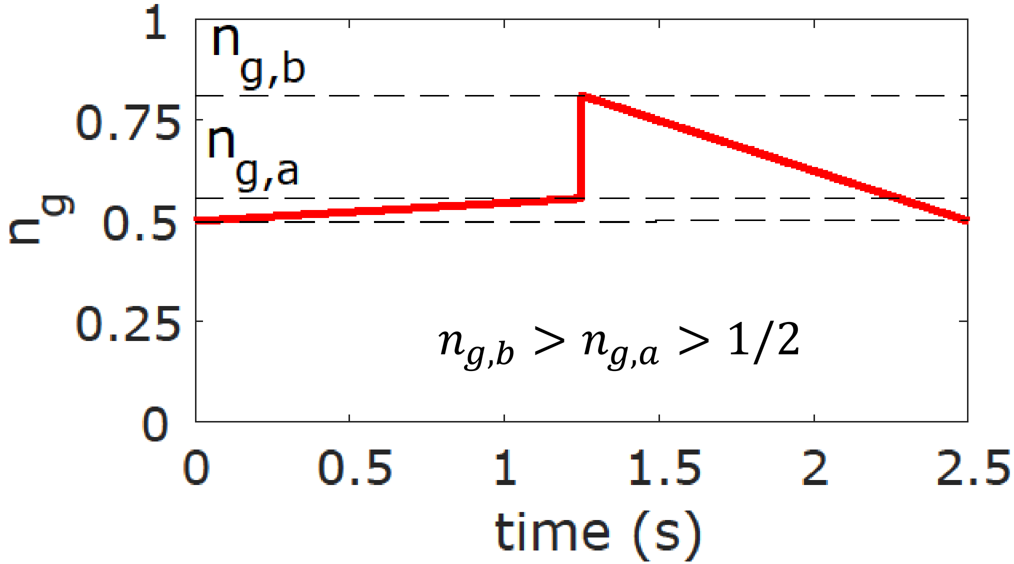
- Maximum probability $p_\infty(W \leq W^-)$ allowed by Jarzynski's equality

$$p_\infty(W \leq W^-) \leq e^{-(W^- - \Delta F)/k_B T} \xrightarrow[W^- \to \Delta F]{} 1$$

A neat limit

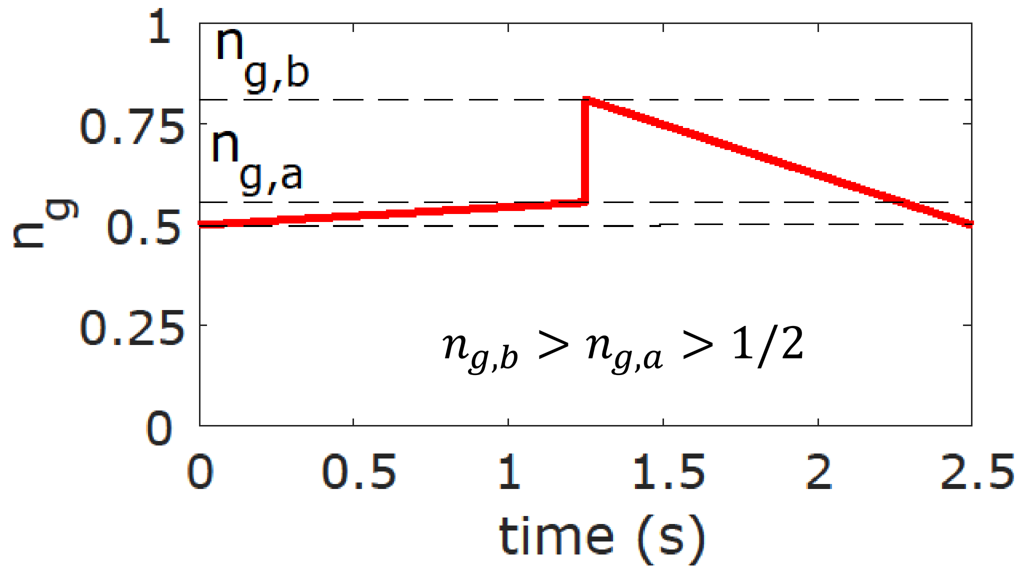


$$W = k_B T(\Delta S)_{quench} + \varphi(n_q)$$



$$W = k_B T(\Delta S)_{quench} + \varphi(n_q)$$

$$(\Delta S)_{quench} = S(n_{g,b}) - S(n_{g,a})$$

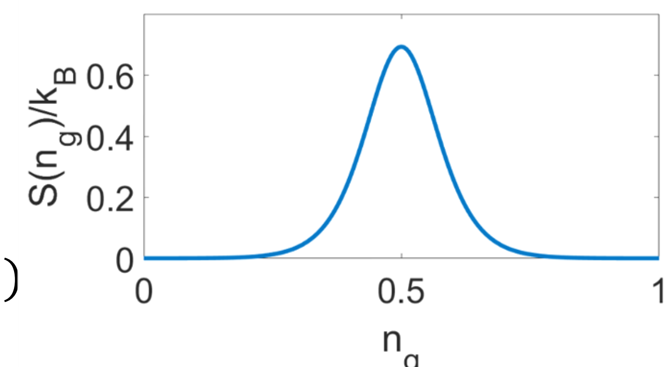


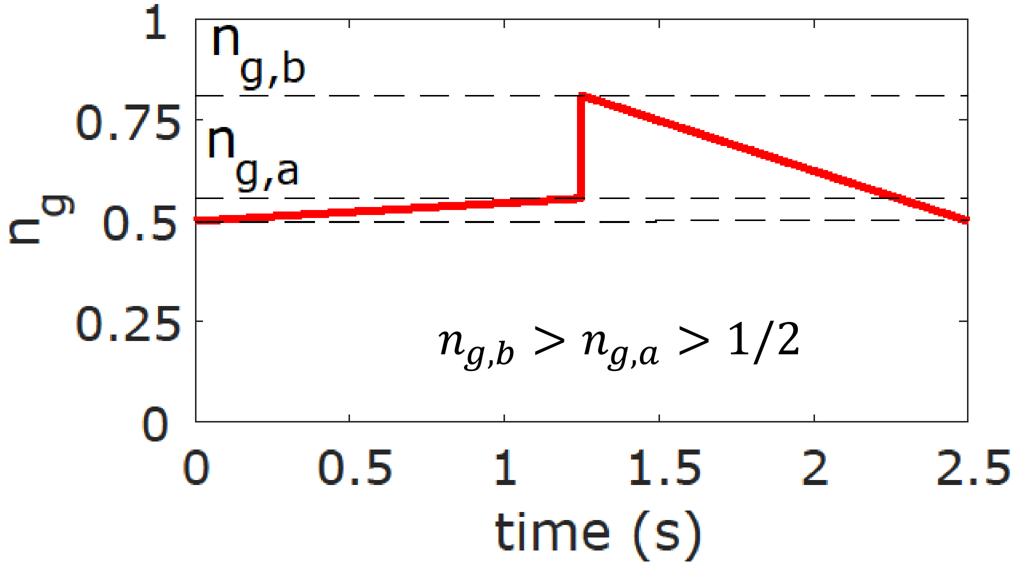
$$W = k_B T(\Delta S)_{quench} + \varphi(n_q)$$

$$(\Delta S)_{quench} = S(n_{g,b}) - S(n_{g,a})$$

S: Shannon entropy for a TLS:

