Phase-coherent heat circulator based on multi-terminal Josephson junctions

Sun-Yong Hwang*, Francesco Giazotto**, and Björn Sothmann*

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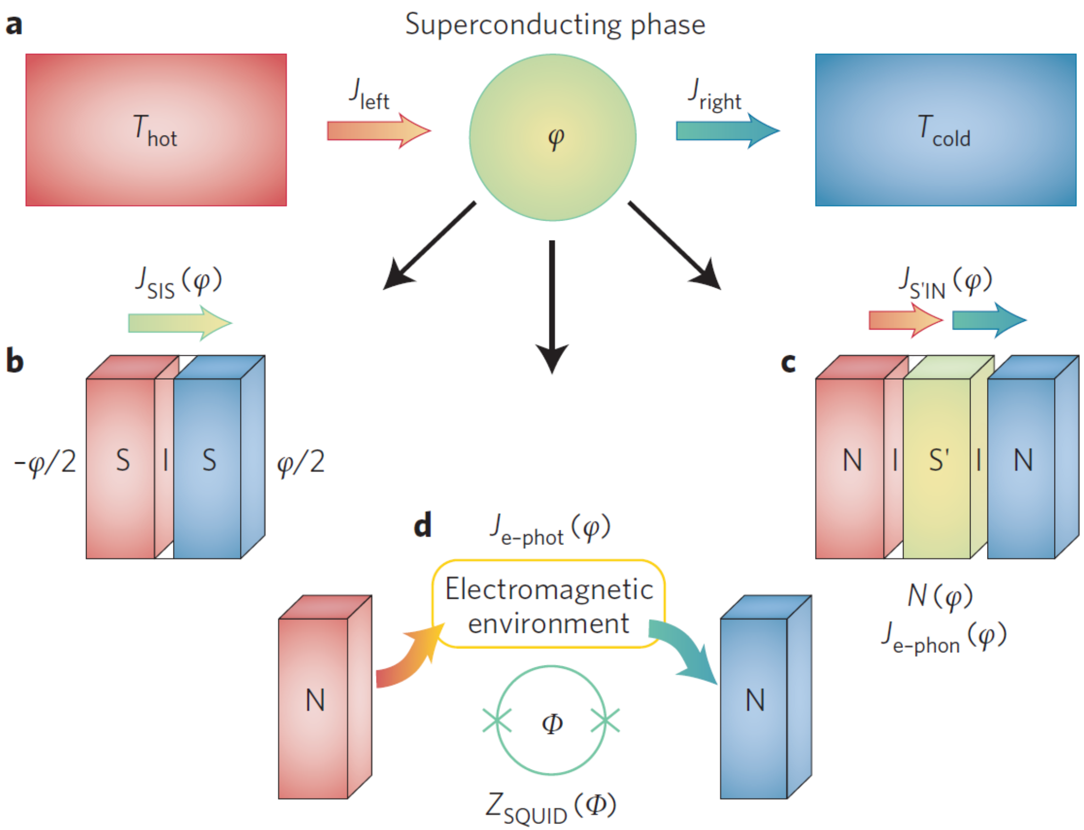
***NEST, Istituto Nanoscienze–CNR and Scuola Normale Superiore, Pisa, Italy*



Energy management

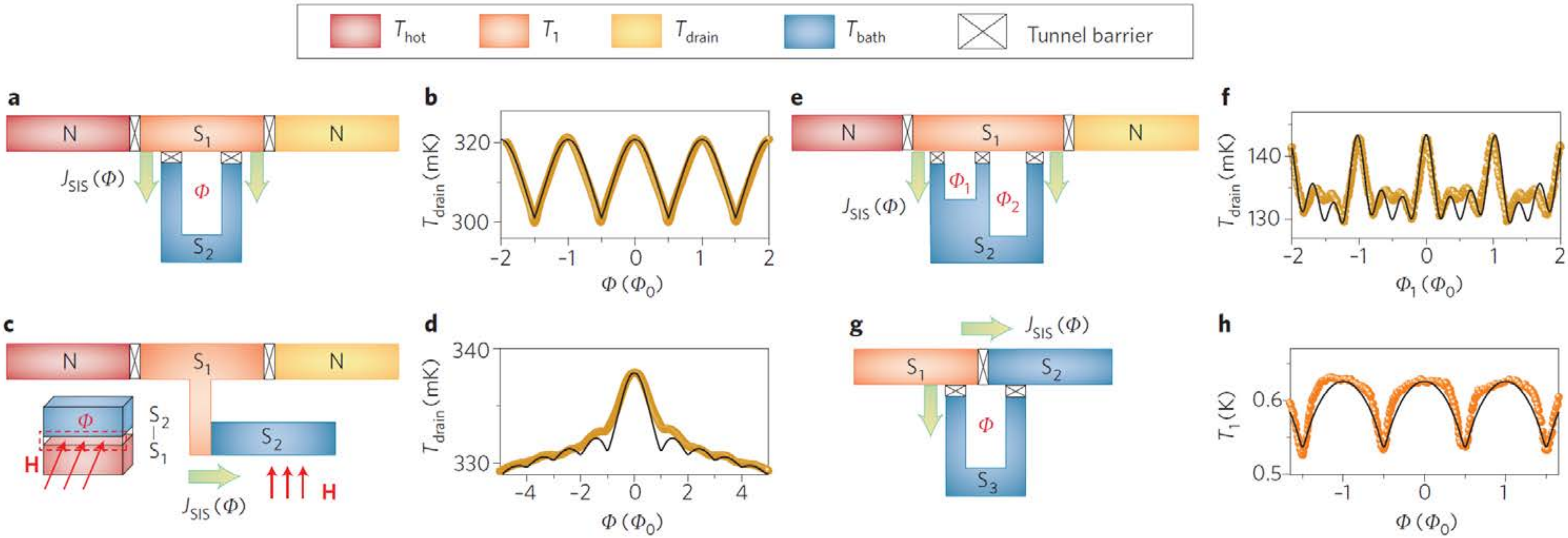
- -Thermal engines
- -Heat diodes and valves
- -Refrigerators
- -Coherent heat splitters

Thermal logic

- -Josephson tunnel circuits
- -Heat interferometers
- -Thermal transistors
- -Solid state memories

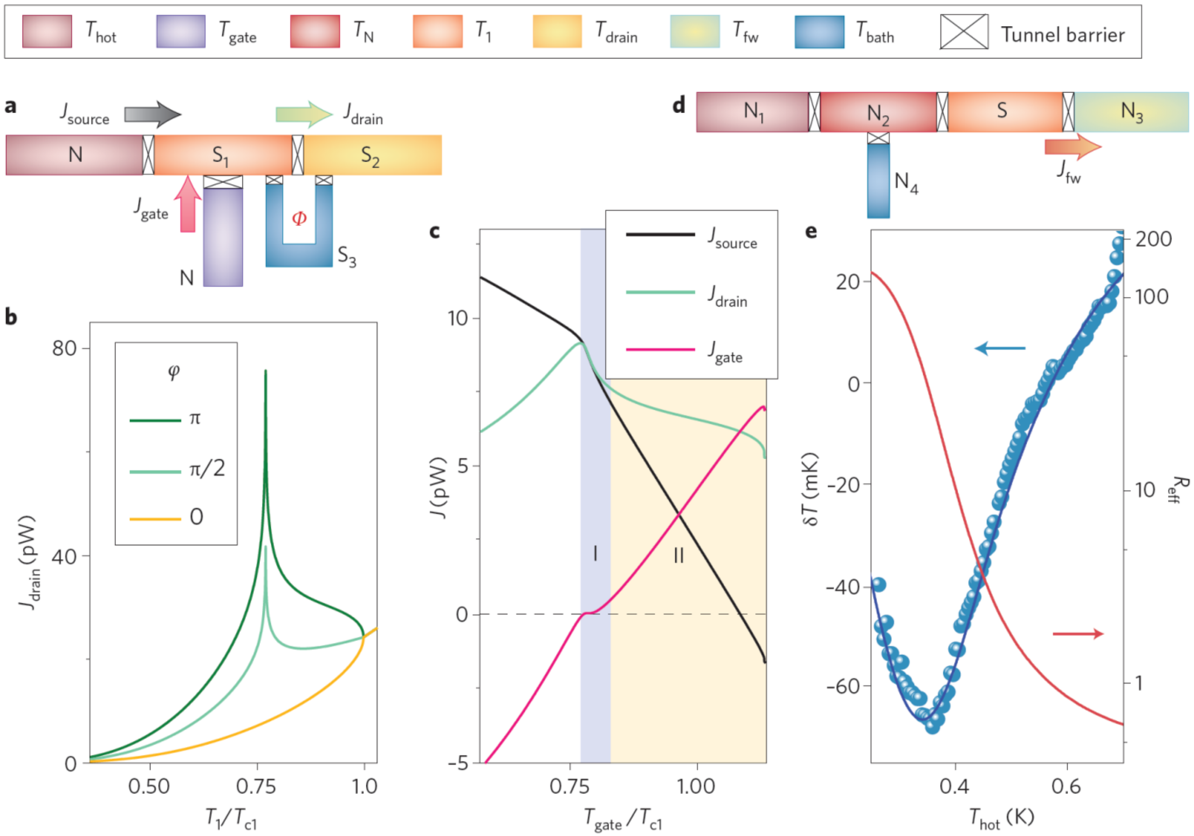
A. Fornieri and F. Giazotto, Nat. Nanotech. 12, 944 (2017).

Josephson heat interferometers



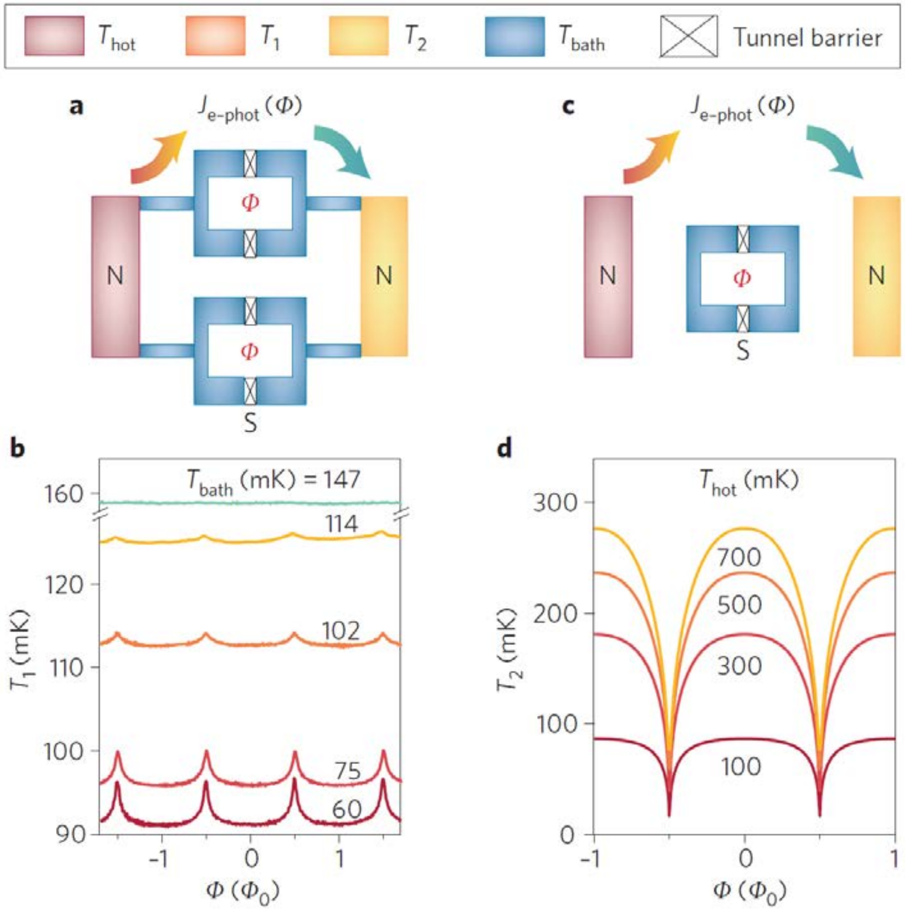
A. Fornieri and F. Giazotto, Nat. Nanotech. 12, 944 (2017).

