UNIVERSITY of ROCHESTER

U.S. DEPARTMENT OF ENERGY
Office of Science

Thermodynamics of Resonance fluorescence

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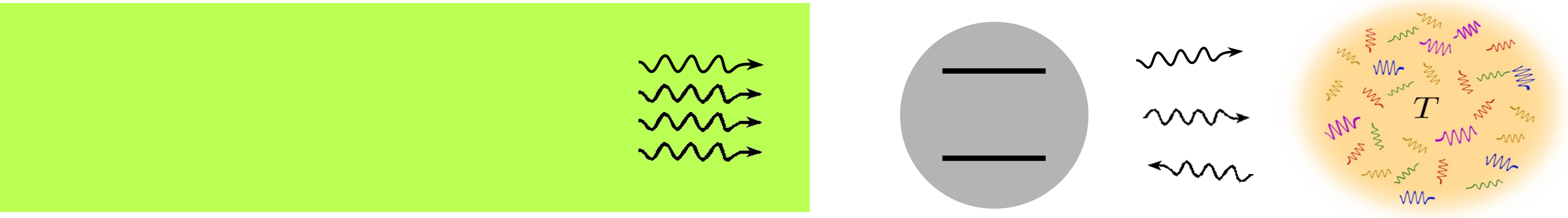


Thermodynamic Consistency of Markovian Master Equations

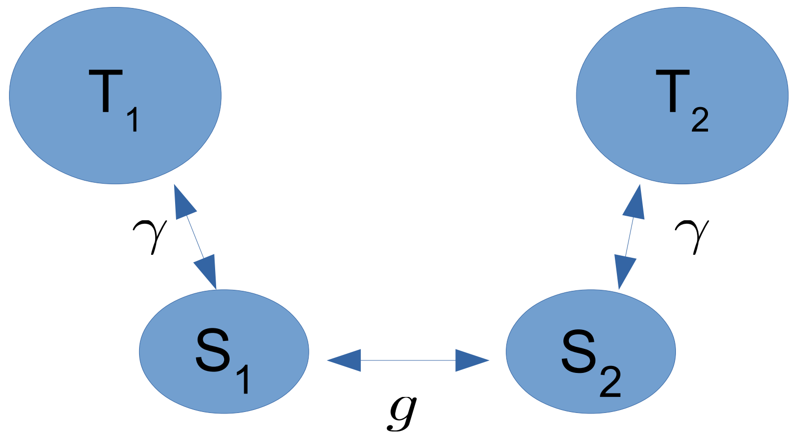
Multipartite Coupled Systems

Long coarse-graining time

“Global limit”

Environment *resolves* all frequencies in system’s spectrum

- Need global eigenstates
- Long time-scale
- Limited to strong coupling



Thermodynamic Consistency of Markovian Master Equations

Multipartite Coupled Systems

Long coarse-graining time

“Global limit”

Environment *resolves* all frequencies in system’s spectrum

- Need global eigenstates
- Long time-scale
- Limited to strong coupling

Shorter coarse-graining time

“Local limit”

Environment *does not resolve* all frequencies in system’s spectrum

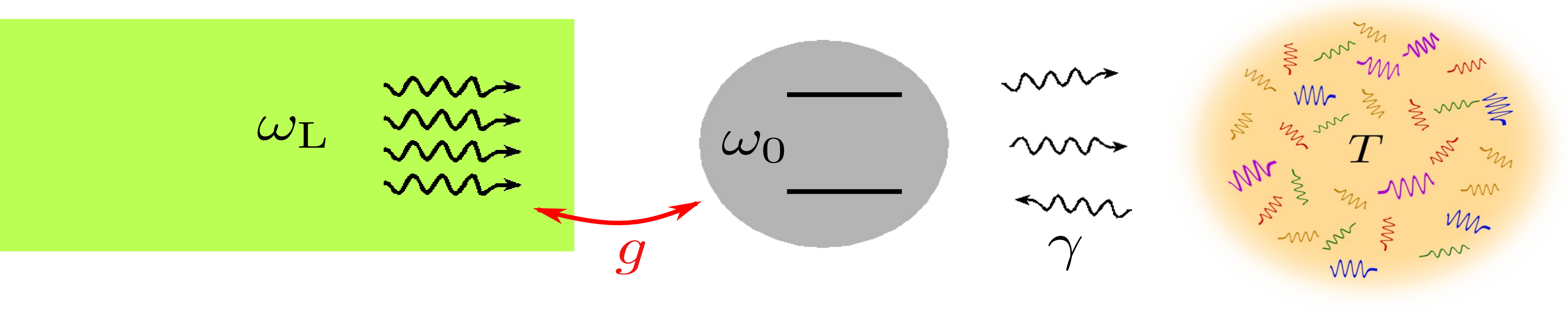
- Short time-scale
- Weak to strong coupling
- Associated to “non autonomous” heat bath ?

Strasberg el al., PRX 7 (2017)
De Chiara et al, NJP 20, 113024 (2018)

Thermodynamic Consistency of Markovian Master Equations

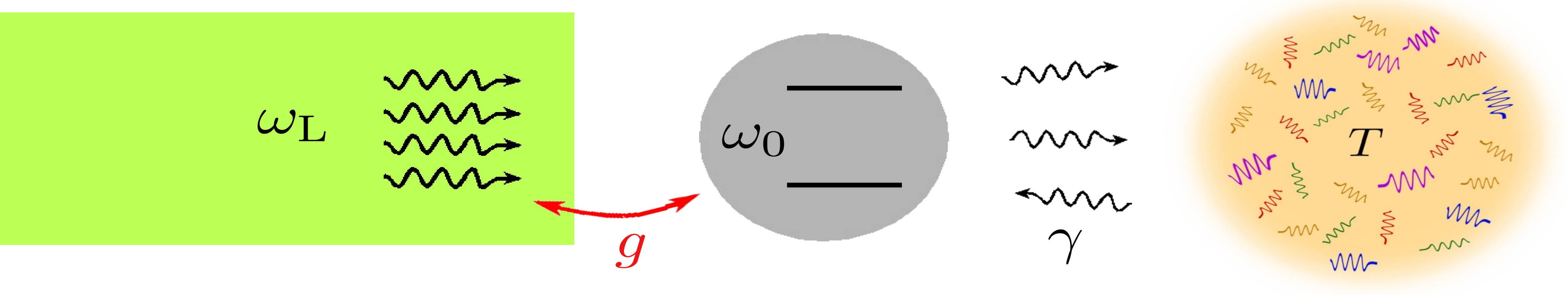
	Long coarse-graining time **Global limit**	**Shorter coarse-graining time** **Local limit**
Multipartite Coupled Systems	Environment *resolves* all frequencies in system's spectrum • Need global eigenstates • Long time-scale • Limited to strong coupling	Environment *does not resolve* all frequencies in system's spectrum • Short time-scale • Weak to strong coupling • Associated to "non autonomous" heat bath ?
Periodically Driven Systems	**Floquet** Environment resolves the coupling to the drive	**"Local"** Environment *does not see* the drive