$$S(n_g) = -(1 - p_0(n_g)) \ln(1 - p_0(n_g)) - p_0(n_g) \ln p_0(n_g)$$

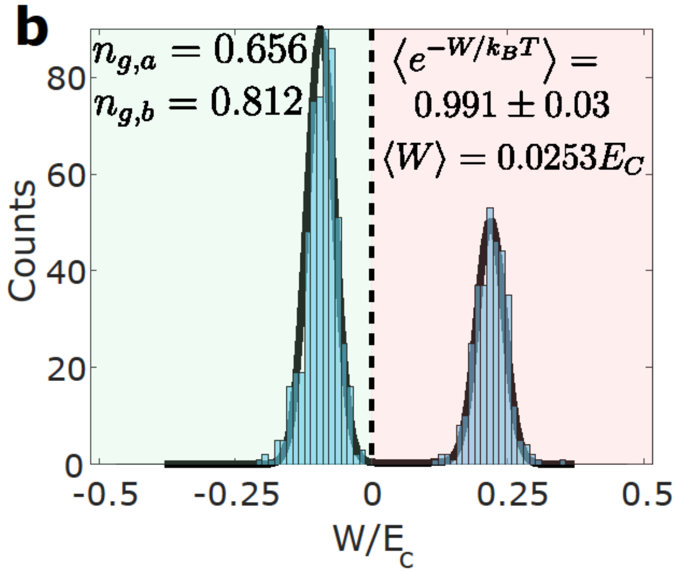




$$W = \underbrace{k_B T (\Delta S)_{quench}}_{<0!} + \varphi(n_q)$$

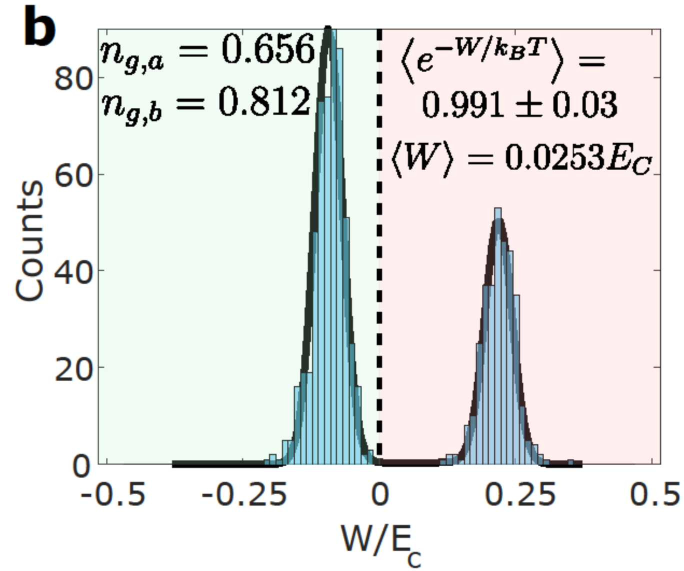
$\varphi(n_q)$: state-dependent offset

- 1st term: Shannon entropy decreases away from degeneracy
- $W < 0$ if ground state before quench:
  probability of work extraction = **ground state probability at the quench onset = favorable!**



O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)

- 65 % of successful events ($W < \Delta F$)



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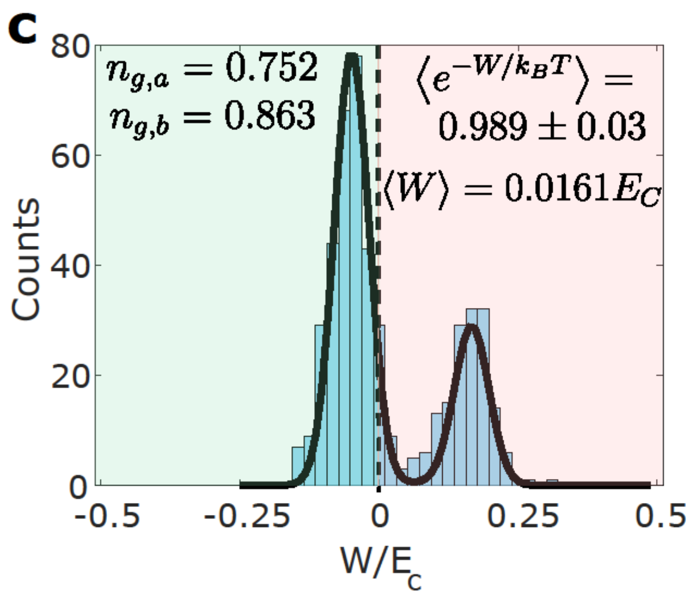
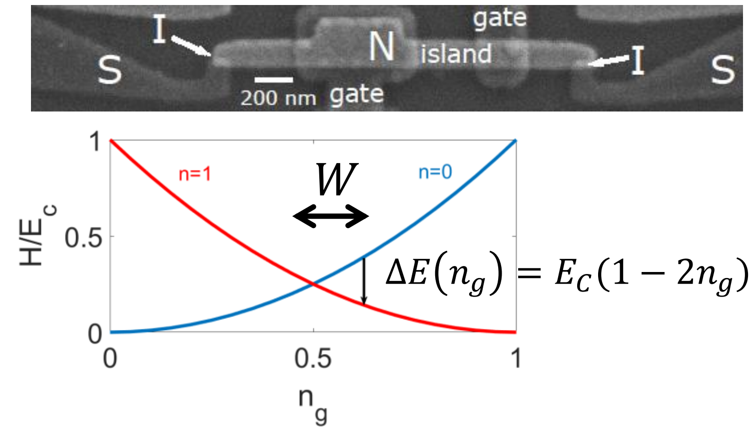


- Efficiency limited by irreversible driving
- No theoretical bound to 99,999… % probable work extraction: optimization required (RF SET, longer ramps…)
- **Statistics matters!**

Summary

- Hybrid normal-superconducting single-electron box with simple energetics as a model system for stochastic thermodynamics



Summary

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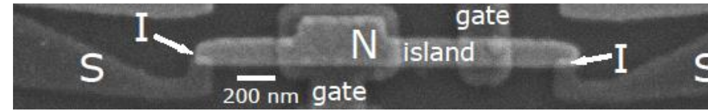
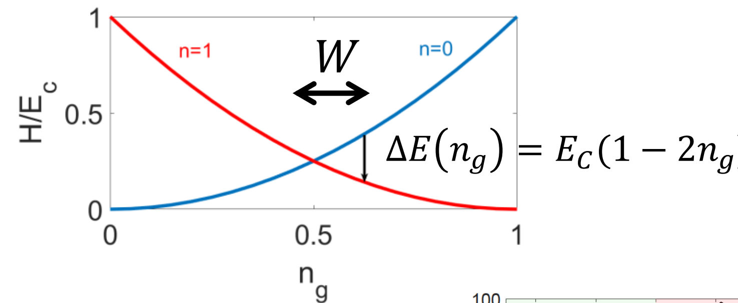
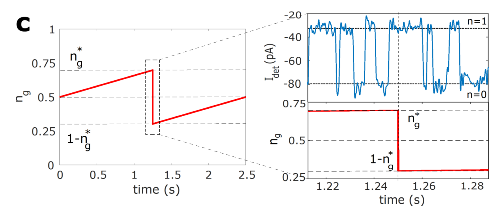
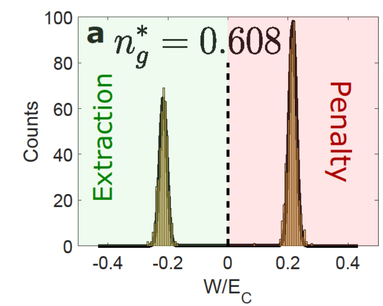




O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)





- Optimal protocol to obtain work fluctuations far beyond the 2nd law prescription

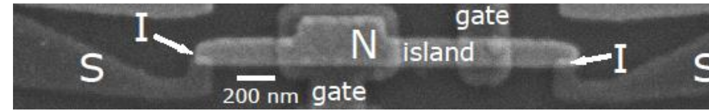
Summary

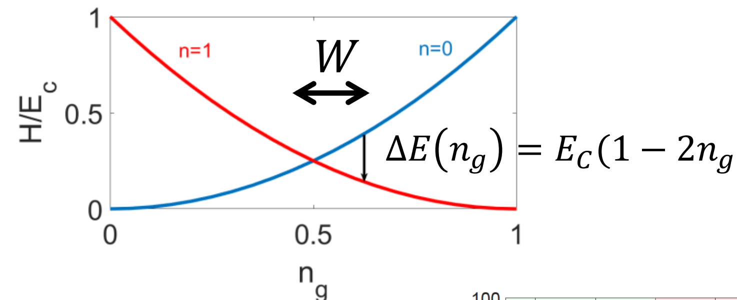
- Hybrid normal-superconducting single-electron box with simple energetics as a model system for stochastic thermodynamics

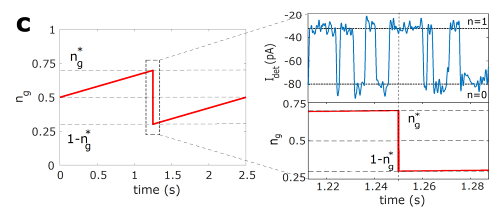
O. Maillet *et al.*, Phys. Rev. Lett. **122,** 150604 (2019)