Thermal transistors and thermal rectifiers



A. Fornieri and F. Giazotto, Nat. Nanotech. 12, 944 (2017).

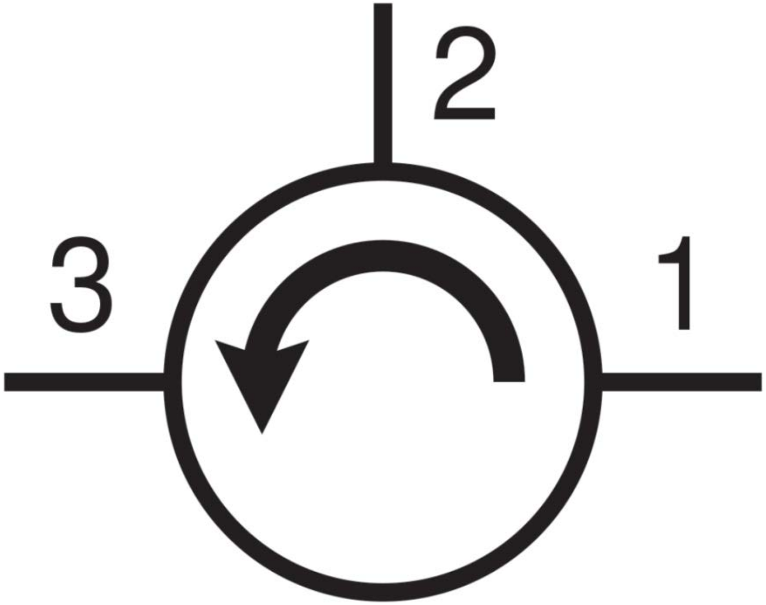
Photonic heat transistors



A. Fornieri and F. Giazotto, Nat. Nanotech. 12, 944 (2017).

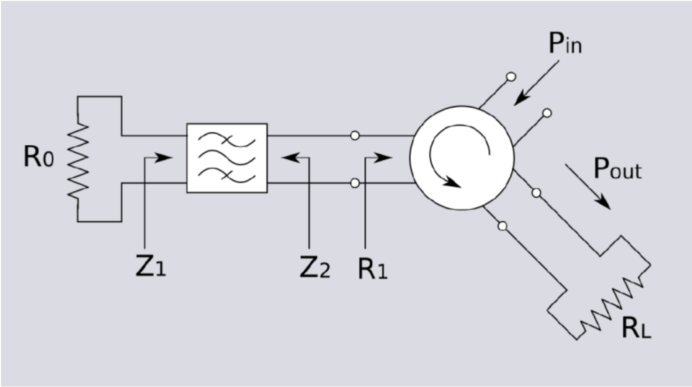
$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