Resonance fluorescence



- Canonical situation of quantum optics

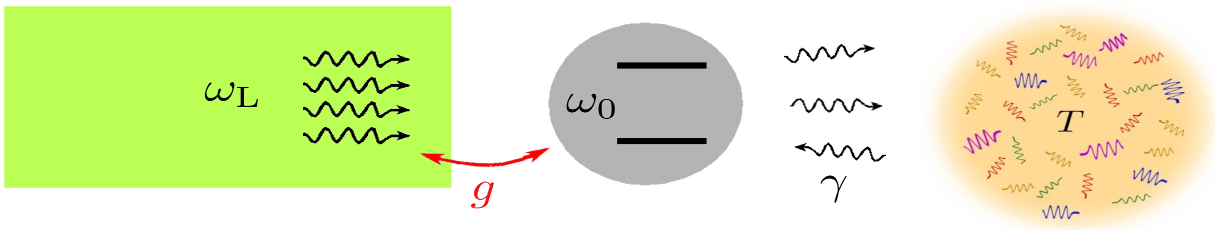
Resonance fluorescence



- Canonical situation of quantum optics
- Thermodynamics known based on Floquet Master Equation
  → Valid **only for** $g \gg \gamma$
  → Require **long coarse-graining** in time $\Delta t \gg g^{-1}$

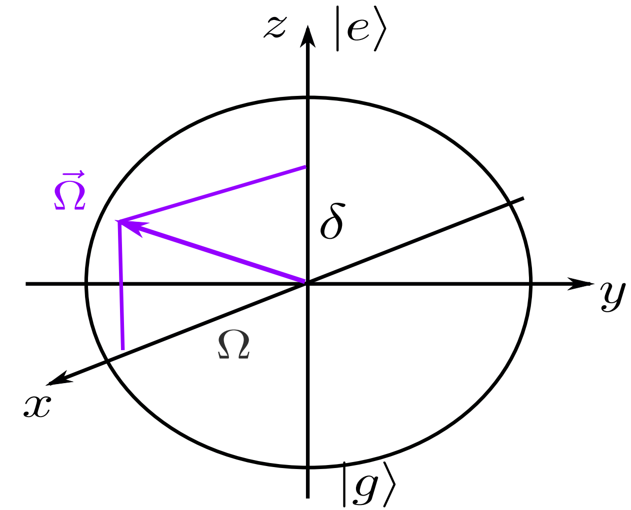
Szczygielski et al., PRE 87(2013); Langemeyer et al., PRE 89(2014); Cuetara et al., NJP 5 (2015); Donvil, J. Stat. Mech. 043104 (2018)

Optical Bloch Equations

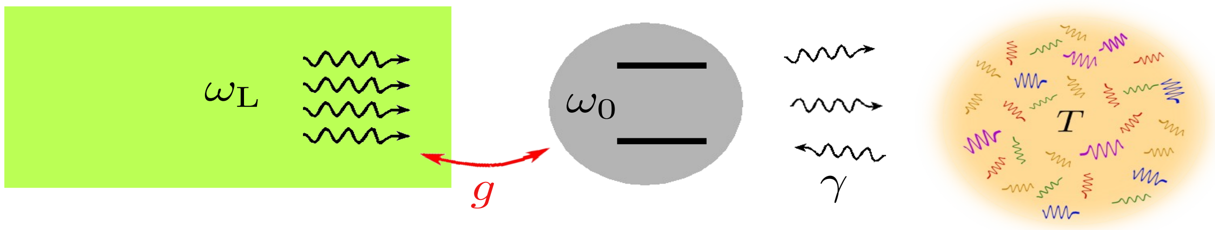


$$\dot{\rho} = -i[H(t), \rho] + \gamma(\bar{n}+1)D[\sigma_-]\rho + \gamma\bar{n}D[\sigma_+]\rho$$

$$H(t) = \frac{\omega_0}{2}\sigma_z + \frac{g}{2}\left(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+\right)$$

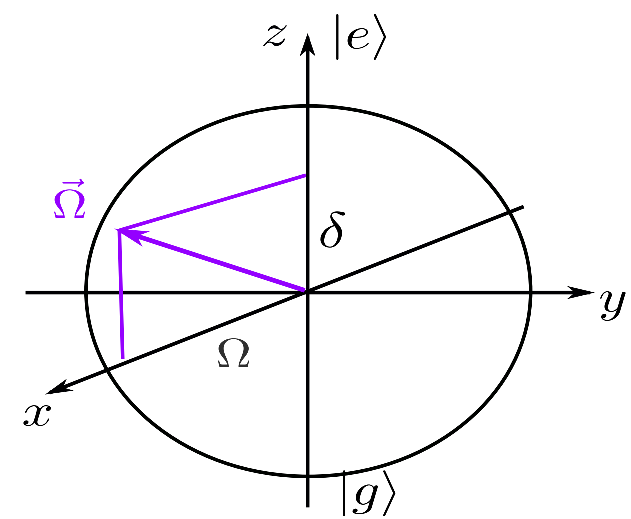


Optical Bloch Equations



$$\dot\rho = -i[H(t),\rho] + \gamma(\bar n + 1)D[\sigma_-]\rho + \gamma\bar n D[\sigma_+]\rho$$

$$H(t) = \frac{\omega_0}{2}\sigma_z + \frac{g}{2}\left(e^{i\omega_{\rm L}t}\sigma_- + e^{-i\omega_{\rm L}t}\sigma_+\right)$$

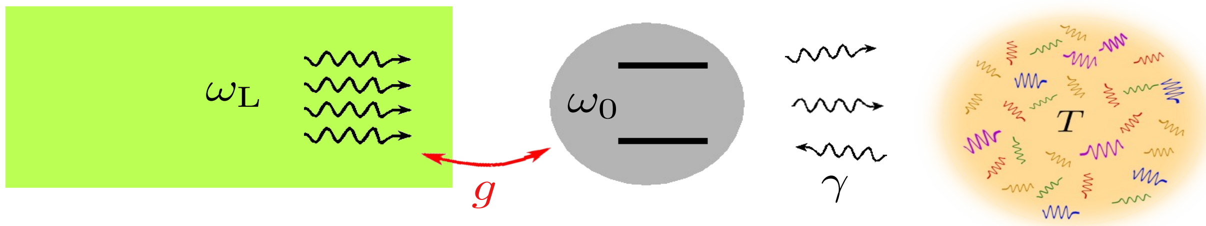


→ Valid for $g \ll \omega_0$

→ Valid on short time-scales $\Delta t \ll g^{-1}$

Carmichael, Statistical Methods in Quantum Optics 1, Springer (1999)

Optical Bloch Equations



$$\dot{\rho} = -i[H(t), \rho] + \gamma(\bar{n}+1)D[\sigma_-]\rho + \gamma\bar{n}D[\sigma_+]\rho$$

Local !

$$H(t) = \frac{\omega_0}{2}\sigma_z + \frac{g}{2}\left(e^{i\omega_{\rm L}t}\sigma_- + e^{-i\omega_{\rm L}t}\sigma_+\right)$$

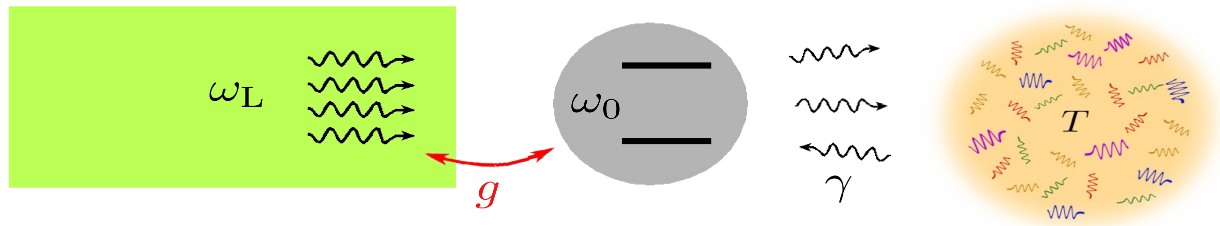
Dissipation and Hamiltonian part do not commute!