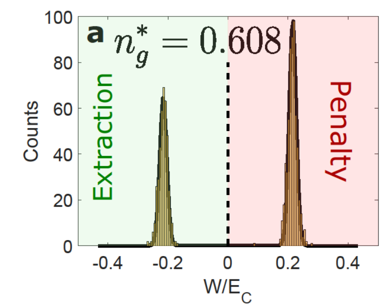
- Optimal protocol to obtain work fluctuations far beyond the 2nd law prescription

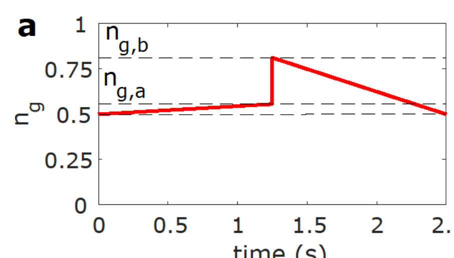
- Can be tuned to make fluctuations beyond the 2nd law prescription arbitrarily probable

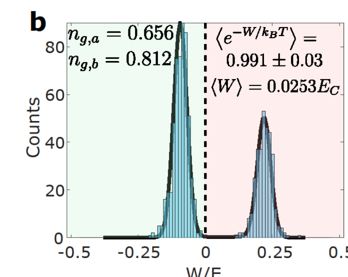










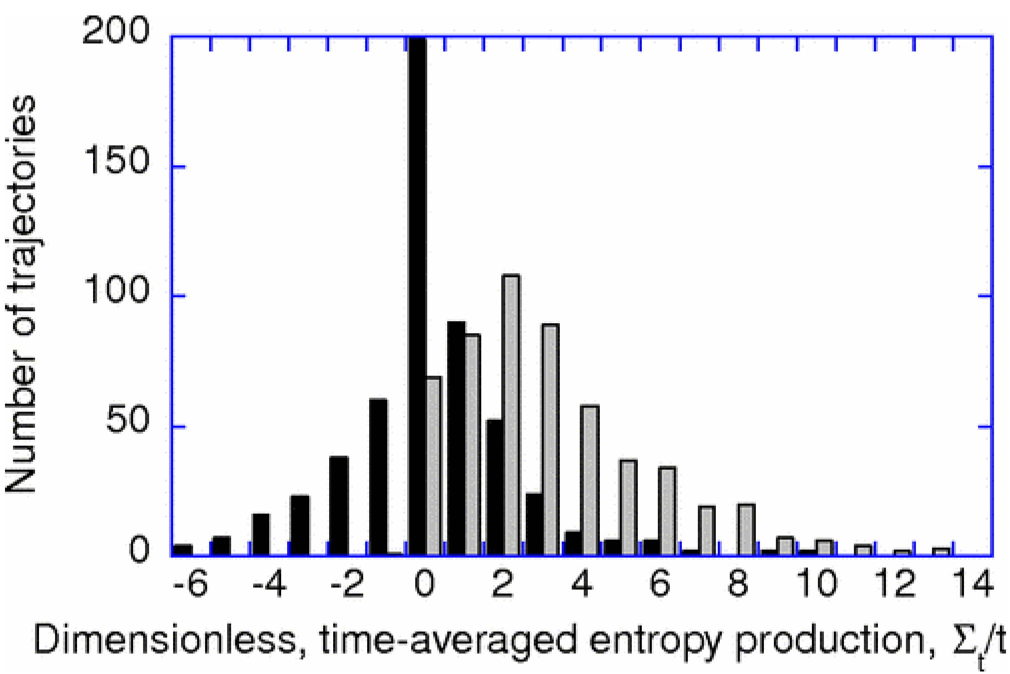


Thank you !

Experiments with mesoscopics

- Colloidal particle in an harmonic trap

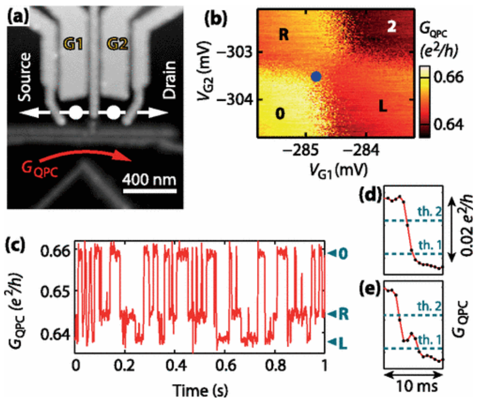
- Experimental demonstration of second law « violations » at short timescales (black data)



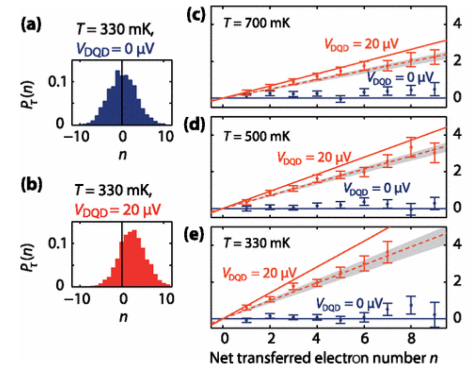
G. M. Wang *et al.*,
Phys. Rev. Lett.
(2002)

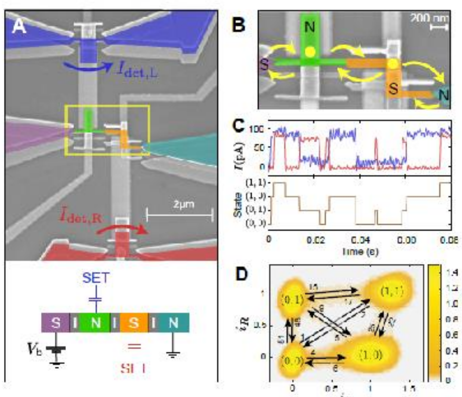
Experiments with mesoscopics

- Double (quantum) dot = direction of electron tunneling = entropy measurements

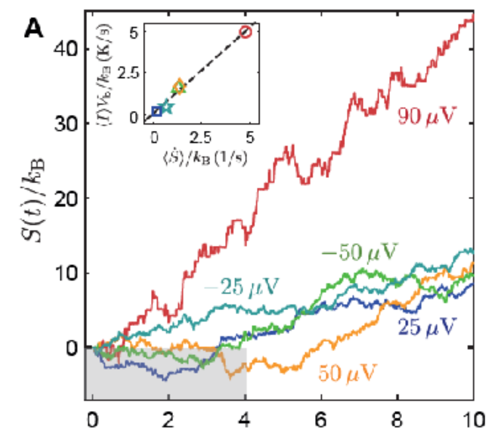


B. Küng *et al.*, Phys. Rev. X (2012)



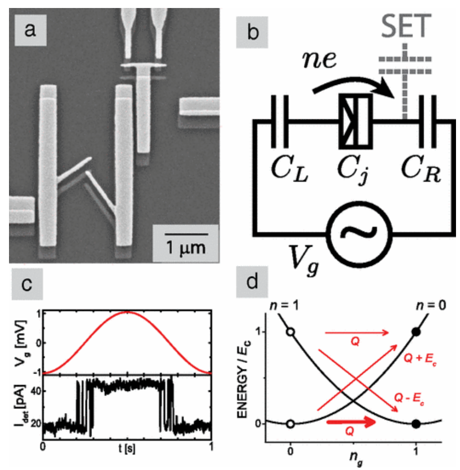


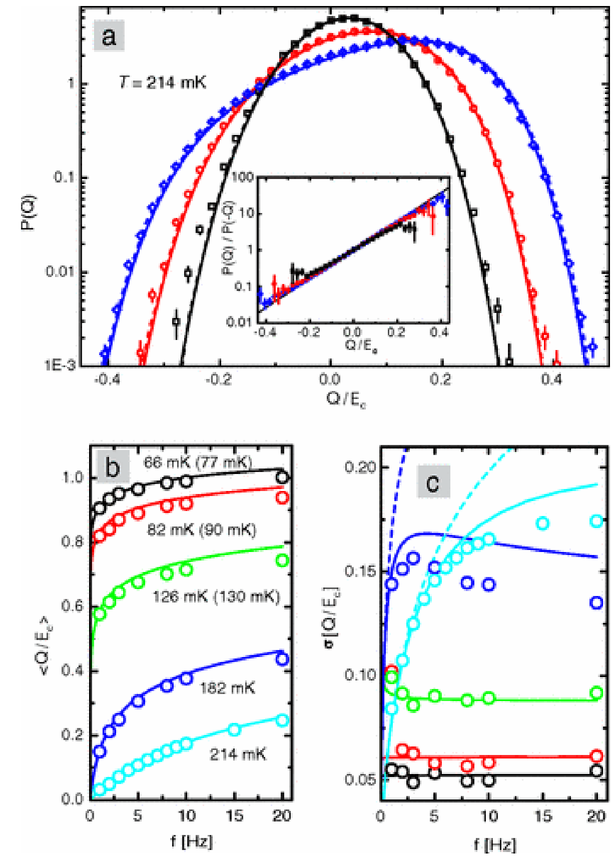
S. Singh *et al.*, ArXiv (2017)