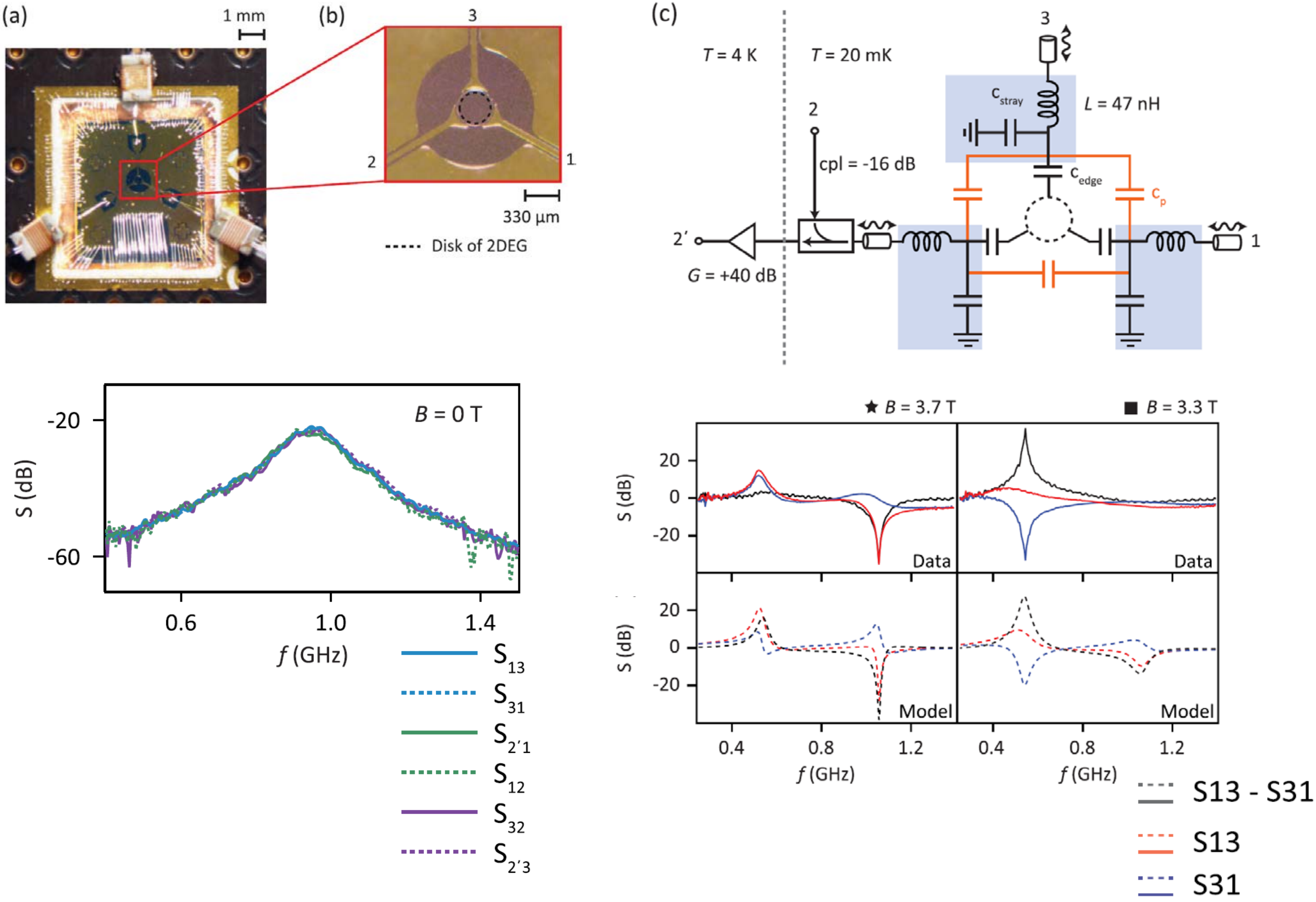
$$S_{ij} = S_{i \leftarrow j}$$



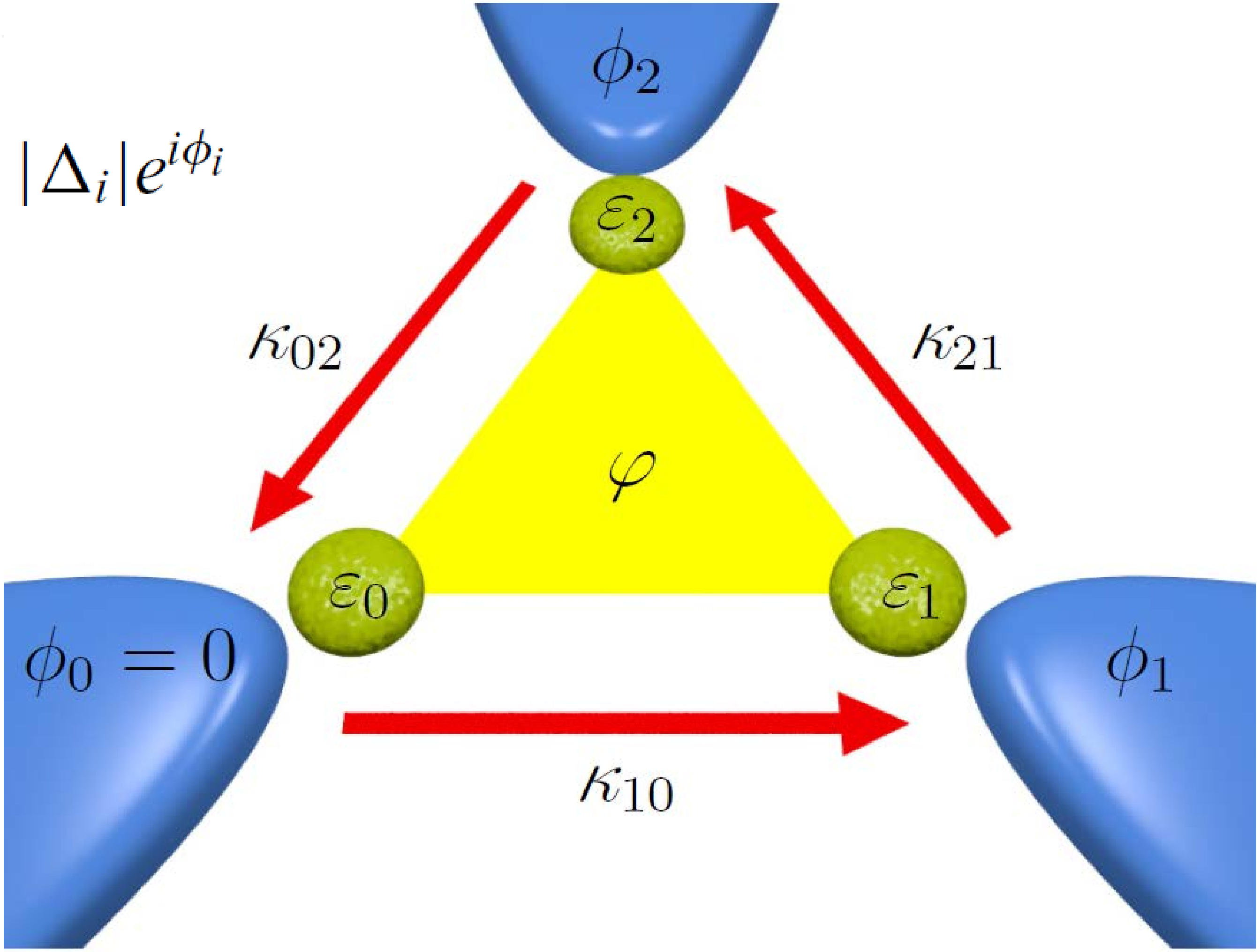
Images from Wikipedia

- Isolator
- Duplexer -> Radar
- Reflection amplifier

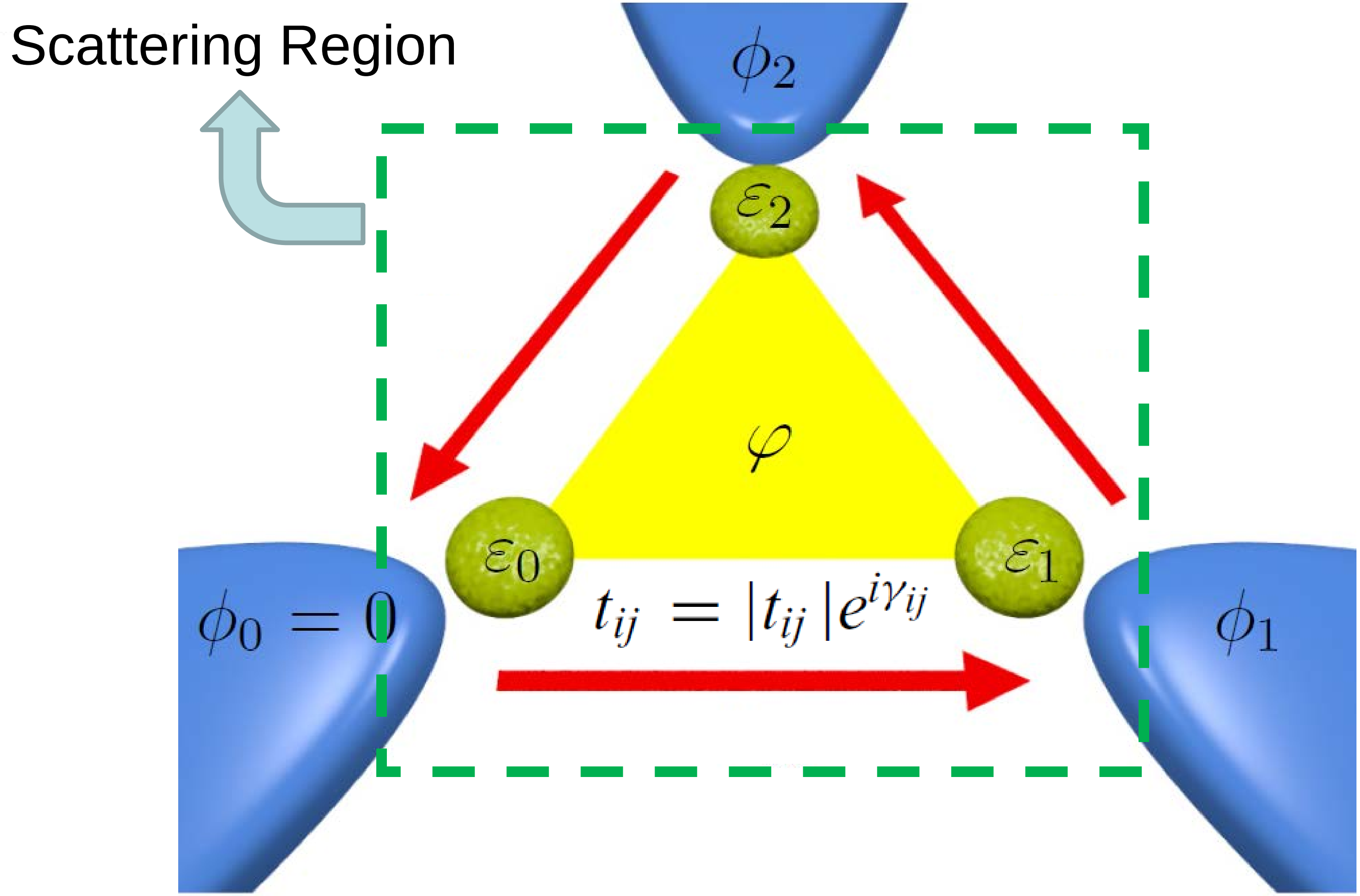



(a)
1 mm
(b)
3
2
1
330 µm
Disk of 2DEG
(c)
T = 4 K
T = 20 mK
3
C_stray
L = 47 nH
2
cpl = -16 dB
C_edge
C_p
2′
1
G = +40 dB
B = 0 T
S (dB)
-20
-60
0.6
1.0
1.4
f (GHz)
S13
S31
S2′1
S12
S32
S2′3
★ B = 3.7 T
■ B = 3.3 T
Data
Model
20
0
-20
0.4
0.8
1.2
S13 - S31
S13
S31

$$\Delta_i = |\Delta_i| e^{i\phi_i}$$

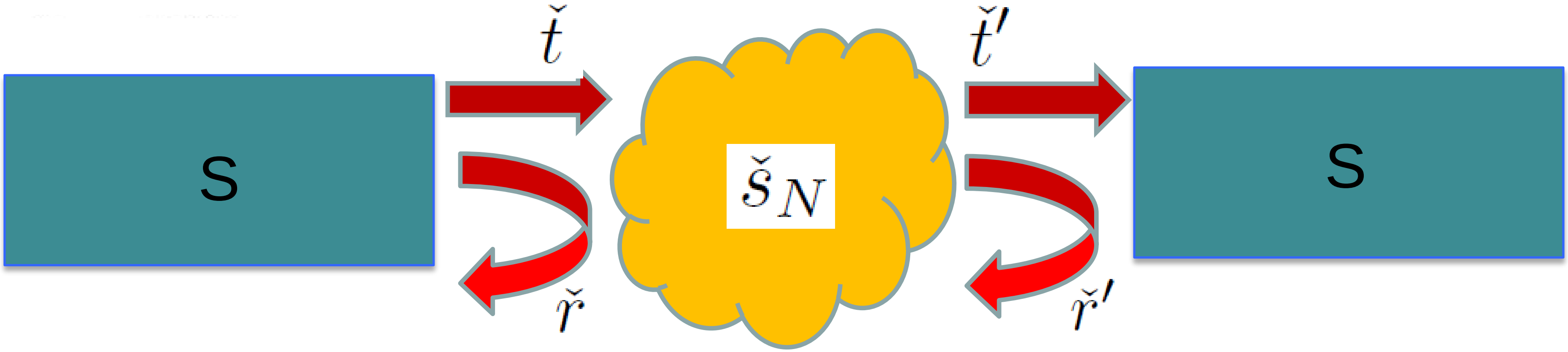


κ_{ij} : **Thermal conductance from *j* to *i***



$$\gamma_{10} + \gamma_{21} + \gamma_{02} = 2\pi\varphi/\varphi_0 \equiv \alpha \qquad \varphi_0 = h/e$$

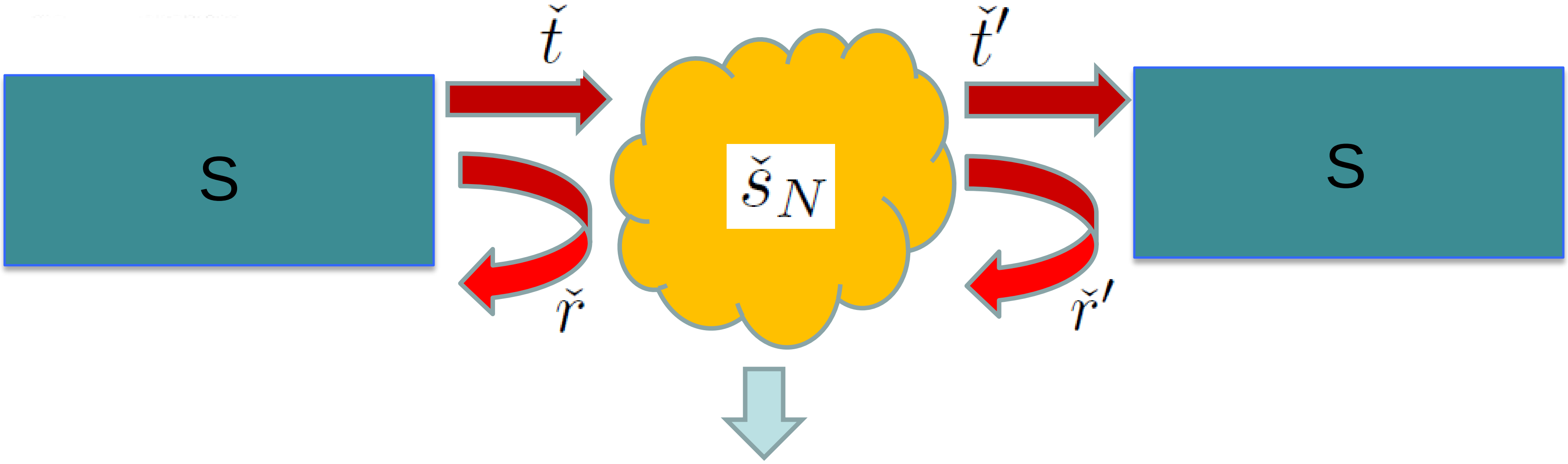
Solve BdG equation



Wavefunction matching

Scattering matrix: $\check{S}$

Solve BdG equation

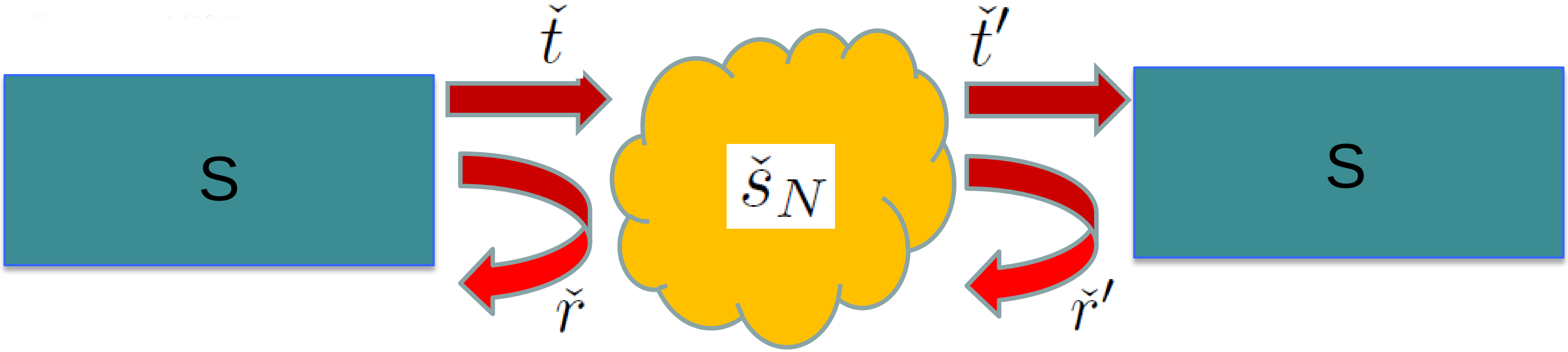


Wavefunction matching

Scattering matrix: $\check{S}$

$$\check{s} = \check{s}_{\mathrm{SNS}} = \check{r} + \check{t}'\check{s}_{\mathrm{N}}(\hat{I} - \check{r}'\check{s}_{\mathrm{N}})^{-1}\check{t}$$

Solve BdG equation



Wavefunction matching

Scattering matrix: $\check{S}$

$$\check{s} = \check{s}_{\mathrm{SNS}} = \check{r} + \check{t}'\check{s}_{\mathrm{N}}\left(\hat{I} - \check{r}'\check{s}_{\mathrm{N}}\right)^{-1}\check{t}$$

$$\hat{I} + \check{r}'\check{s}_{\mathrm{N}} + \check{r}'\check{s}_{\mathrm{N}}\check{r}'\check{s}_{\mathrm{N}} + \cdots$$

Scattering matrix: $\check{S}$

$$\mathcal{T}_{ij}(\omega) = \mathrm{Tr}[\check{s}^{\dagger}_{ij}(\omega)\check{s}_{ij}(\omega)] = |\check{s}_{ie,je}|^2 + |\check{s}_{ie,jh}|^2 + |\check{s}_{ih,je}|^2 + |\check{s}_{ih,jh}|^2$$

Thermal conductance from *j* to *i*

$$\kappa_{ij} = \frac{1}{hT}\int d\omega\ \omega^2[2\delta_{ij} - \mathcal{T}_{ij}(\omega)](-\partial_\omega f)$$

κ_{ij} **: Thermal conductance from *j* to *i***

$$\kappa_{ij}(\phi_0, \phi_1, \phi_2, \varphi) = \kappa_{ji}(-\phi_0, -\phi_1, -\phi_2, -\varphi)$$