→ Steady state with coherences, different from thermal equilibrium state

→ Same property true also for non-adiabatic master equations

Dann et al., PRA 98 (2018)

Optical Bloch Equations



$$\dot{\rho} = -i[H(t), \rho] + \gamma(\bar{n}+1)D[\sigma_-]\rho + \gamma\bar{n}D[\sigma_+]\rho$$

Local !

$$H(t) = \frac{\omega_0}{2}\sigma_z + \frac{g}{2}\left(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+\right)$$

What is the corresponding thermodynamic description?
(is there a consistent, ***autonomous***, thermodynamics description?)

First law: work from the drive

$$H(t) = \frac{\omega_0}{2}\sigma_z + \frac{g}{2}\left(e^{i\omega_{\mathrm{L}}t}\sigma_- + e^{-i\omega_{\mathrm{L}}t}\sigma_+\right)$$

- Internal energy $U(t) = \mathrm{Tr}\{\rho(t)H(t)\}$

- Work flow
$$\dot{W} = \mathrm{Tr}\left\{\rho(t)\frac{d}{dt}H(t)\right\}$$
$$= -\hbar\omega_{\mathrm{L}}\frac{g}{2}\,\mathrm{Im}\left\langle\sigma_- e^{i\omega_{\mathrm{L}}t}\right\rangle$$

Non-zero (positive) at steady state

Alicki, R., J. Phys. A 12 (1979)
Vinjanampathy, S. & Anders, J., Cont. Phys. 57 (2016)

First law: other contributions

$$H(t) = H_0 + H_{\mathrm{L}}(t)$$
$$\dot{\rho}(t) = -i[H(t), \rho] + \mathcal{L}[\rho(t)]$$

- Total ``heat''
$$\frac{d}{dt}U - \dot{W} = \mathrm{Tr}\{H(t)\mathcal{L}[\rho(t)]\}$$

- Two parts
$$\dot{Q}_{\mathrm{cl}} = \mathrm{Tr}\{H_0\mathcal{L}[\rho(t)]\}$$
$$\dot{Q}_{\mathrm{q}} = \mathrm{Tr}\{H_{\mathrm{L}}(t)\mathcal{L}[\rho(t)]\}$$

"Classical" contribution

$$H(t) = H_0 + H_{\mathrm{L}}(t)$$

$$\dot{\rho}(t) = -i[H(t), \rho] + \mathcal{L}[\rho(t)]$$

$$\dot{Q}_{\mathrm{cl}} = \mathrm{Tr}\{H_0 \mathcal{L}[\rho(t)]\}$$
$$= \hbar\omega_0\Big(- \gamma(\bar{n}+1)P_e(t) + \gamma\bar{n}P_g(t)\Big)$$

Counts photons exchanged
with the bath at
the atomic frequency

"Classical" contribution

$$H(t) = H_0 + H_{\mathrm{L}}(t)$$

$$\dot{\rho}(t) = -i[H(t), \rho] + \mathcal{L}[\rho(t)]$$

Corresponding entropy production looks like classical and "usual" global Lindblad equation analysis

$$\dot{\sigma}_{\mathrm{cl}} = \frac{d}{dt} S_{\mathrm{VN}} - \frac{\dot{Q}_{\mathrm{cl}}}{T}$$

$$= -\mathrm{Tr}\left\{ \mathcal{L}[\rho(t)] \Big(\log \rho(t) - \log \left(e^{-\frac{H_0}{k_B T}} Z \right) \Big) \right\} \geq 0$$

Alicki, J. Phys. A 12 (1979)
Breuer et al., Oxford University Press (2007)

“Quantum” contribution

$$H(t) = H_0 + H_\mathrm{L}(t)$$

$$\dot{\rho}(t) = -i[H(t), \rho] + \mathcal{L}[\rho(t)]$$

$$\dot{Q}_\mathrm{q} = \mathrm{Tr}\{H_\mathrm{L}(t)\mathcal{L}[\rho(t)]\}$$
$$= -\hbar g \frac{\gamma}{2} \mathrm{Re}\left\langle \sigma_- e^{i\omega_\mathrm{L} t} \right\rangle$$

Energy change from coherence erasure by the reservoir

"Quantum" contribution

$$H(t) = H_0 + H_{\mathrm{L}}(t)$$

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Energy change from coherence erasure by the reservoir

$$\dot{\sigma} = \frac{d}{dt} S_{\mathrm{VN}} - \frac{\dot{Q}_{\mathrm{cl}}}{T} - \frac{\dot{Q}_{\mathrm{q}}}{T} \geq 0$$

Another candidate second law ???

Not expressed as
a Spohn inequality...

"Quantum" contribution

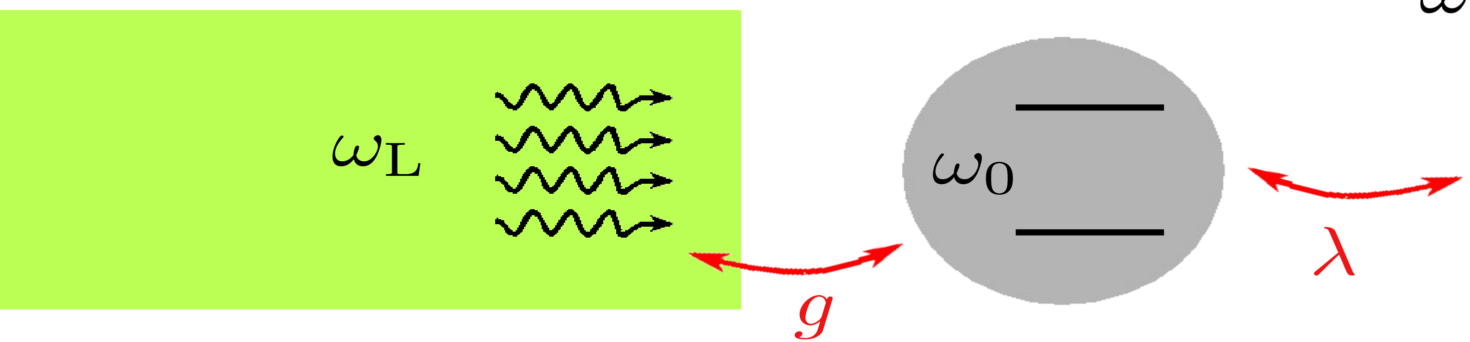
$$H(t) = H_0 + H_\mathrm{L}(t)$$
$$\dot{\rho}(t) = -i[H(t), \rho] + \mathcal{L}[\rho(t)]$$

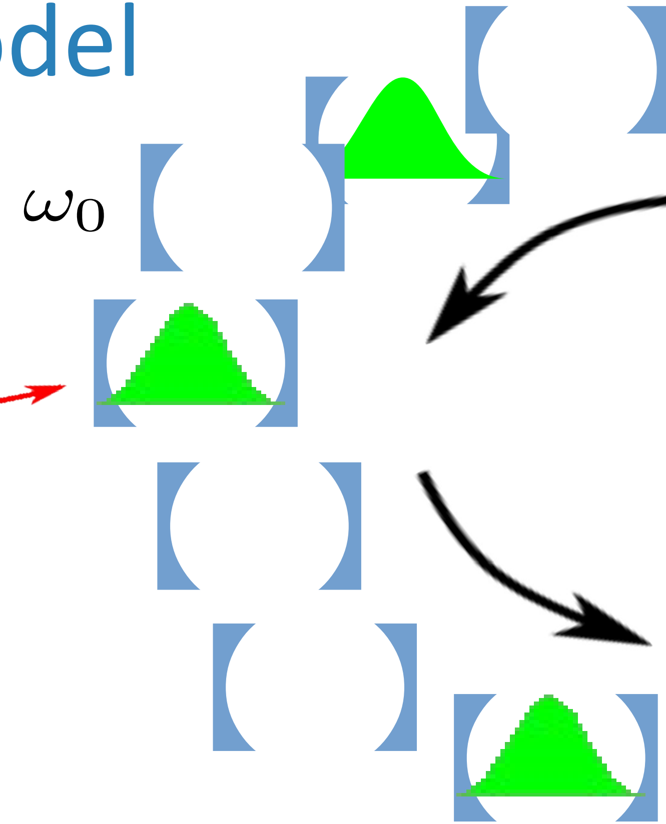
$$\dot{Q}_\mathrm{q} = \mathrm{Tr}\{H_\mathrm{L}(t)\mathcal{L}[\rho(t)]\}$$
$$= -\hbar g \frac{\gamma}{2} \mathrm{Re}\,\langle \sigma_- e^{i\omega_\mathrm{L} t}\rangle$$