Experiments with mesoscopics

- Test of Jarzynski equality with a single electron box
- Gate driving cycle, measurement of heat exchange (tunneling events) during the cycle



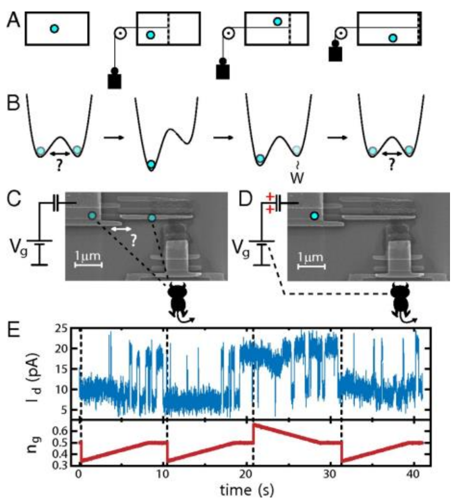


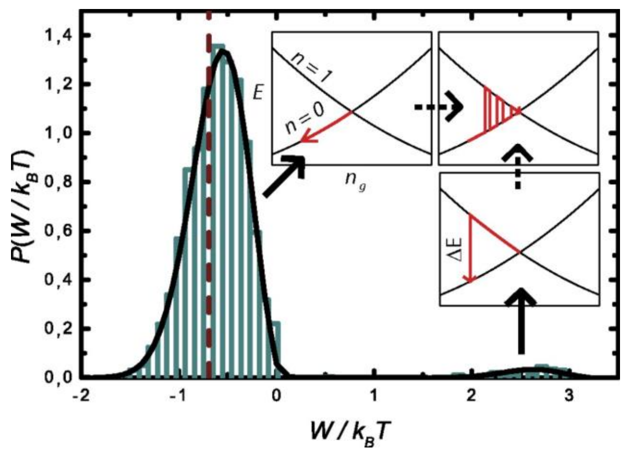
O.-P. Saira *et al.*,
Phys. Rev. Lett. (2012)

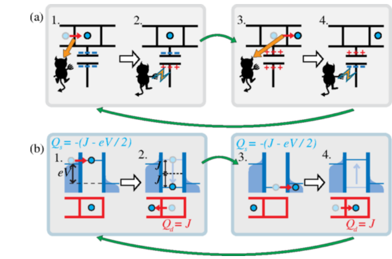
Experiments with mesoscopics

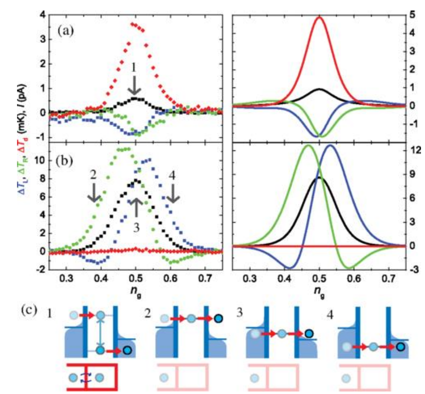
- Szilard engine: feedback on system driving applied using the information gained by a detector (= Maxwell Demon operation)
- Work extracted from the system on average, close to Landauer limit (-kTlog2)
- Not a true violation of 2nd law: entropy created in the Demon («cost of information»)