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$$\frac{\kappa_{ij}}{\kappa_{ji}} \gg 1$$

<u>Heat Circulator</u>

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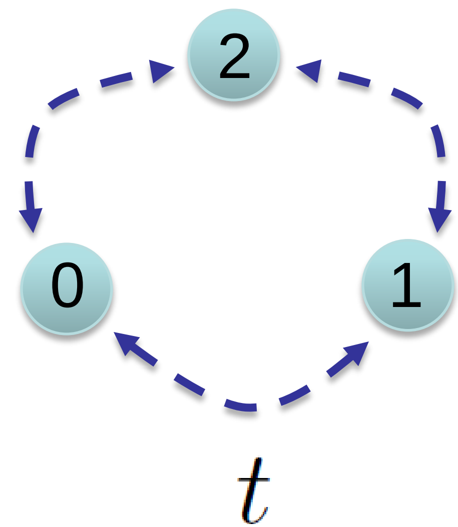
Heat Circulator

$$\kappa_{ij}(\phi_1, \phi_2, \varphi) \gg \kappa_{ji}(\phi_1, \phi_2, \varphi) \qquad \phi_0 \equiv 0$$

$$R = \frac{\kappa_{02}\kappa_{21}\kappa_{10} - \kappa_{01}\kappa_{12}\kappa_{20}}{\kappa_{02}\kappa_{21}\kappa_{10} + \kappa_{01}\kappa_{12}\kappa_{20}} = \frac{\circlearrowleft - \circlearrowright}{\circlearrowleft + \circlearrowright}$$



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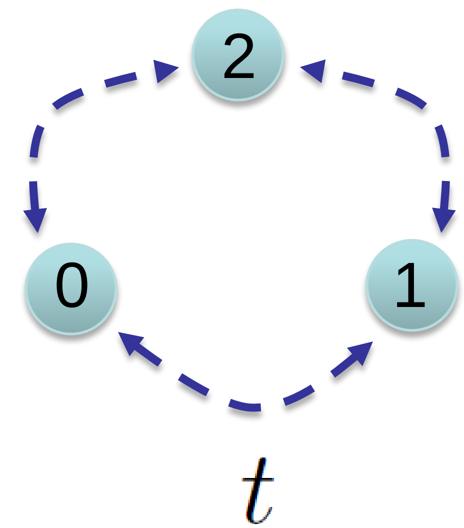
$R = 0$ No circulation



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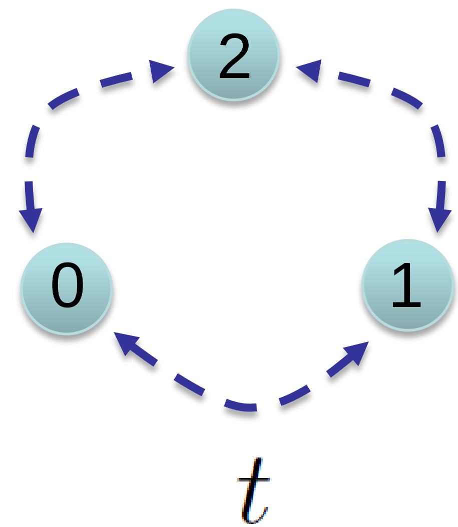
$R = 0$ No circulation

$R = -1$ Perfect clockwise circulation



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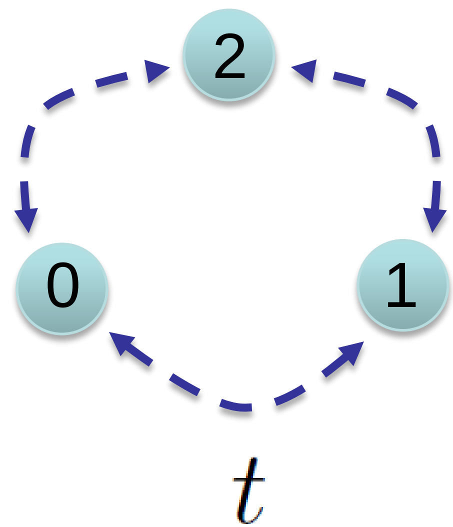
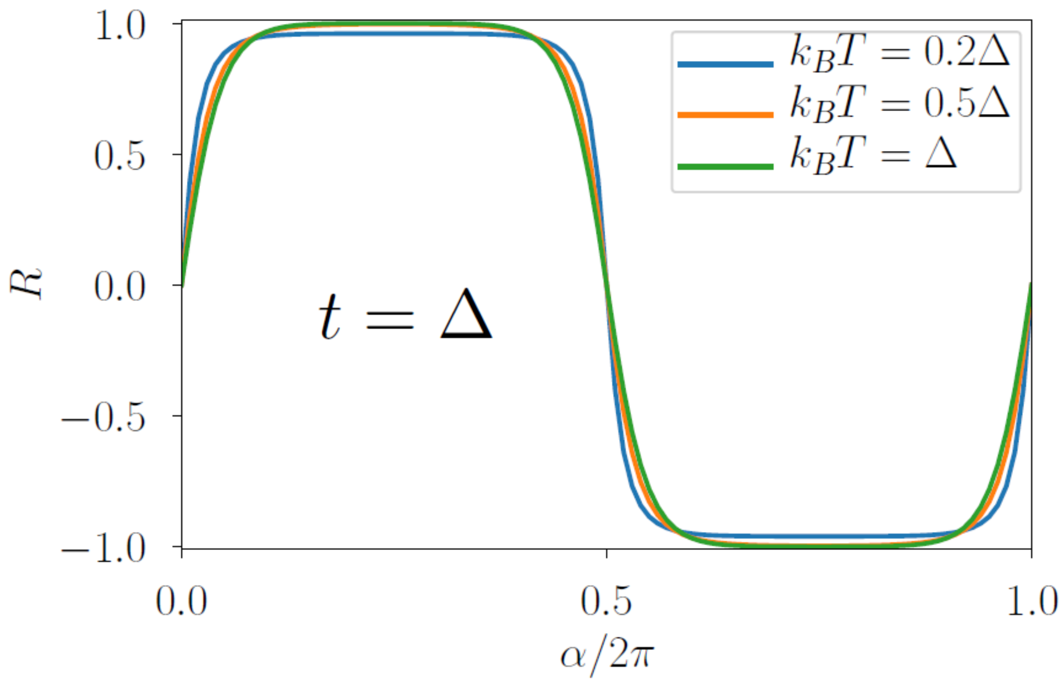


$R = 0$ No circulation

$R = -1$ Perfect <u>clockwise</u> circulation

$R = 1$ Perfect <u>counterclockwise</u> circulation

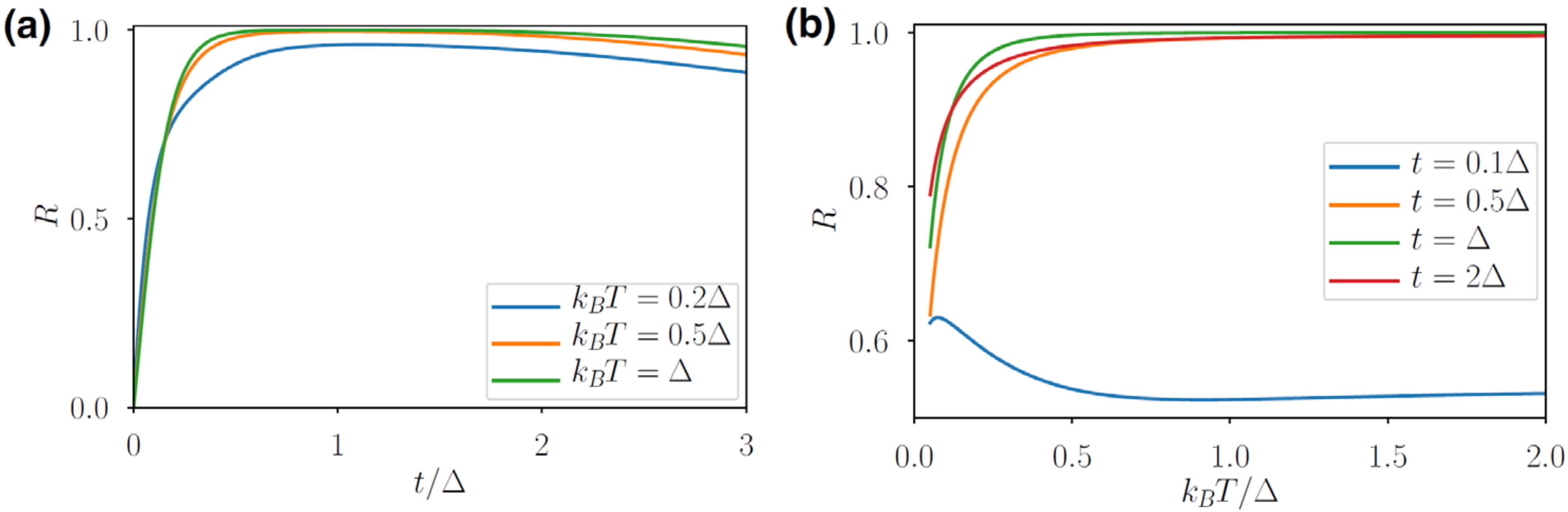


$$\varepsilon = E_F \equiv 0$$

Magnetic field of the order of mT

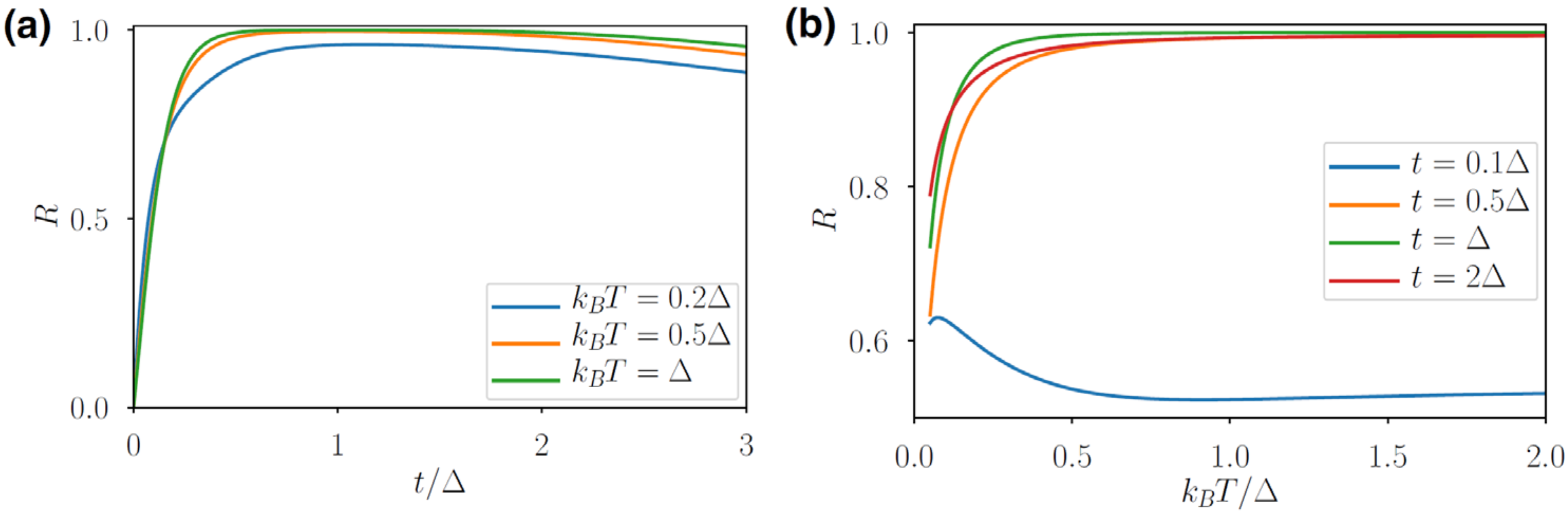
Phys. Rev. Applied **10**, 044062 (2018).