Energy change from coherence erasure by the reservoir

Need to find who provides this energy
→ Requires to precise more the environment

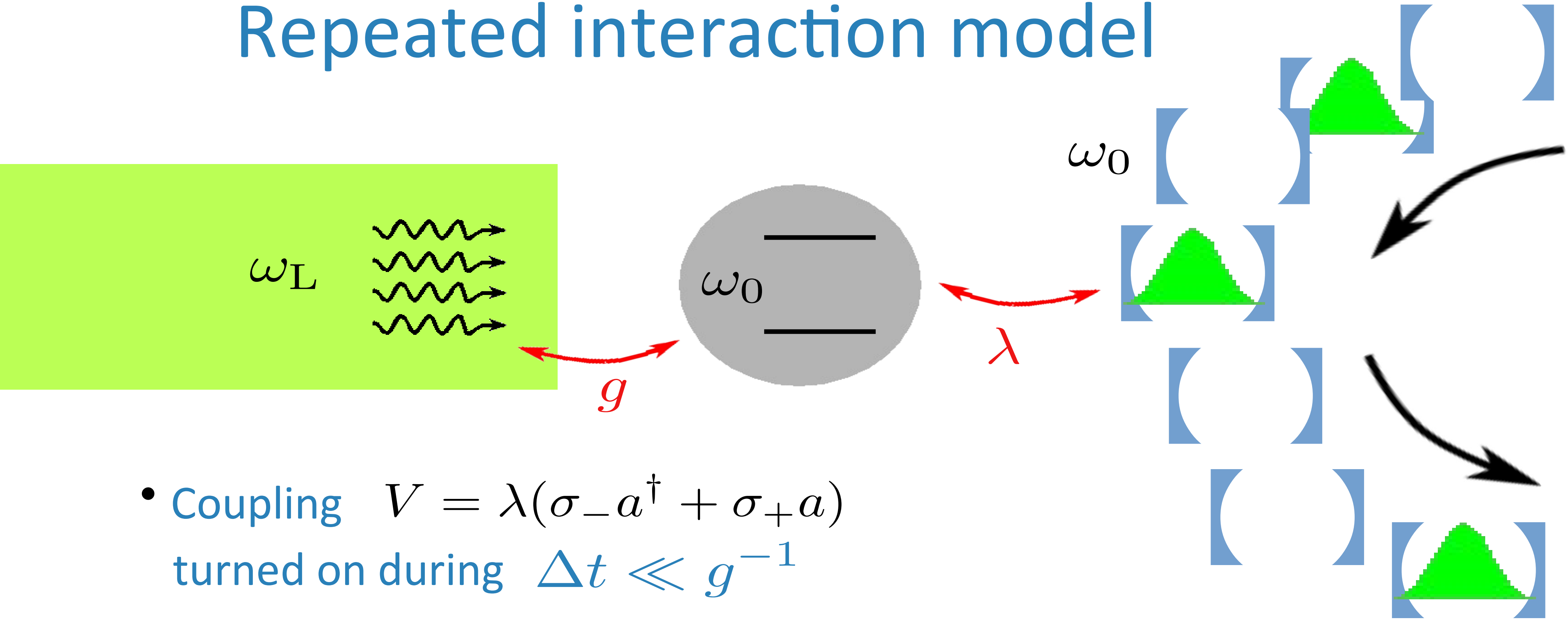
Repeated interaction model





- Coupling $V = \lambda(\sigma_- a^\dagger + \sigma_+ a)$
  turned on during $\Delta t \ll g^{-1}$

Repeated interaction model



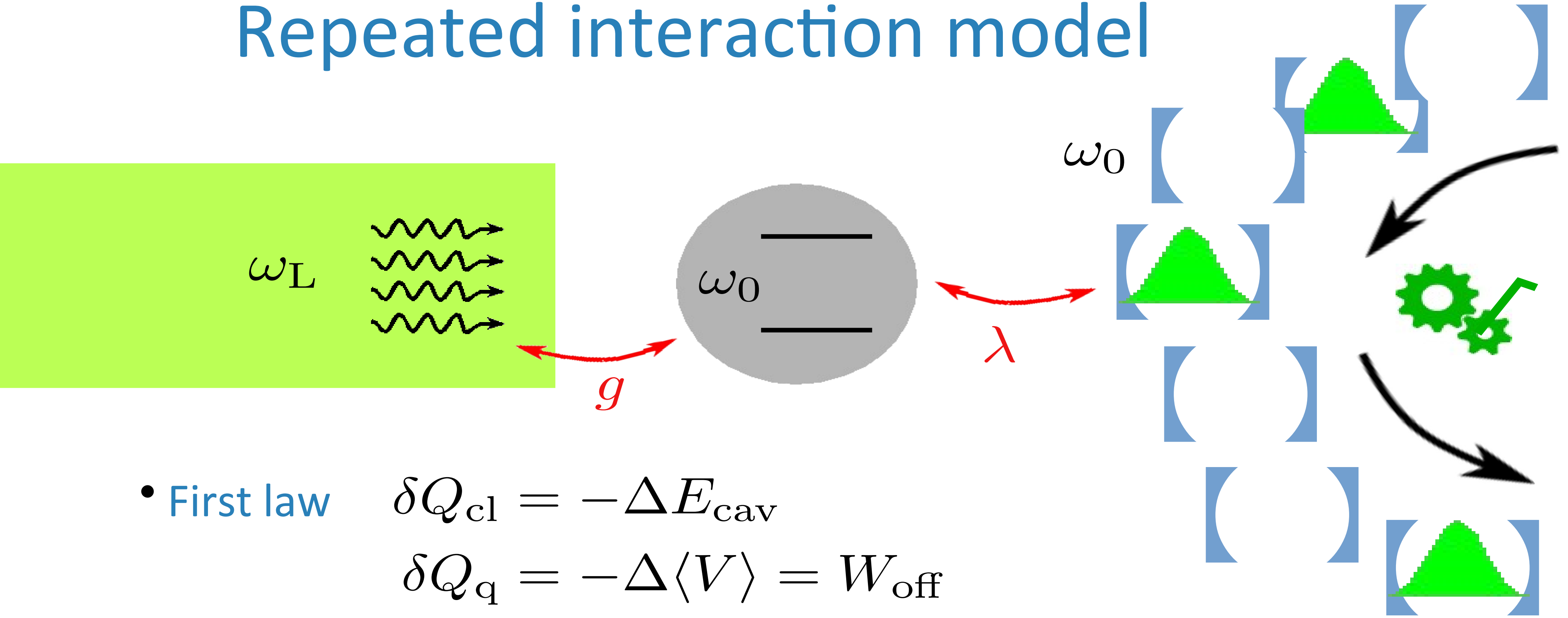
- Coupling $V = \lambda(\sigma_- a^\dagger + \sigma_+ a)$
  turned on during $\Delta t \ll g^{-1}$