J. V. Koski *et al.*, PNAS (2014)
J. V. Koski *et al.*, Phys. Rev. Lett. (2015)

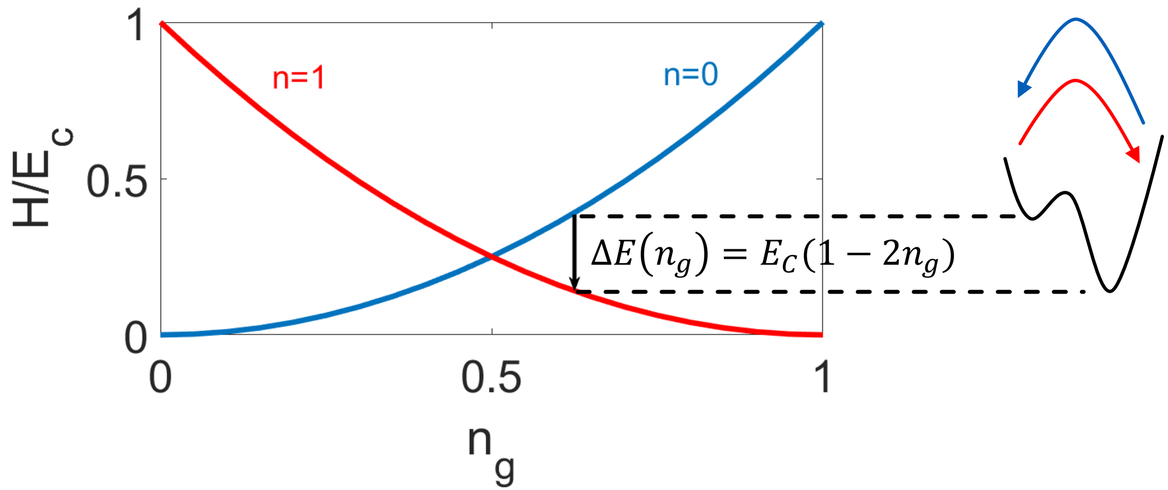


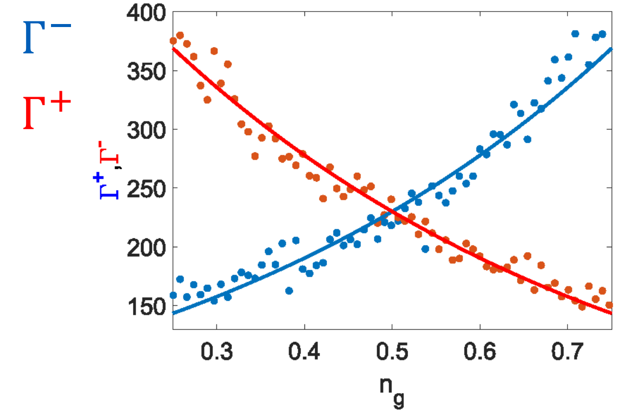


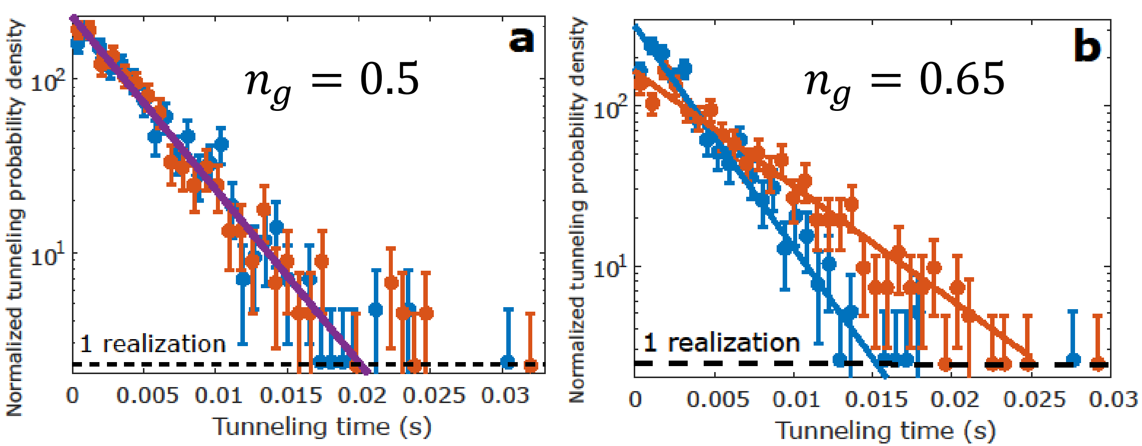




Single electron box

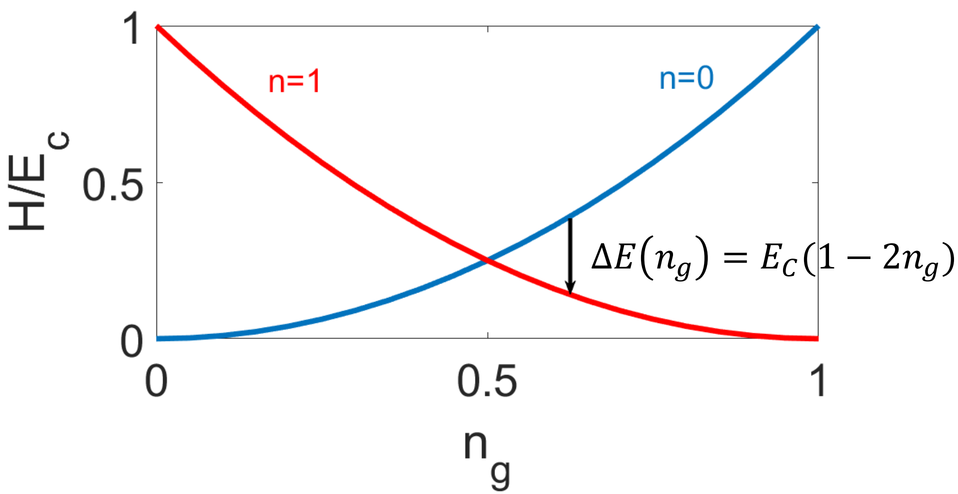


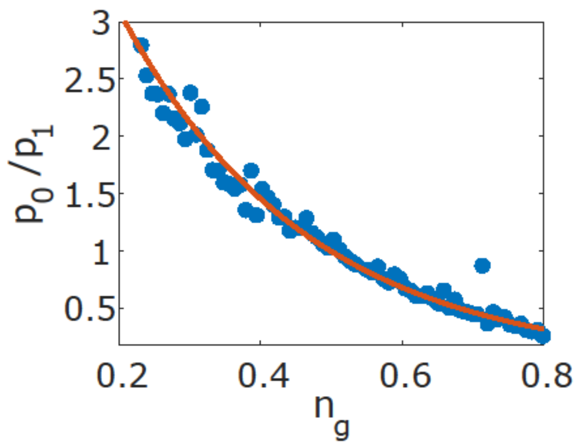




- Typical tunneling rates ~ 100-300 Hz
- Waiting time distribution for a Poisson process

Single electron box





- Rate equations:

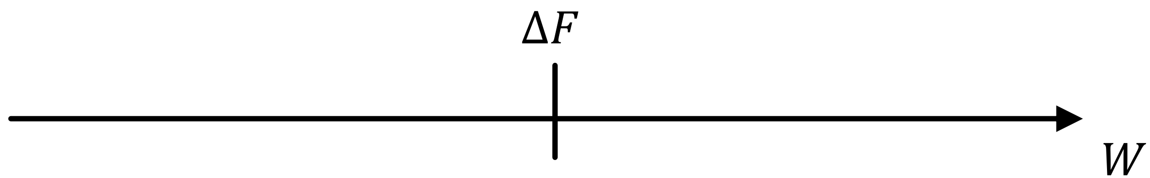
$$\begin{cases} \partial_t p_0 = -\Gamma^+ p_0 + \Gamma^- p_1 \\ \partial_t p_1 = -\Gamma^- p_1 + \Gamma^+ p_0 \end{cases}$$

- Detailed balance at equilibrium:

$$\frac{p_0}{p_1} = \frac{\Gamma^-}{\Gamma^+} = e^{\Delta E(n_g)/k_B T}$$

Probabilistic work extraction

Work applied $W[n(t), n_g(t)] = \int_0^T dt \dot{n}_g(t) \frac{\partial H}{\partial n_g}$



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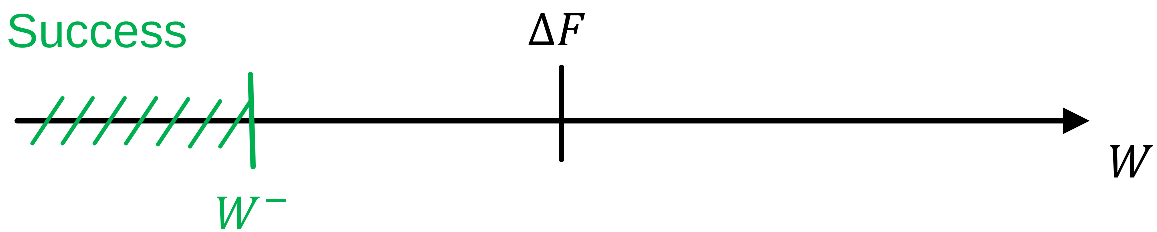

Requirements for the driving protocol over a cycle:

- Bound on minimum work extracted $W^- - \Delta F$ if successful attempt

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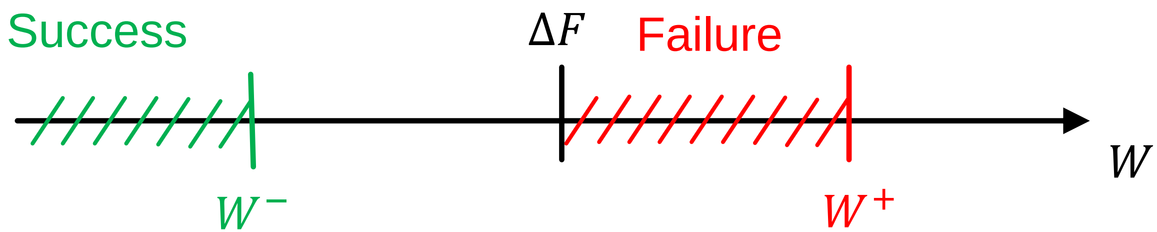

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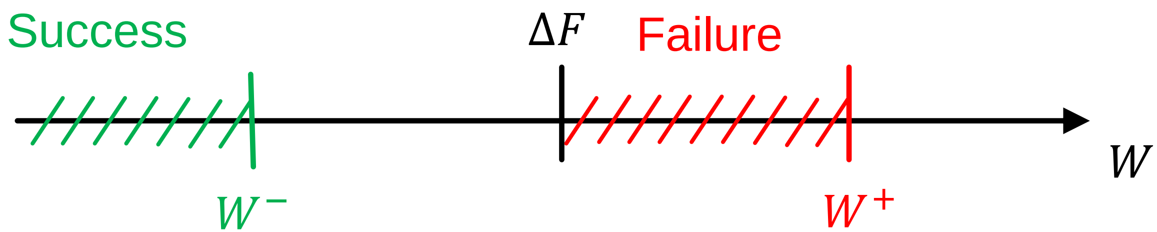

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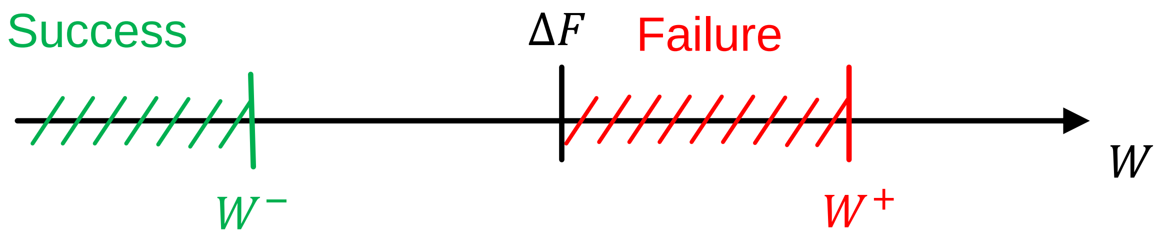

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V. Cavina et al., Sci. Rep. (2016)