Flux-controlled heat circulator



$$\alpha = \pi/2 \qquad \varepsilon = E_F \equiv 0$$

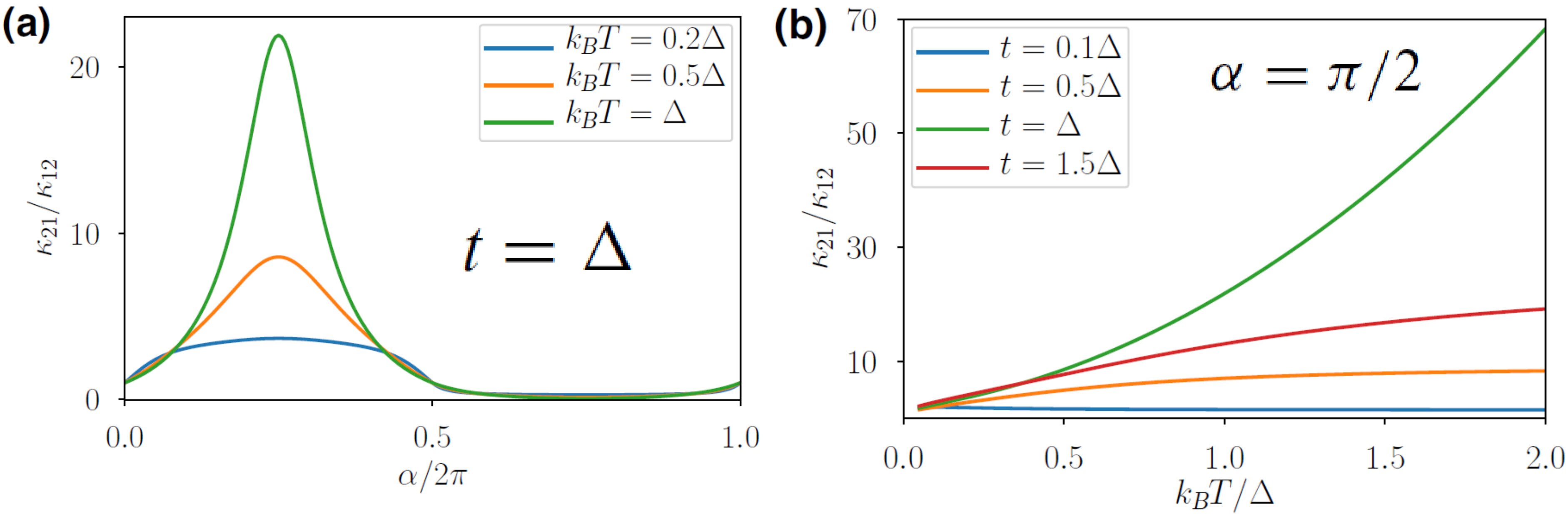
Phys. Rev. Applied **10**, 044062 (2018).

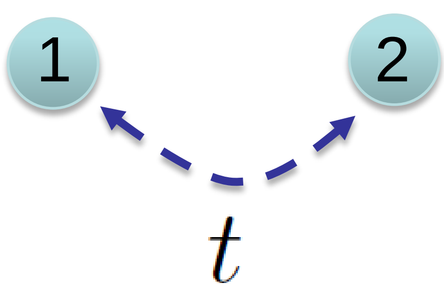


$$\alpha = \pi/2 \qquad \varepsilon = E_F \equiv 0$$

$$R \sim 1 \quad \text{for} \quad t > 0.5\Delta,\ k_BT > 0.2\Delta$$

Phys. Rev. Applied **10**, 044062 (2018).





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Phys. Rev. Applied **10**, 044062 (2018).

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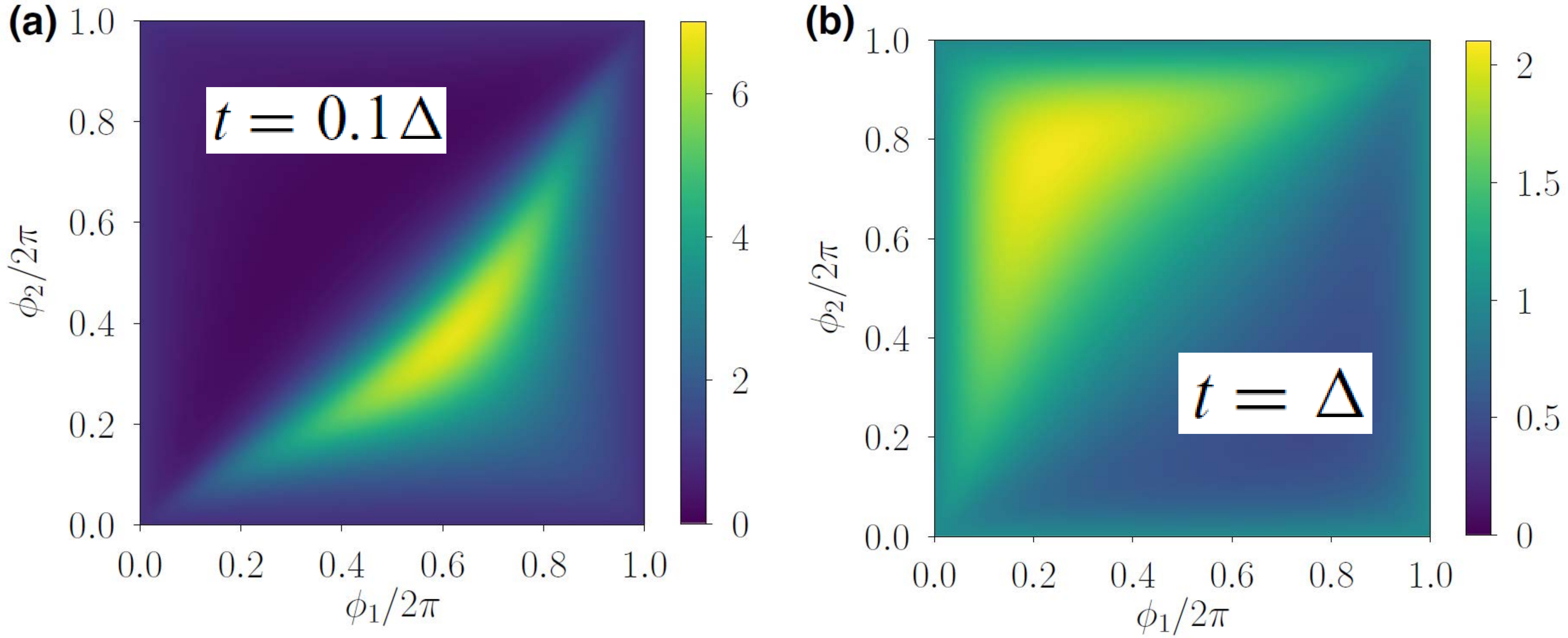
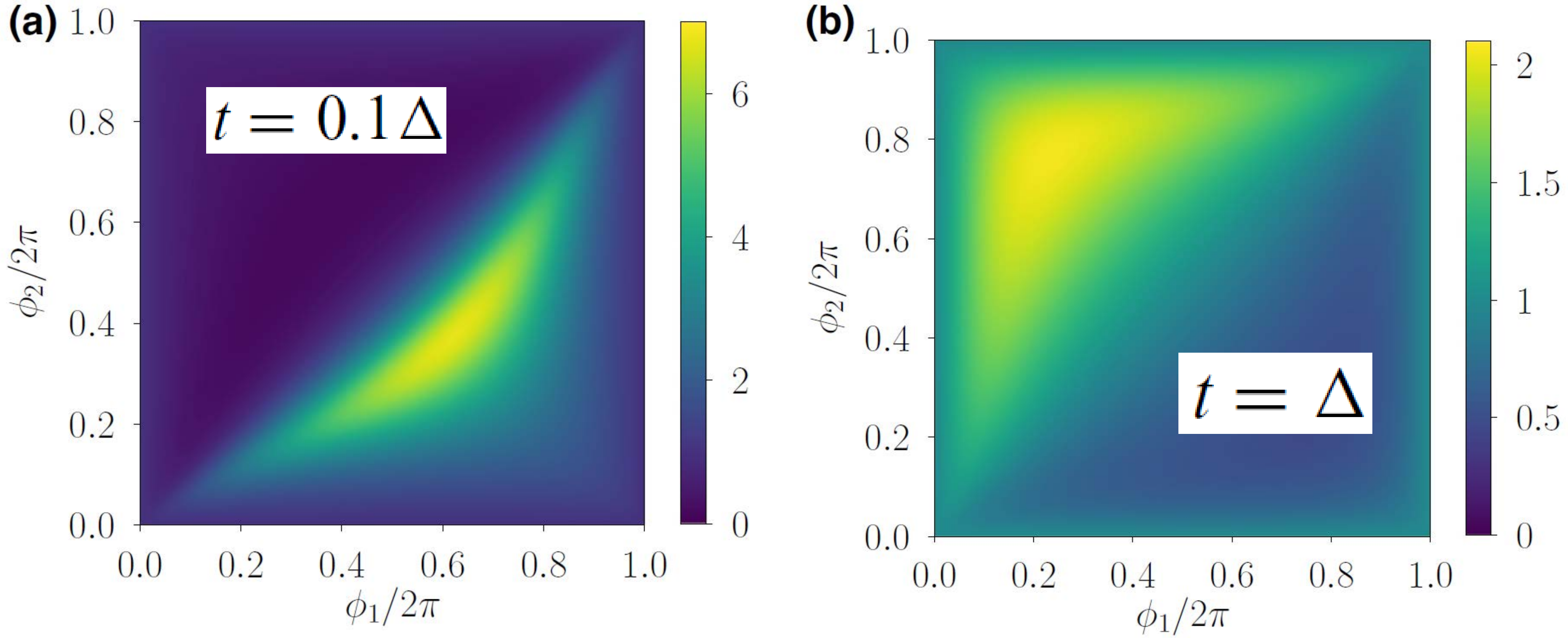
Phase-controlled rectification



$k_B T = 0.1\,\Delta$ $\quad$ $\varepsilon = E_F \equiv 0$ and $\alpha = 0$.

Phys. Rev. Applied **10**, 044062 (2018).