- After interaction $\langle V \rangle \neq 0$

  → Turning off coupling requires to perform some work

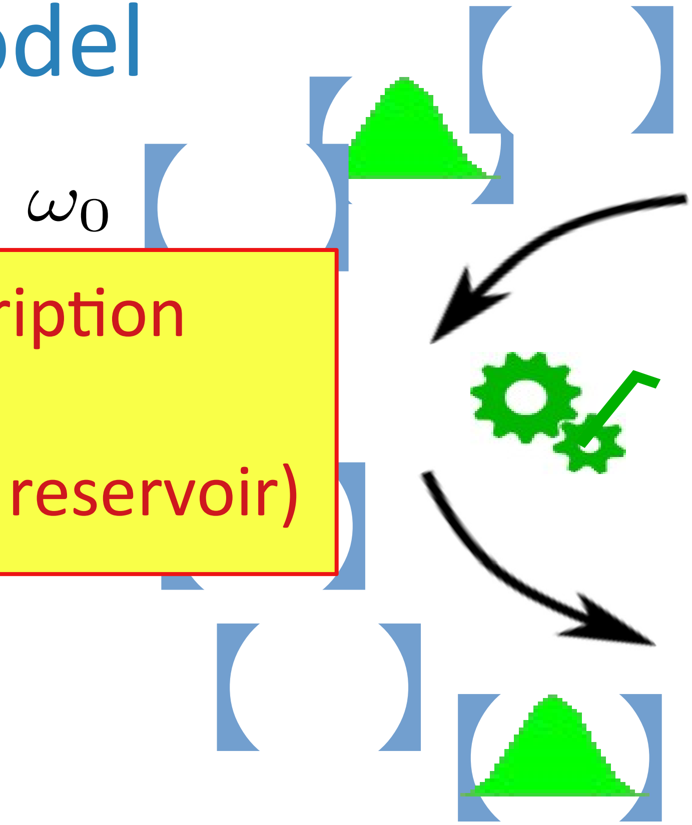
Strasberg el al., PRX 7 (2017)

Repeated interaction model



- First law $$\delta Q_{\rm cl} = -\Delta E_{\rm cav}$$ $$\delta Q_{\rm q} = -\Delta \langle V \rangle = W_{\rm off}$$
- Second law $$\sigma_{\rm cl} = \Delta S_{\rm VN} - Q_{\rm cl}/T \geq 0$$

Repeated interaction model



Consistent thermodynamic description

BUT

Non autonomous (source of work in reservoir)

- First law $$\delta Q_{\rm cl} = -\Delta E_{\rm cav}$$ $$\delta Q_{\rm q} = -\Delta \langle V \rangle = W_{\rm off}$$
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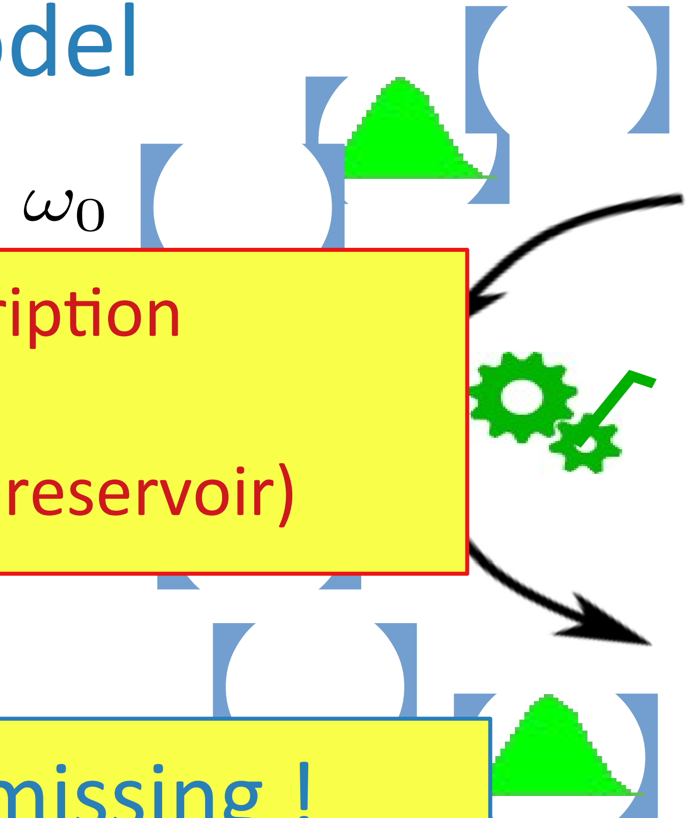
Repeated interaction model



Consistent thermodynamic description

BUT

Non autonomous (source of work in reservoir)

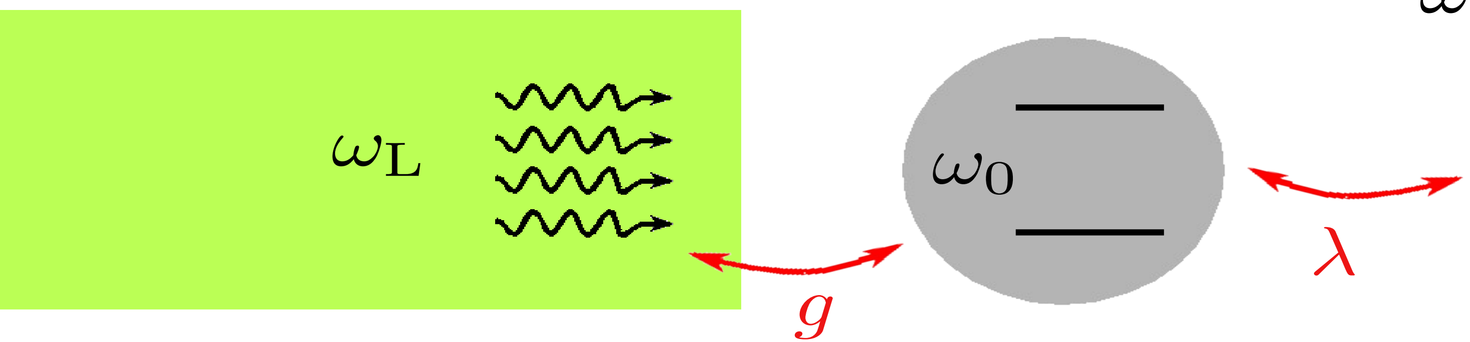
- First law $\delta Q_{\rm cl} = -\Delta E_{\rm cav}$

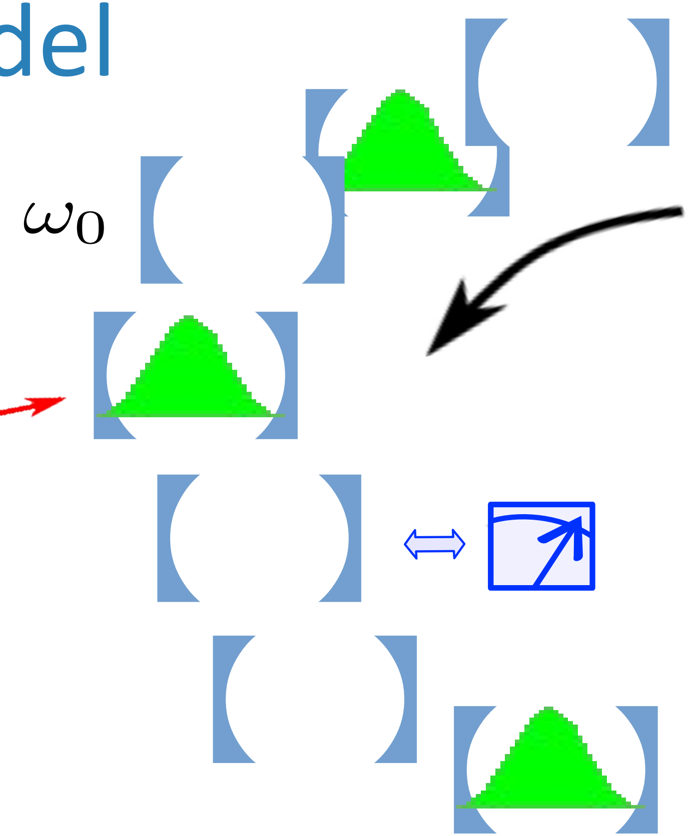
Decoherence of the bath is missing !