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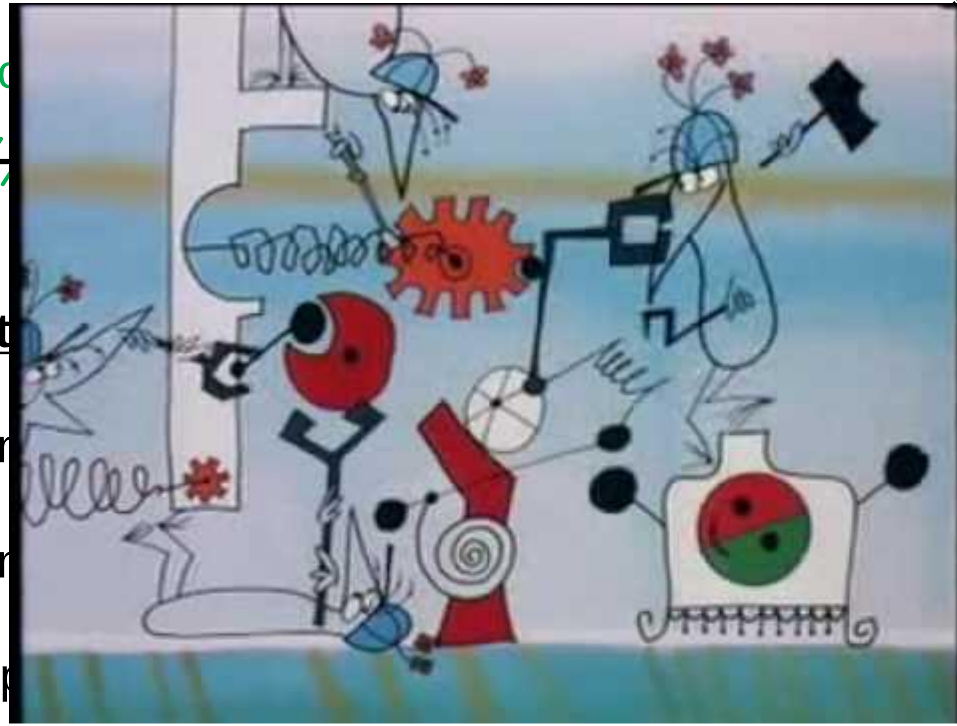
Succ

W



Requirement

➢ Bound on r ... ul attempt

➢ Bound on r

➢ Maximum p ... **ski's equality**

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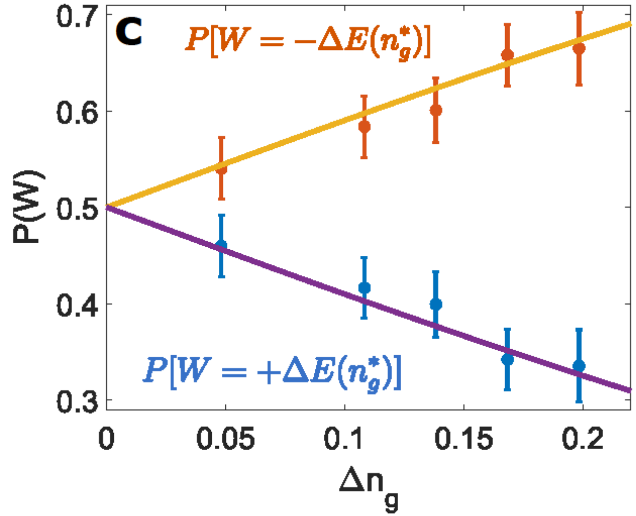
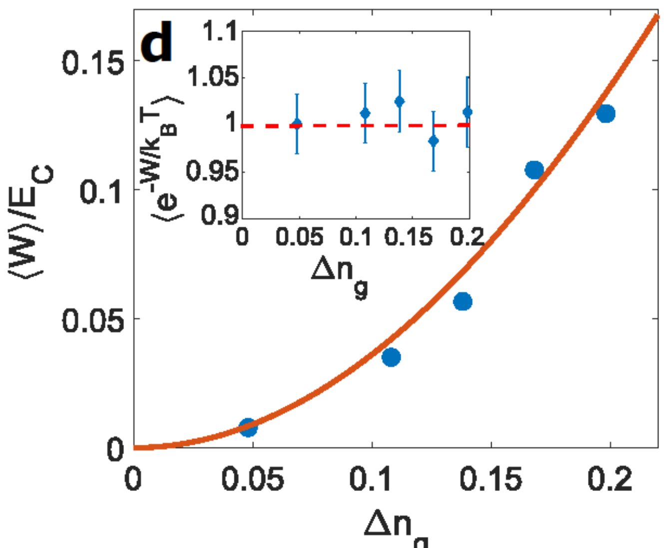
V. Cavina et al., Sci. Rep. (2016)





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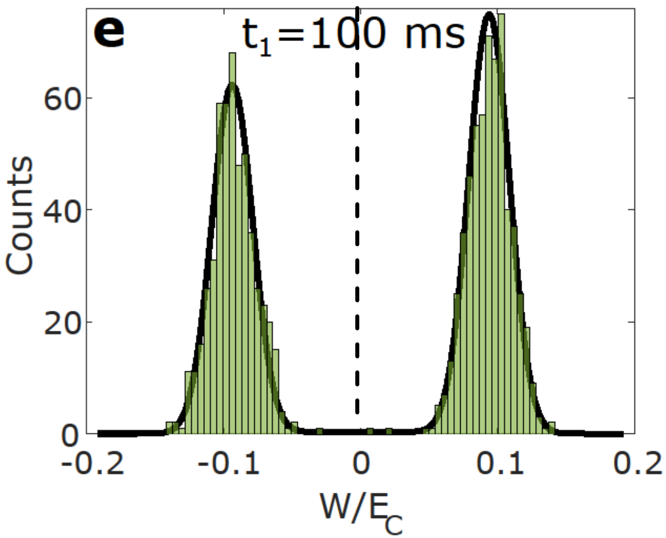
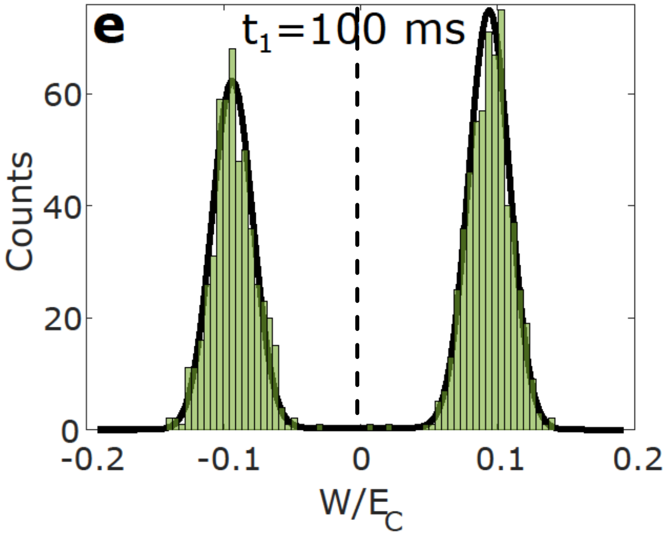
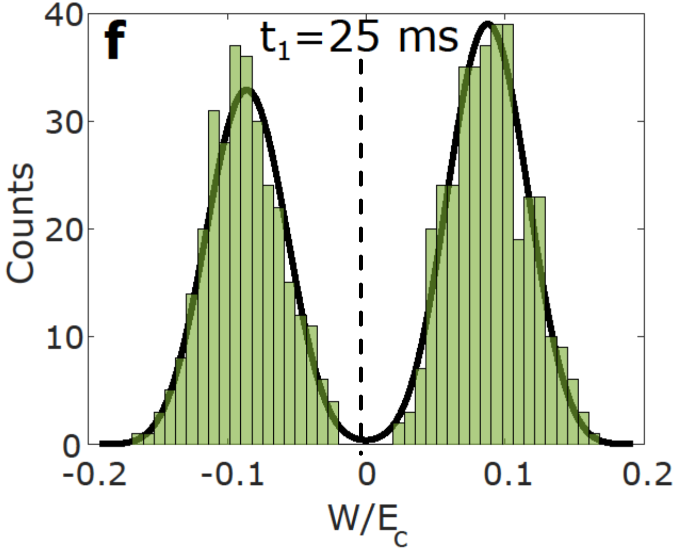
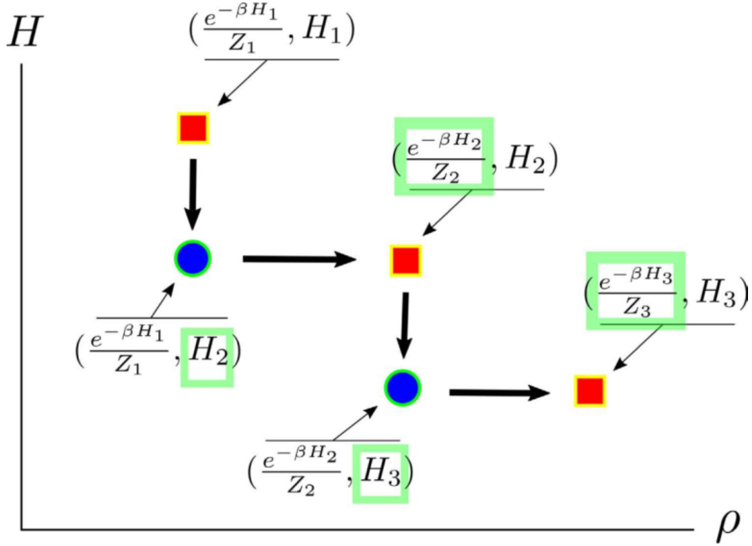