Phase-controlled rectification

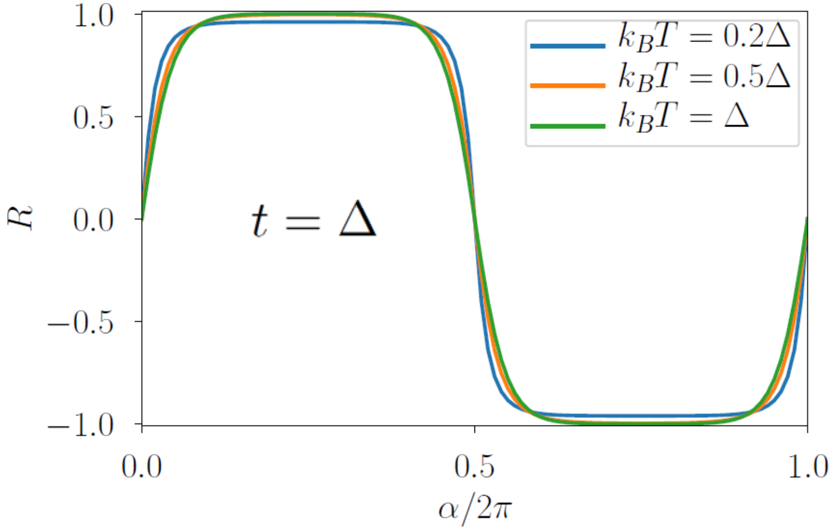


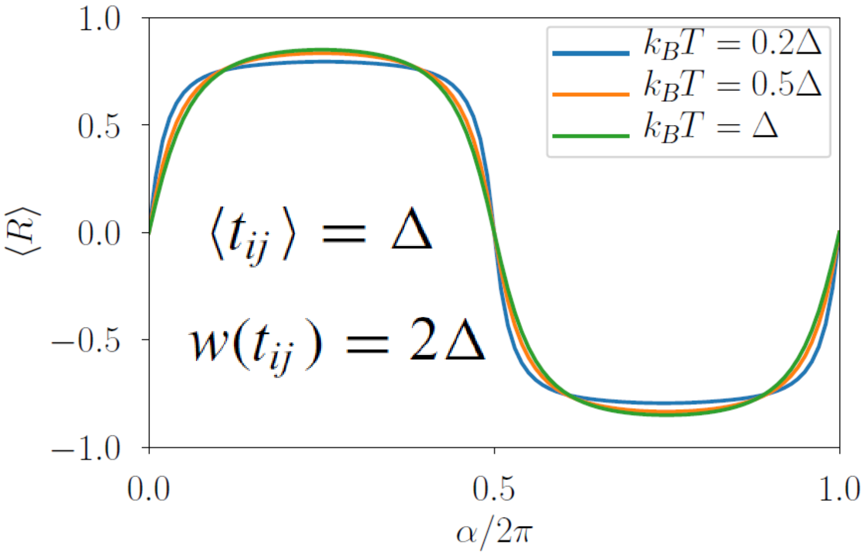
$k_B T = 0.1\,\Delta$ $\varepsilon = E_F \equiv 0$ and $\alpha = 0$

Superconducting <u>phase bias</u> can also generate large rectification <u>even without magnetic field.</u>

$$\varepsilon = E_F \equiv 0$$

$$\langle \varepsilon_i \rangle = E_F \equiv 0$$
$$w(\varepsilon_i) = 2\Delta$$





Only less than 15% efficiency drop with strong disorder

Summary

- **Heat circulator based on multi-terminal Josephson junctions**

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- Highly efficient with an almost ideal efficiency $|R|\sim 1$

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- **Highly efficient with an almost ideal efficiency |R|~1**

- **Tunable by magnetic flux and superconducting phases**

- **Experimentally relevant**
  -robust with respect to the disorder
  -magnetic field of the order of militesla

Summary

- Heat circulator based on multi-terminal Josephson junctions
- Highly efficient with an almost ideal efficiency $|R|\sim 1$
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  - -robust with respect to the disorder
  - -magnetic field of the order of militesla

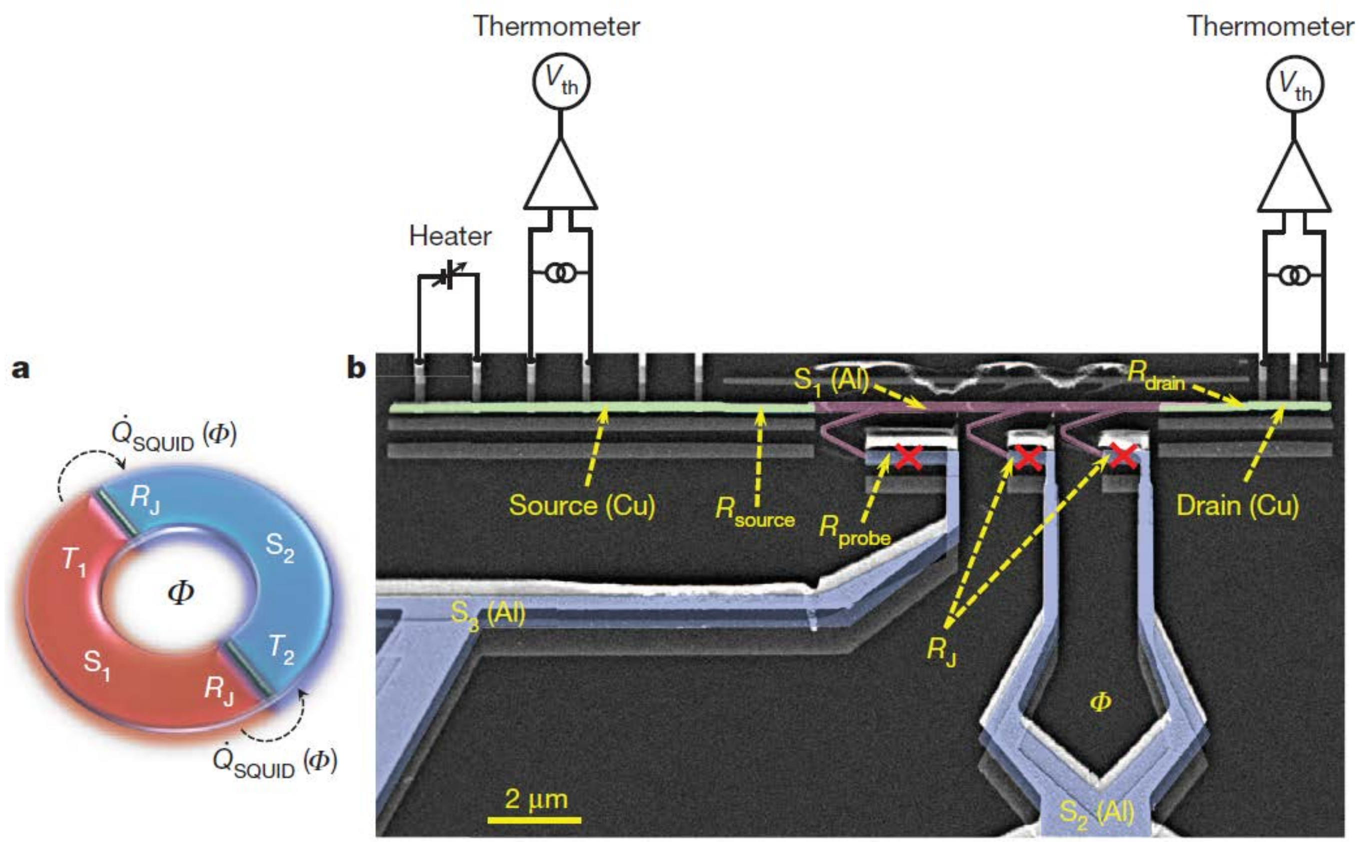
Hwang, Giazotto, and Sothmann, Phys. Rev. Applied **10**, 044062 (2018).

doi:10.1038/nature11702

The Josephson heat interferometer

Francesco Giazotto[1] & María José Martínez-Pérez[1]

[1]NEST, Istituto Nanoscienze—CNR and Scuola Normale Superiore, Piazza San Silvestro 12, I-56127 Pisa, Italy.



Faraday Effect

$$\beta = \nu B d$$

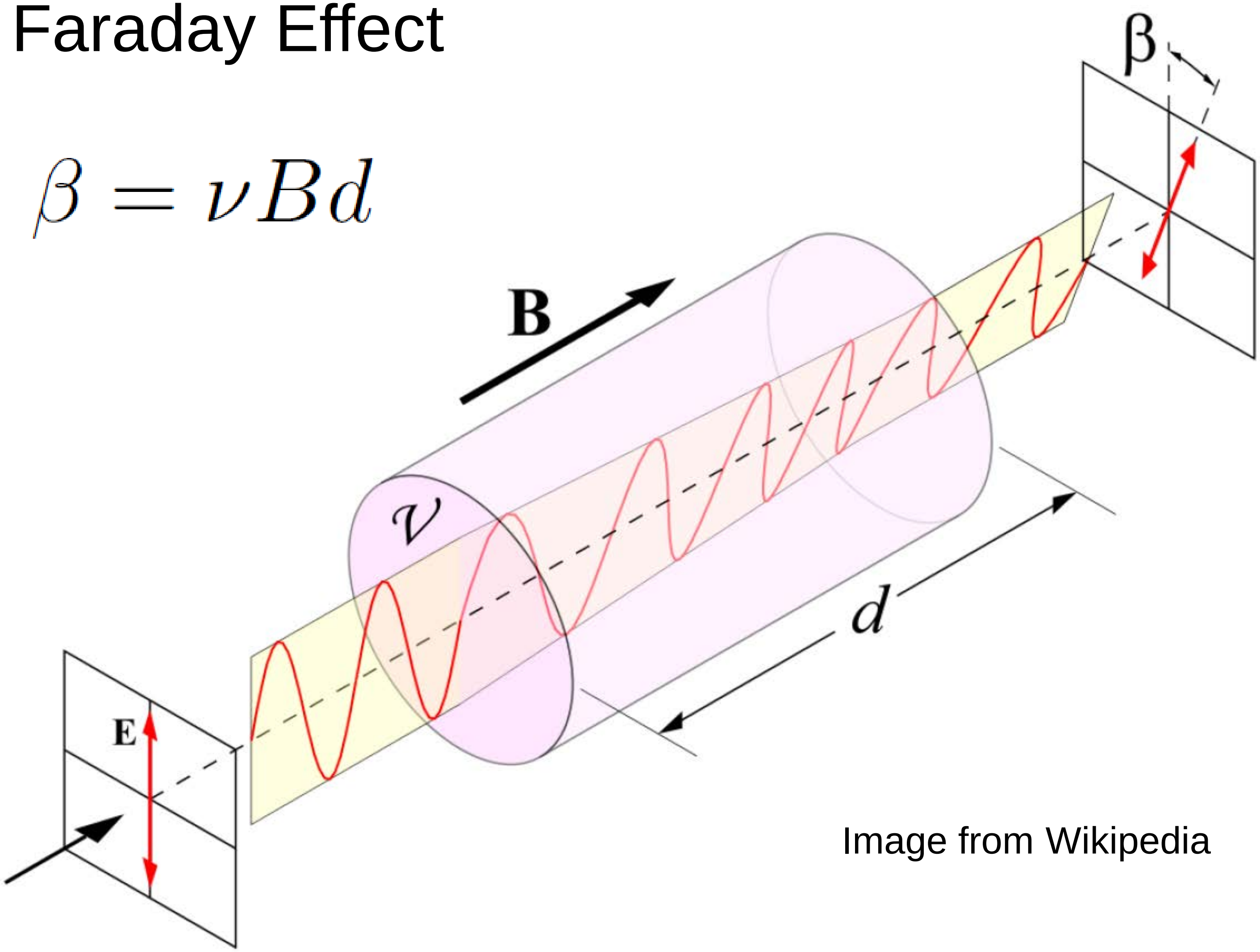
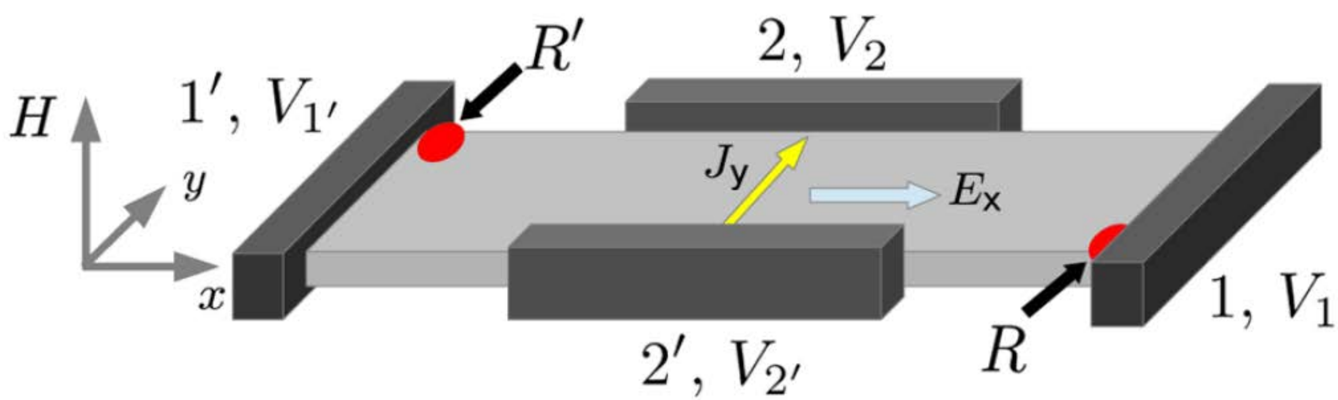