- Second law $\sigma_{\rm cl} = \Delta S_{\rm VN} - Q_{\rm cl}/T \geq 0$

Repeated interaction model

+ measurement





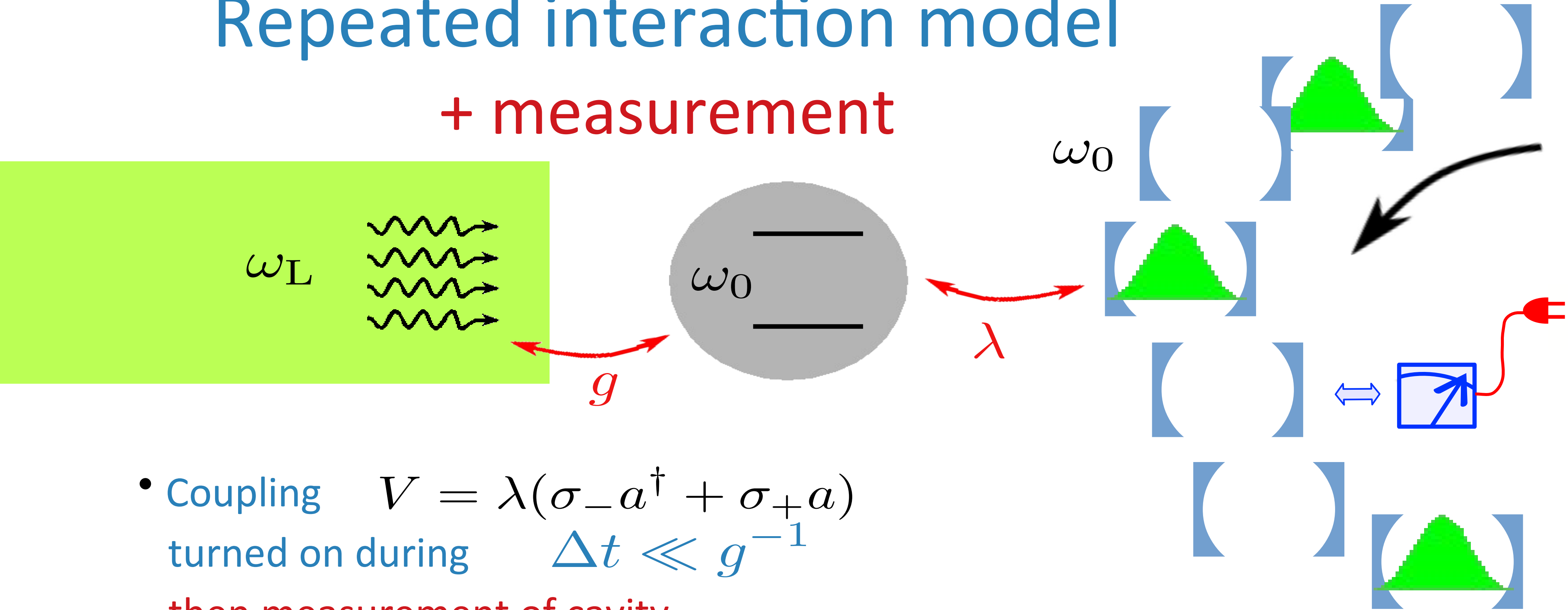
- Coupling $V = \lambda(\sigma_- a^\dagger + \sigma_+ a)$

  turned on during $\Delta t \ll g^{-1}$

  then measurement of cavity

Repeated interaction model

+ measurement

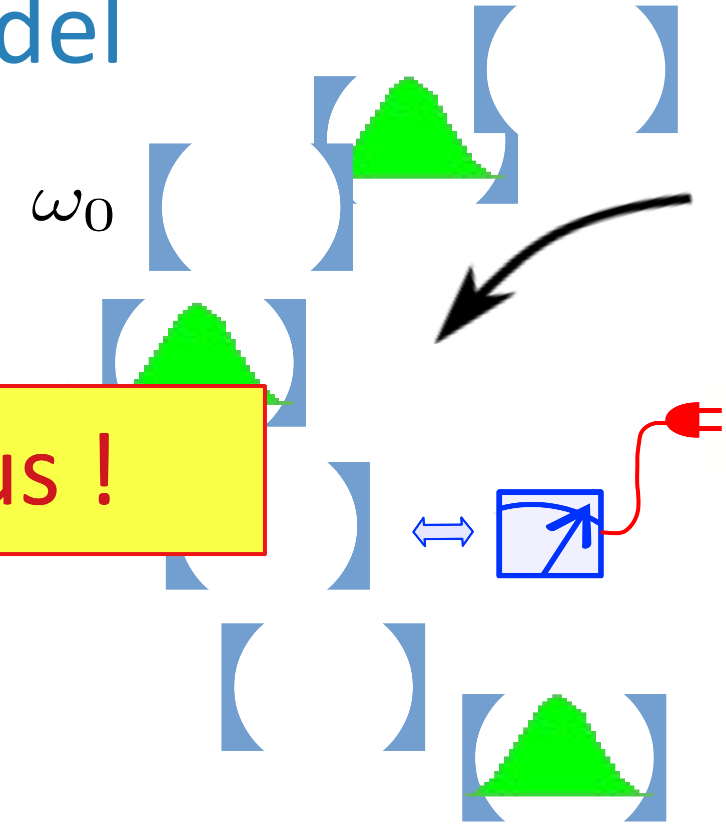


- Coupling $V = \lambda(\sigma_- a^\dagger + \sigma_+ a)$
  turned on during $\Delta t \ll g^{-1}$
  then measurement of cavity

- After measurement $\langle V \rangle = 0$
- Energy from measuring device $\Delta E_{\text{meas}} = -\Delta V = Q_{\text{q}}$

Repeated interaction model

+ measurement



Still non autonomous !