- Heat dissipated by imperfect quasi-static driving: broadening
- Additional irreversibility scales with the ramp slope



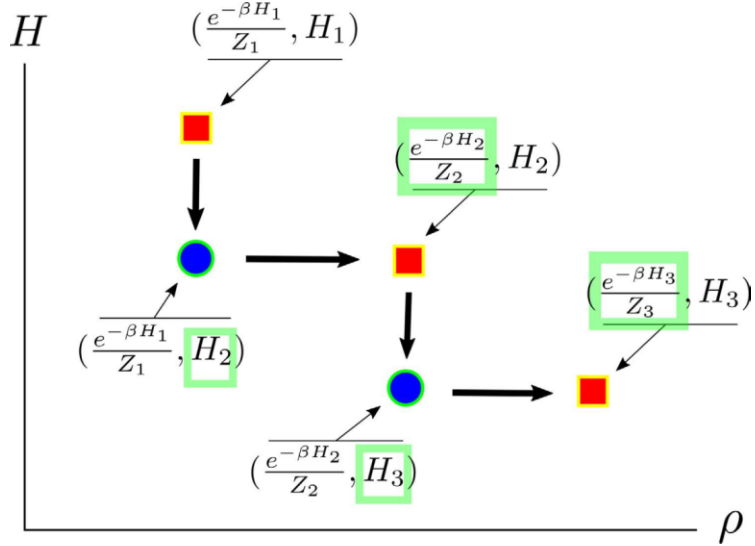


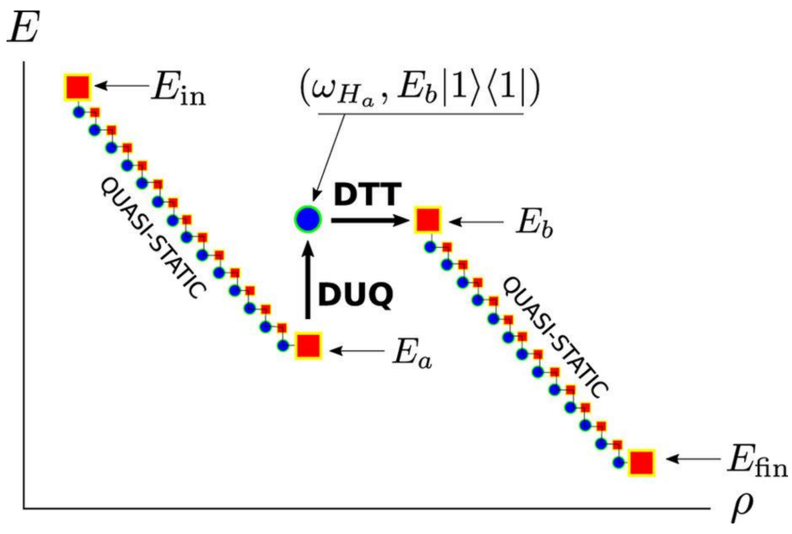
- Heat dissipated by imperfect quasi-static driving: broadening
- Additional irreversibility scales with the ramp slope
- Master equation approach: time evolution of work probability distribution $\rho(W,t)$
- Good agreement with data, **no free parameter**



V. Cavina et al., Sci. Rep. (2016): sequence of « discrete » steps describing a transformation:

- **Quench**: fast = no heat exchanged, only work
- **Thermalization**: no work done, heat removed from/released to the bath





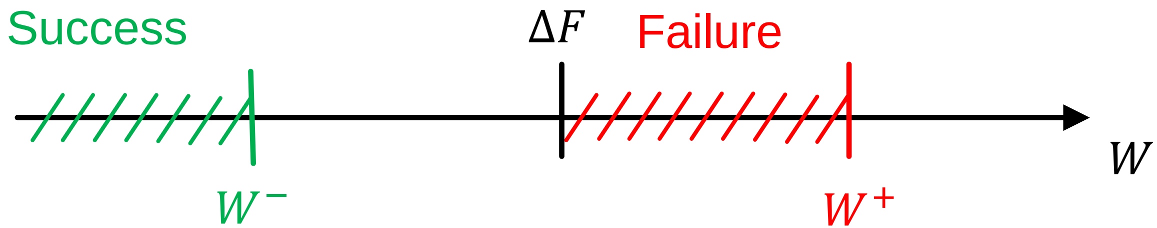
V. Cavina et al., Sci. Rep. (2016): sequence of « discrete » steps describing a transformation:

- **Quench**: fast = no heat exchanged, only work
- **Thermalization**: no work done, heat removed from/released to the bath
- Large quench = splits in two the work distribution

$$p(W) = p_0\delta(W - W^+) + (\mathbf{1} - \boldsymbol{p_0})\boldsymbol{\delta}(\boldsymbol{W} - \boldsymbol{W^-})$$

- $p_0 = \dfrac{1}{e^{-(E_b - E_a)/k_B T} + 1}$ (Gibbs)

A neat limit



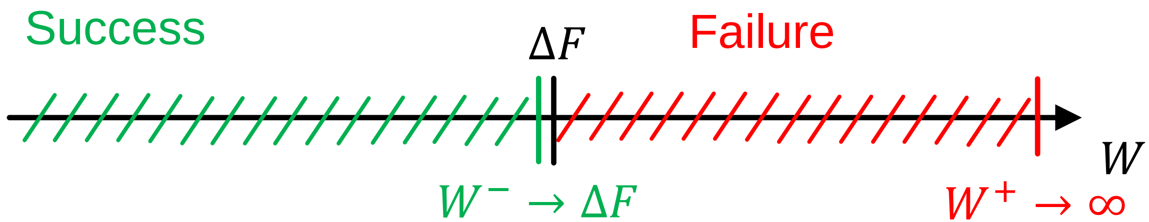
Requirements:

- Bound on minimum work extracted $W^- - \Delta F$ if successful attempt
- Bound on maximum work paid $W^+ - \Delta F$ if failed attempt
- Maximum probability $p_{W^+}(W \leq W^-)$ allowed by Jarzynski's equality

$$p_{W^+}(W \leq W^-) \leq \frac{e^{\Delta F/k_B T} - e^{-W^+/k_B T}}{e^{-W^-/k_B T} - e^{-W^+/k_B T}}$$

V. Cavina et al., Sci. Rep. (2016)

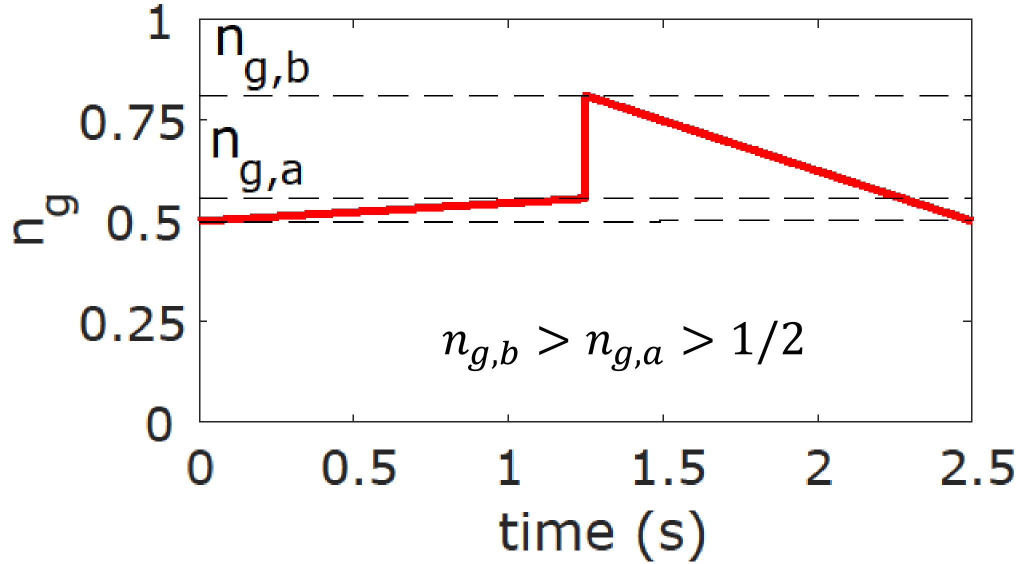
A neat limit



If we loosen requirements: $W^- \to \Delta F$, $W^+ \to \infty$

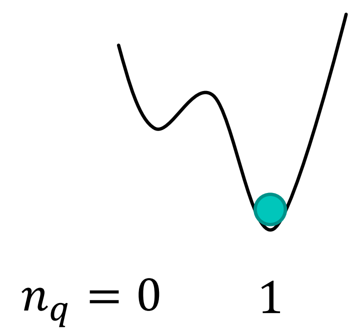
- Maximum probability $p_\infty(W \le W^-)$ allowed by Jarzynski's equality