Image from Wikipedia

Hall Effect Gyrators and Circulators

Giovanni Viola[1,*] and David P. DiVincenzo[1,2,3]
[1]*Institute for Quantum Information, RWTH Aachen University, D-52056 Aachen, Germany*
[2]*Peter Grünberg Institute, Theoretical Nanoelectronics, Forschungszentrum Jülich, D-52428 Jülich, Germany*
[3]*Jülich-Aachen Research Alliance (JARA), Fundamentals of Future Information Technologies, D-52425 Jülich, Germany*
(Received 20 December 2013; published 2 May 2014; publisher error corrected 12 September 2014)

The electronic circulator and its close relative the gyrator are invaluable tools for noise management and signal routing in the current generation of low-temperature microwave systems for the implementation of new quantum technologies. The current implementation of these devices using the Faraday effect is satisfactory but requires a bulky structure whose physical dimension is close to the microwave wavelength



PHYSICAL REVIEW X **7,** 011007 (2017)

On-Chip Microwave Quantum Hall Circulator

A. C. Mahoney,[1,2] J. I. Colless,[1,2,*] S. J. Pauka,[1,2] J. M. Hornibrook,[1,2] J. D. Watson,[3,4] G. C. Gardner,[4,5] M. J. Manfra,[3,4,5] A. C. Doherty,[1] and D. J. Reilly[1,2,†]

[1]*ARC Centre of Excellence for Engineered Quantum Systems, School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia*
[2]*Station Q Sydney, The University of Sydney, Sydney, New South Wales 2006, Australia*
[3]*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA*
[4]*Birck Nanotechnology Center, School of Materials Engineering and School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*
[5]*Station Q Purdue, Purdue University, West Lafayette, Indiana 47907, USA*

(Received 10 September 2016; revised manuscript received 15 November 2016; published 24 January 2017)

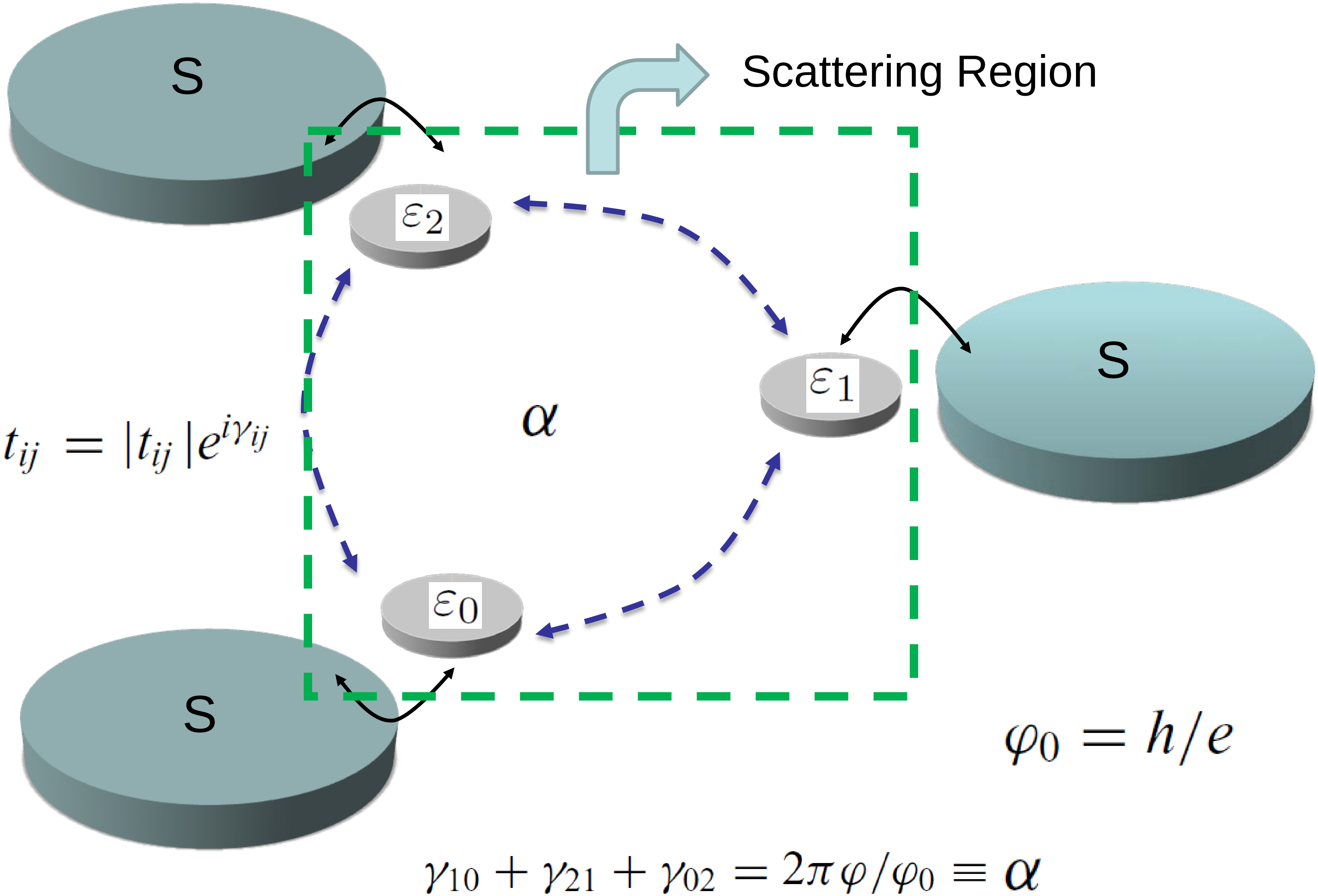
Circulators are nonreciprocal circuit elements that are integral to technologies including radar systems, microwave communication transceivers, and the readout of quantum information devices. Their non-reciprocity arises from the interference of microwaves over the centimeter scale of the signal wavelength, in the presence of bulky magnetic media that breaks time-reversal symmetry. Here, we realize a completely

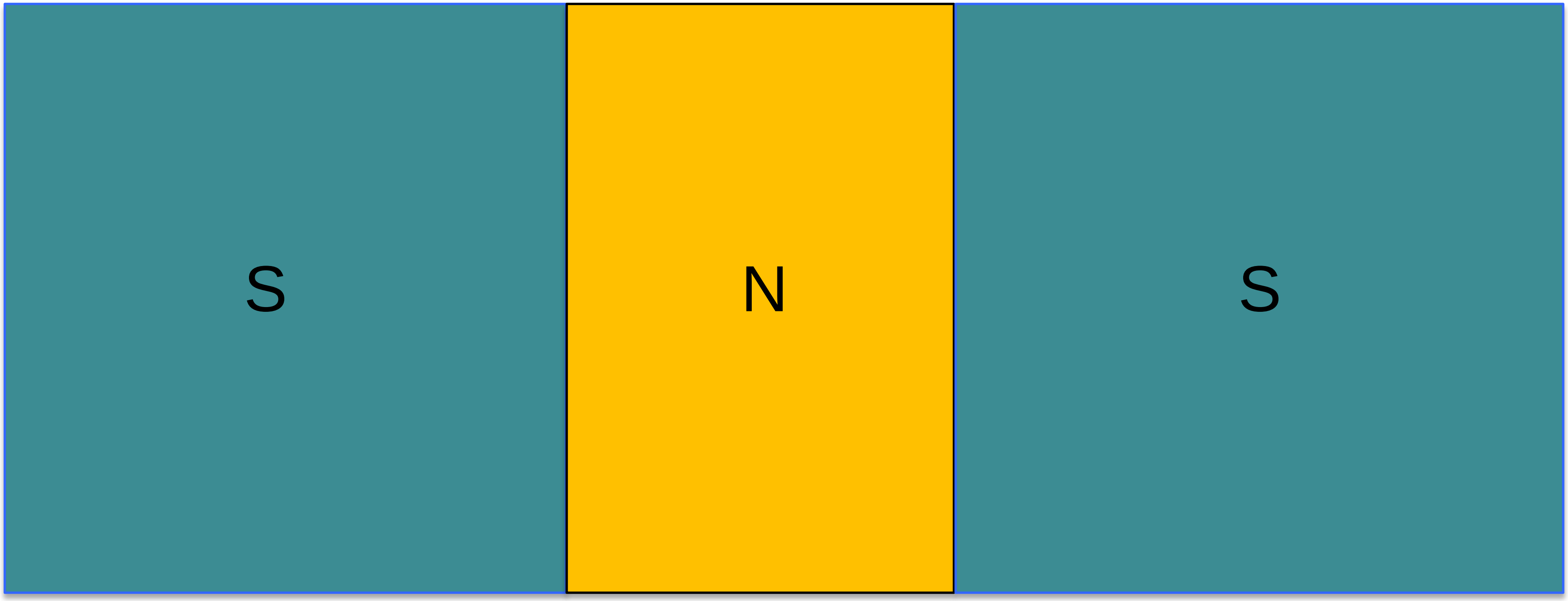
ENTROPY TRANSPORT BETWEEN TWO SUPERCONDUCTORS BY ELECTRON TUNNELING*

Kazumi Maki† and Allan Griffin

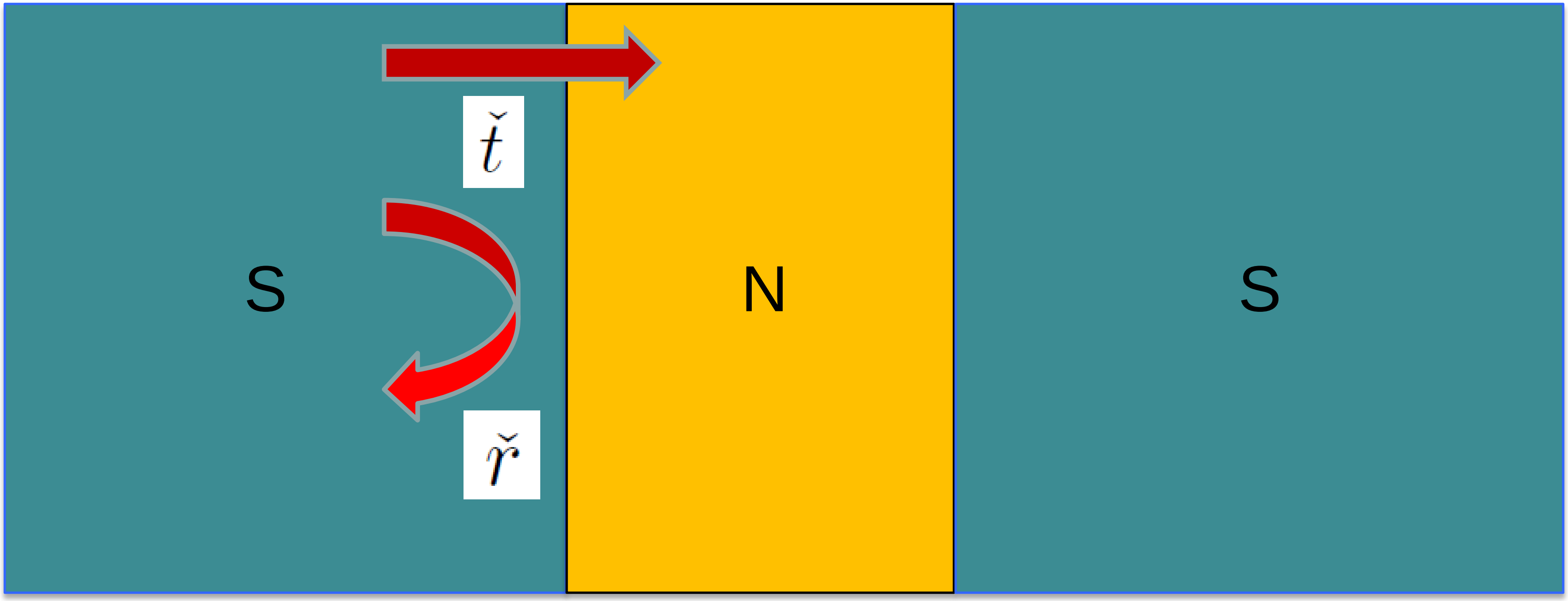
Department of Physics, University of California, San Diego, La Jolla, California
(Received 5 November 1965)