- Coupling $V = \lambda(\sigma_- a^\dagger + \sigma_+ a)$
  turned on during $\Delta t \ll g^{-1}$
  then measurement of cavity

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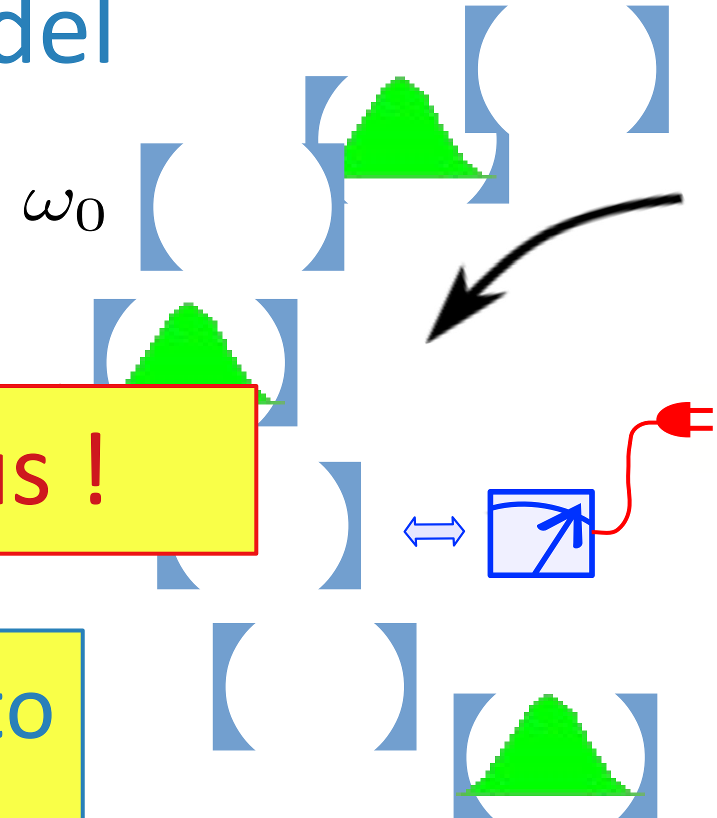
Repeated interaction model

+ measurement



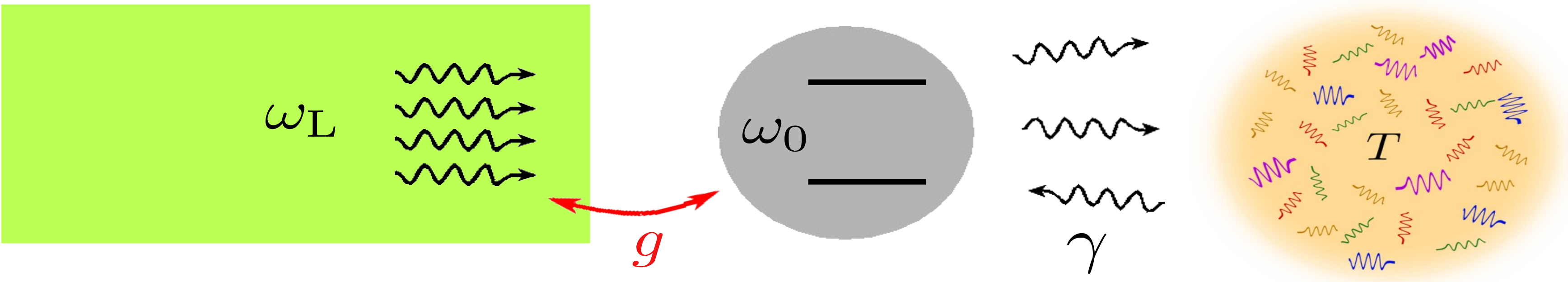
Still non autonomous !

- Coupling V
  turned on durin
  then measurem

A bath is supposed to "measure itself" ?

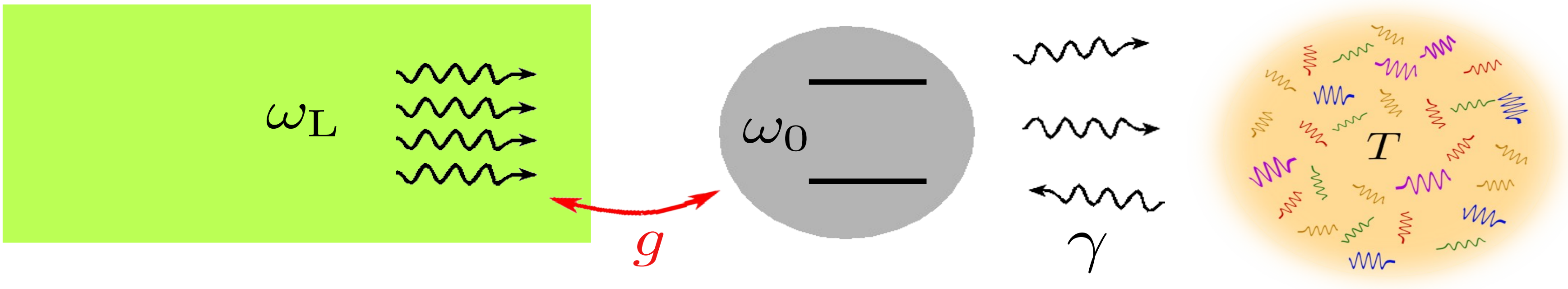
- After measurement $\langle V \rangle = 0$
- Energy from measuring device $\Delta E_{\mathrm{meas}} = -\Delta V = Q_{\mathrm{q}}$

Macroscopic reservoir



- Coupling $V = \int d\omega g(\omega)\left(\sigma_- a^\dagger_\omega + \sigma_+ a_\omega\right)$ always on

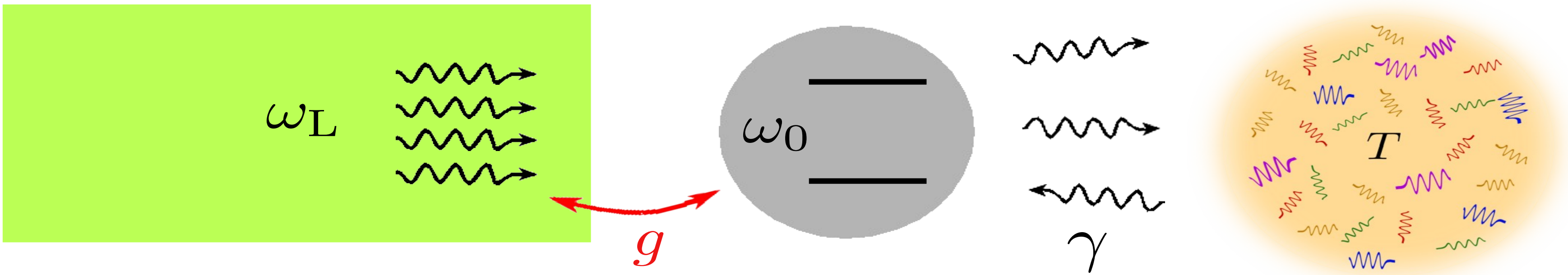
Macroscopic reservoir



- Coupling $V = \int d\omega g(\omega) \left(\sigma_- a^\dagger_\omega + \sigma_+ a_\omega\right)$ always on

- Compute $\Delta E_{\text{meas}}, \quad \Delta\langle V \rangle$ using the approximations leading to optical Bloch equations

Macroscopic reservoir



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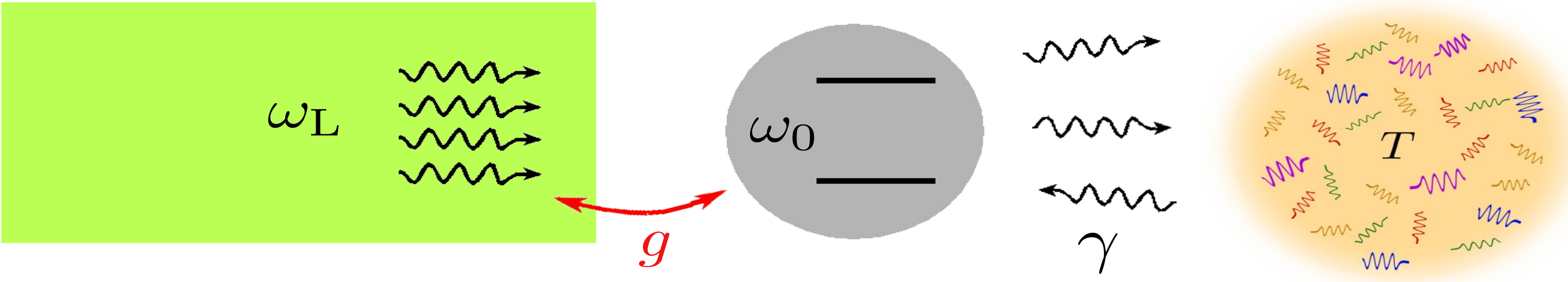
- Compute $\Delta E_{\mathrm{meas}}, \quad \Delta\langle V\rangle$ using the approximations leading to optical Bloch equations
- Results:

$$\boxed{\begin{aligned} \Delta E_{\mathrm{res}} &= \delta Q_{\mathrm{cl}} + \delta Q_{\mathrm{q}} \\ \Delta\langle V\rangle &= 0 \end{aligned}}$$