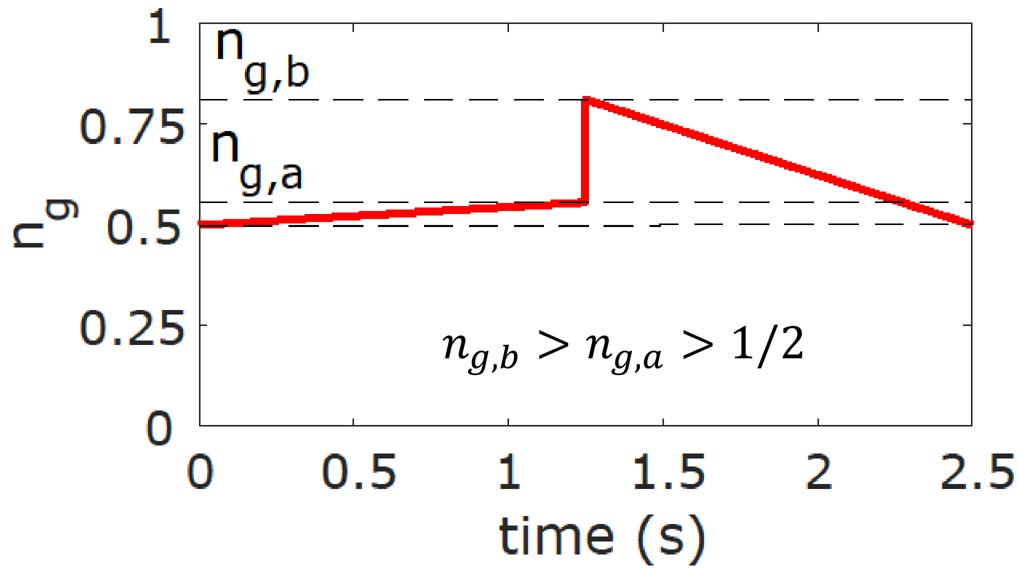
$$p_\infty(W \le W^-) \le e^{-(W^- - \Delta F)/k_B T} \xrightarrow[W^- \to \Delta F]{} 1$$



$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

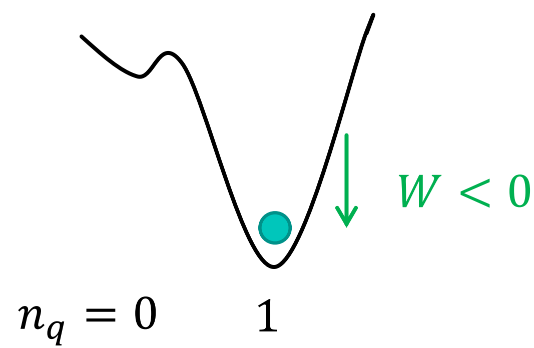
- $\Sigma(n_q) > 0$ if $n_{g,b} > n_{g,a} > 1/2$
- $\Sigma(n_q = 1) < -k_B T(\Delta S)_{quench}$

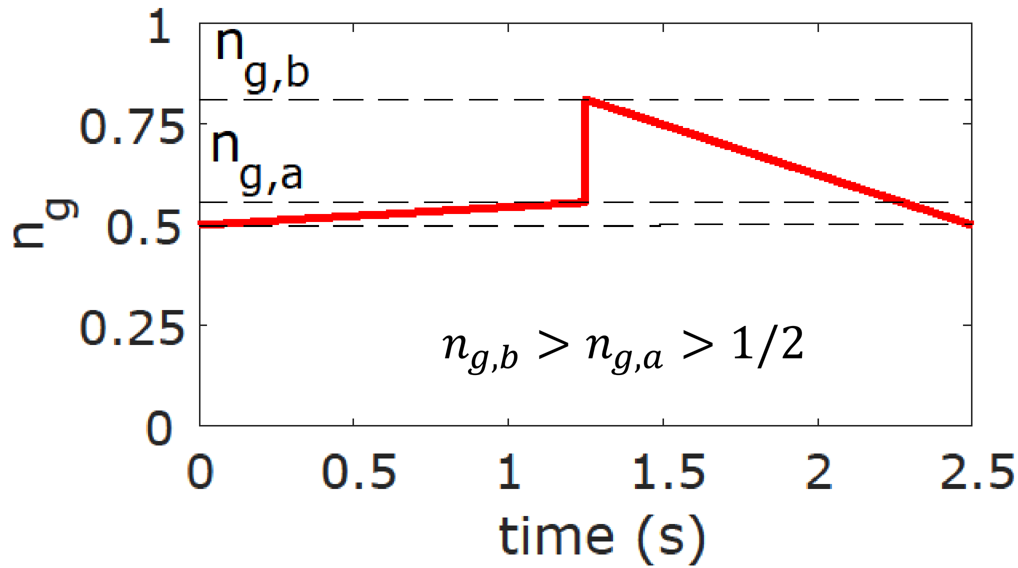




$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

- $\Sigma(n_q) > 0$ if $n_{g,b} > n_{g,a} > 1/2$
- $\Sigma(n_q = 1) < -k_B T(\Delta S)_{quench} \rightarrow W(n_q = 1) < 0$





$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

- $\Sigma(n_q) > 0$ if $n_{g,b} > n_{g,a} > 1/2$
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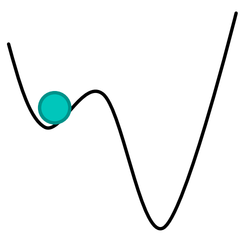
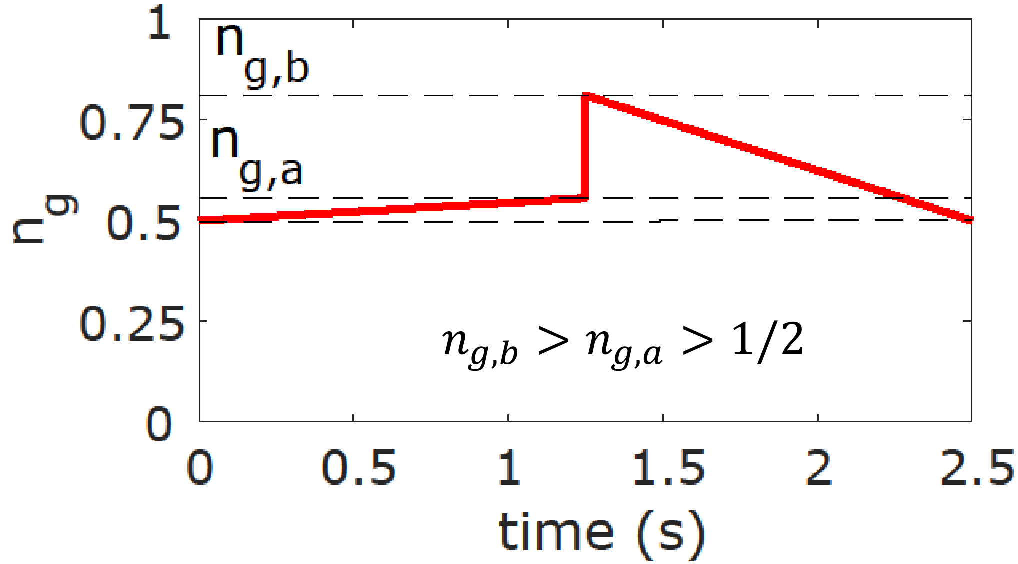
- $\Sigma(n_q = 0) > -k_B T(\Delta S)_{quench}$



$n_q = 0 \qquad 1$



$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

- $\Sigma(n_q) > 0$ if $n_{g,b} > n_{g,a} > 1/2$
- $\Sigma(n_q = 1) < -k_B T(\Delta S)_{quench} \rightarrow W(n_q = 1) < 0$
- $\Sigma(n_q = 0) > -k_B T(\Delta S)_{quench} \rightarrow W(n_q = 0) > 0$