If an electronic heat current $\langle Q \rangle$ is made to flow from one metal to another across a thin insulating barrier (for example, an oxide layer), there will appear a temperature drop δT and an associated surface thermal resistance ($R \equiv \delta T / \langle Q \rangle$, if the cross section of the metals is unity). In this Letter, we report some preliminary calculations of this tunneling heat current. We find that R depends very much on whether the metals are in the superconducting or normal phases. In addition there is an oscillatory heat flux due to fluctuations in the quasiparticle tunneling between two superconductors. We might also add that the change in chemical potential due to the temperature difference δT can give rise to the ac and dc Josephson supercurrents.[1,2] However, our calculations show that no entropy is carried by these currents, which is in agreement with one's physical expectations.

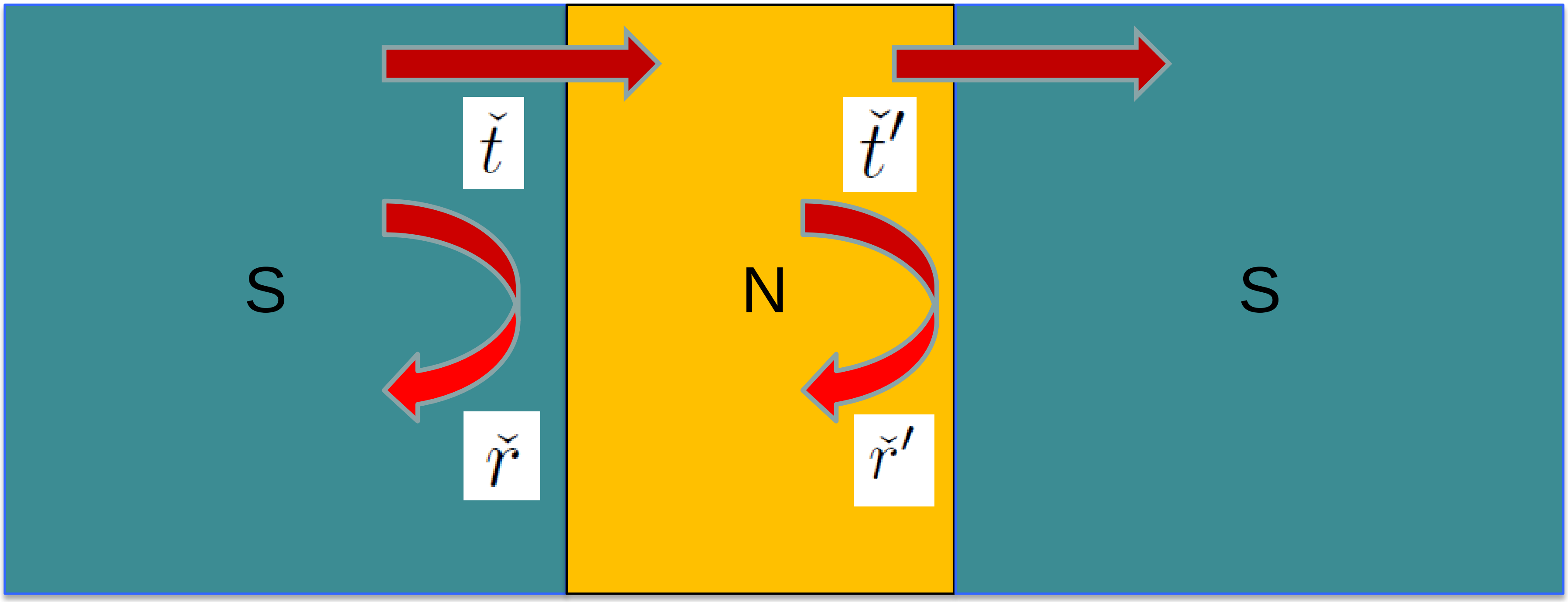

S
Scattering Region
ε_2
S
ε_1
α
$t_{ij} = |t_{ij}|e^{i\gamma_{ij}}$
ε_0
S
$\varphi_0 = h/e$
$\gamma_{10} + \gamma_{21} + \gamma_{02} = 2\pi\varphi/\varphi_0 \equiv \alpha$


S
N
S

Solve BdG equation

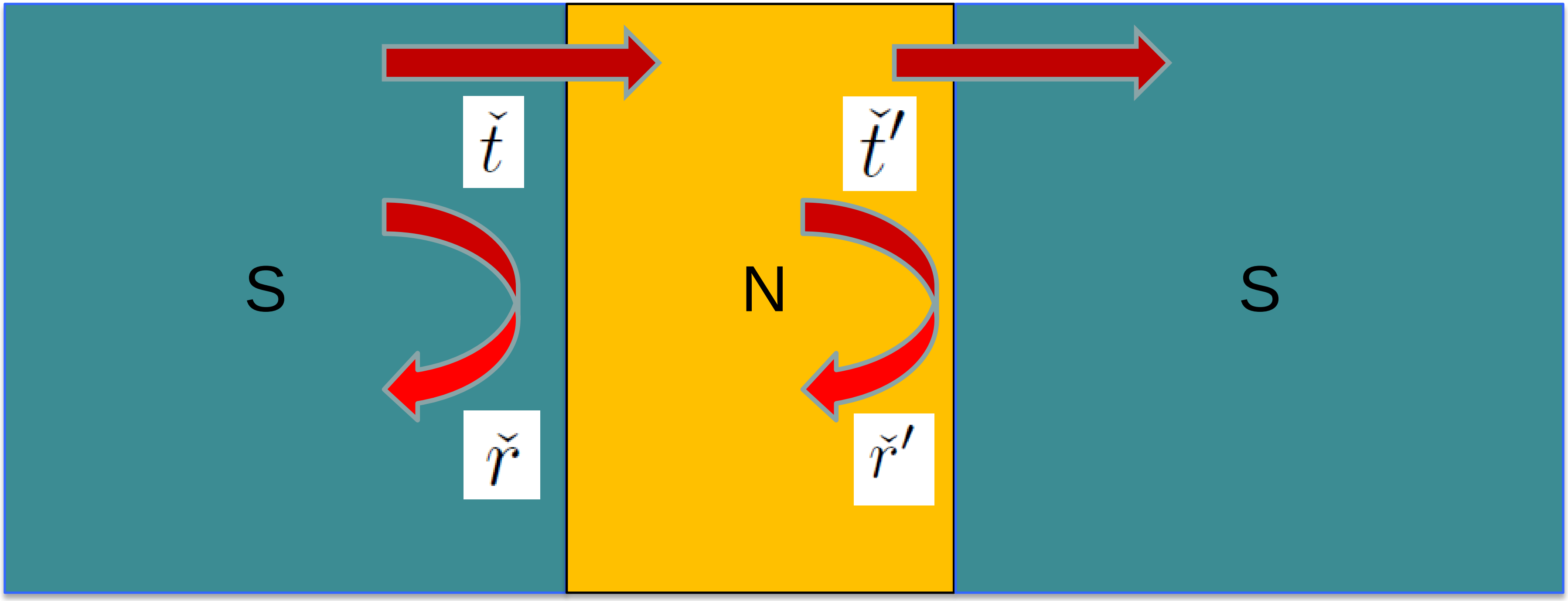


Solve BdG equation



Solve BdG equation

Scattering Matrix: $\check{s}_N$



$$\check{s} = \check{s}_{\mathrm{SNS}} = \check{r} + \check{t}'\check{s}_{\mathrm{N}}(\hat{I} - \check{r}'\check{s}_{\mathrm{N}})^{-1}\check{t}$$

$$\check{s} = \left(\begin{array}{ccc|ccc} \check{s}_{0e,0e} & \check{s}_{0e,1e} & \check{s}_{0e,2e} & \check{s}_{0e,0h} & \check{s}_{0e,1h} & \check{s}_{0e,2h} \\ \check{s}_{1e,0e} & \check{s}_{1e,1e} & \check{s}_{1e,2e} & \check{s}_{1e,0h} & \check{s}_{1e,1h} & \check{s}_{1e,2h} \\ \check{s}_{2e,0e} & \check{s}_{2e,1e} & \check{s}_{2e,2e} & \check{s}_{2e,0h} & \check{s}_{2e,1h} & \check{s}_{2e,2h} \\ \hline \check{s}_{0h,0e} & \check{s}_{0h,1e} & \check{s}_{0h,2e} & \check{s}_{0h,0h} & \check{s}_{0h,1h} & \check{s}_{0h,2h} \\ \check{s}_{1h,0e} & \check{s}_{1h,1e} & \check{s}_{1h,2e} & \check{s}_{1h,0h} & \check{s}_{1h,1h} & \check{s}_{1h,2h} \\ \check{s}_{2h,0e} & \check{s}_{2h,1e} & \check{s}_{2h,2e} & \check{s}_{2h,0h} & \check{s}_{2h,1h} & \check{s}_{2h,2h} \end{array}\right)$$

$$\check{s} = \left(\begin{array}{ccc|ccc} \check{s}_{0e,0e} & \check{s}_{0e,1e} & \check{s}_{0e,2e} & \check{s}_{0e,0h} & \check{s}_{0e,1h} & \check{s}_{0e,2h} \\ \check{s}_{1e,0e} & \check{s}_{1e,1e} & \check{s}_{1e,2e} & \check{s}_{1e,0h} & \check{s}_{1e,1h} & \check{s}_{1e,2h} \\ \check{s}_{2e,0e} & \check{s}_{2e,1e} & \check{s}_{2e,2e} & \check{s}_{2e,0h} & \check{s}_{2e,1h} & \check{s}_{2e,2h} \\ \hline \check{s}_{0h,0e} & \check{s}_{0h,1e} & \check{s}_{0h,2e} & \check{s}_{0h,0h} & \check{s}_{0h,1h} & \check{s}_{0h,2h} \\ \check{s}_{1h,0e} & \check{s}_{1h,1e} & \check{s}_{1h,2e} & \check{s}_{1h,0h} & \check{s}_{1h,1h} & \check{s}_{1h,2h} \\ \check{s}_{2h,0e} & \check{s}_{2h,1e} & \check{s}_{2h,2e} & \check{s}_{2h,0h} & \check{s}_{2h,1h} & \check{s}_{2h,2h} \end{array}\right)$$

$$\mathcal{T}_{ij}(\omega) = \mathrm{Tr}[\check{s}^{\dagger}_{ij}(\omega)\check{s}_{ij}(\omega)] = \left|\check{s}_{ie,je}\right|^2 + \left|\check{s}_{ie,jh}\right|^2 + \left|\check{s}_{ih,je}\right|^2 + \left|\check{s}_{ih,jh}\right|^2$$