Macroscopic reservoir



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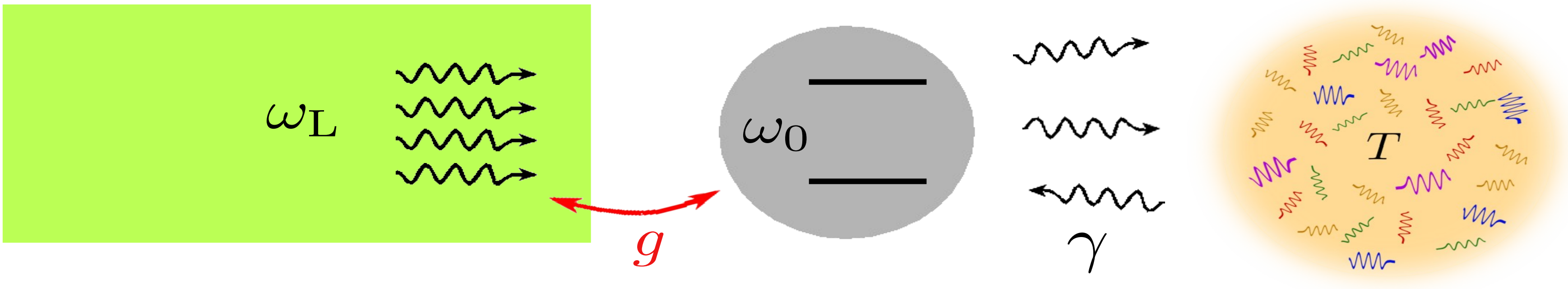
- Results:

$$\boxed{\begin{aligned}\Delta E_{\mathrm{res}} &= \delta Q_{\mathrm{cl}} + \delta Q_{\mathrm{q}} \\ \Delta\langle V\rangle &= 0\end{aligned}}$$

The bath measures itself… and pays for it

Macroscopic reservoir



- The bath provides both classical and quantum contributions

$$\Delta E_{\mathrm{res}} = -\delta Q_{\mathrm{cl}} - \delta Q_{\mathrm{q}}$$

- Corresponding second law

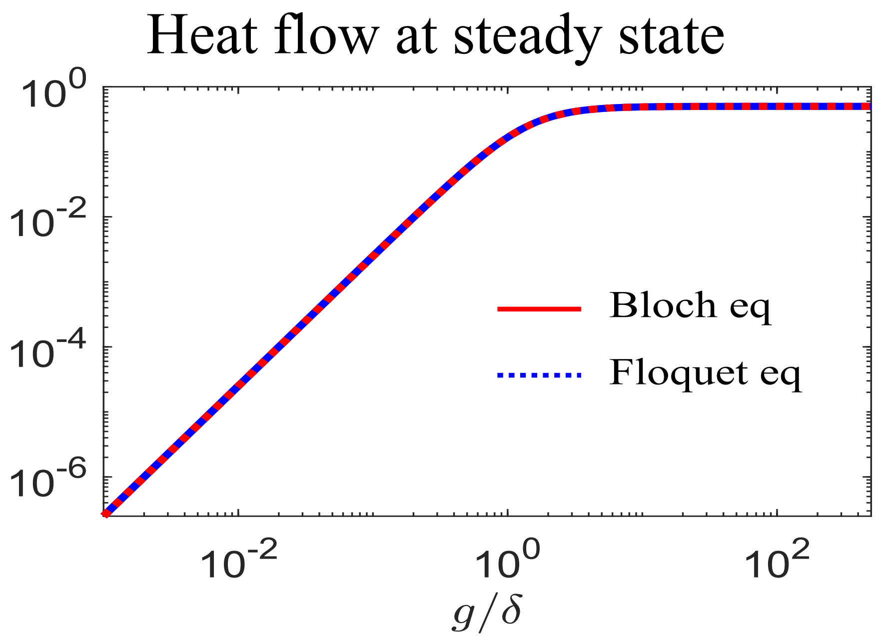
$$\sigma = \Delta S_{\mathrm{VN}} - \frac{Q_{\mathrm{cl}} + Q_{\mathrm{q}}}{T}$$
$$= D(\rho_{\mathrm{tot}}(\Delta t) || \rho(\Delta t) \otimes \rho_{\mathrm{res}}^{\mathrm{eq}}) \geq 0$$

M. Esposito et al., NJP 12, 013013 (2010)

Agreement with Floquet description

Common regime of validity of both descriptions

$$\gamma \ll g \ll \omega_0, \omega_\mathrm{L}$$



Perfect agreement for the value of total heat flow

Summary

- Optical Bloch Equations are compatible w/ thermodynamics
- Two different Second laws

  → Autonomous macroscopic bath

  → Repeated interaction w/ micro bath (requires extra work)

  → Needs to know more than dynamics to find thermodynamics
- The difference corresponds to coherence erasure cost

  → Quantum effect
- Test: measure average energy change of the bath

Outlook

- Intermediate Second laws when increasing the bath size.
- Stochastic thermodynamics from quantum jump trajectories → fluctuation theorem.
- Thermodynamics of local master equations
  → Sometimes they require an external source of work, but not always.

Thank you for your attention!

Soon on the arXiv

Thermodynamic Consistency of the Optical Bloch Equations

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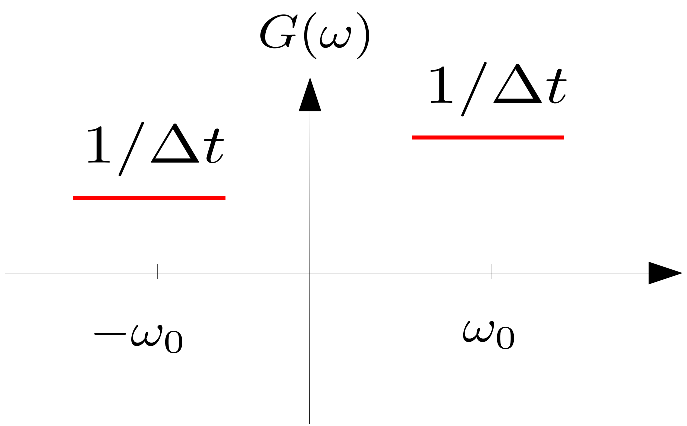
Supplementary slides

Derivation of Bloch Equations

Environment Spectral Density



Kubo-Martin-Schwinger relation

$$G(\omega) = e^{\omega/T} G(-\omega)$$

Breuer, et al., Oxford UniV. Press (2007)
Alicki et al., PRA 73 (2006)

We use a generalization
of the singular coupling limit
which preserves the thermodynamic
properties of the bath

$$\tau_c, \omega_0^{-1} \ll \Delta t \ll g^{-1}, \gamma^{-1}$$

$$G(\omega_0 \pm g) \simeq G(\omega_0)$$