

# GABRIELE DE CHIARA

LOCAL MASTER EQUATIONS ARE CONSISTENT WITH THERMODYNAMICS

+ 3-QUBIT REFRIGERATORS





Common models:

- Caldeira-Leggett
- Spin-boson
- Spin star/networks
- Fermionic baths



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APPROXIMATIONS: E.G. BORN-MARKOV Lindblad Master Equation





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Computation See Marco

Cattaneo's

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#### Comparisons Global vs Local & Reviews:

Rivas et al., NJP 2010; Correa et al., PRE 2013; Guimarães et al., PRE 2016; Gonzalez et al., OSID, 2017; Hofer et al., NJP 2017; Stockburger&Motz FP 2017; Mitchison&Plenio, NJP 2018; Naseem et al., PRA 2018; Shammah et al., PRA 2018, S. Seah et al. PRE 2018, M. Cattaneo et al. 2019...& many more!

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Current  $1 \rightarrow 2$ 

$$\begin{cases} \propto T_2 - T_1 \\ \propto n_2 - n_1 \end{cases}$$
$$n_i = \frac{1}{e^{\beta \cdot \omega_i} - 1}$$

 $e^{p_i\omega_i} - 1$ 

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The sign of the current depends on the sign of:

$$\frac{T_1}{\omega_1} - \frac{T_2}{\omega_2}$$

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Current 1→2   

$$\begin{cases}
\propto T_2 - T_1 & \text{The sign of the current} \\
\alpha n_2 - n_1 & \text{depends on the sign of:} & \frac{T_1}{\omega_1} - \frac{T_2}{\omega_2} \\
\text{LME predicts} \\
\text{spontaneous cold} \\
\text{to hot current!} & \text{to hot current!} & \text{to hot current} \\
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S

 $E_3$ 

 $E_2$ 

 $E_{I}$ 

 $E_4$ 

 One possibility is to use collisional models (aka repeated interactions):

The goal of this talk is to show: **Thermodynamic laws** are observed provided one includes all energy/entropy contributions.

## OUTLINE

- Collisional models
- Thermodynamic cost of Local Master Equations
- Further considerations
- Bonus: Three-qubit refrigerators with two-body interactions (see Adam's poster)
- Conclusions





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#### System-Unit interaction:



$$\rho_{S}[(n+1)\tau] = \operatorname{Tr}_{E}\left\{ e^{-i\tau H_{\text{tot}}} \rho_{S}(n\tau) \otimes \rho_{E_{n}} e^{i\tau H_{\text{tot}}} \right\}$$

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Next steps: 1)Assume thermal ancillas:  $\rho_1$ 

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3)Limit τ→0

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$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_{i=1}^N D_i(\rho_S), \qquad D_i(\rho_S) = -\frac{1}{2} \operatorname{Tr}_{E_i}[V_i, [V_i, \rho_S \otimes \rho_{E_i}]].$$

## EXAMPLE: HARMONIC OSCILLATORS

 $H_S = \sum_i \omega_i a_i^{\dagger} a_i + \sum_{i \neq j} K_{ij} a_i^{\dagger} a_j + L_{ij} a_i a_j$ 



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#### Local Master equation!

 $\mathbf{J}$ 

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_{i=1}^{N_{\text{baths}}} D_i(\rho_S),$$



$$n_i = \frac{1}{e^{\beta_i \omega_i} - 1} \qquad \gamma_i = g_i^2$$

 $D_i(\rho_S) = \gamma_i(n_i + 1) \mathscr{L}_{a_i}(\rho_S) + \gamma_i n_i \mathscr{L}_{a_i^{\dagger}}(\rho_S)$ 

$$\mathscr{L}_{\sigma}(\rho_{S}) = 2\sigma\rho_{S}\sigma^{\dagger} - \sigma^{\dagger}\sigma\rho_{S} - \rho_{S}\sigma^{\dagger}\sigma$$

#### THERMODYNAMICS

At steady state:  $\dot{U} = 0$  $\dot{Q}_i = \operatorname{tr} \left[ D_i(\rho_S) H_i \right] = \gamma_i \omega_i (n_i - \langle a_i^{\dagger} a_i \rangle)$ 

De Chiara, Landi, Hewgill, Reid, Ferraro, Roncaglia, Antezza, NJP 20, 113024 (2018) see also: Barra, Sci. Rep. 2015; Strasberg et al., PRX 2017

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Since  $H_{tot}$  is time dependent  $\Rightarrow$  external work

$$\delta W_{\text{ext}} = \int_{n\tau}^{(n+1)\tau} \left\langle \frac{\partial H_{\text{tot}}}{\partial t} \right\rangle dt \,.$$

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Lack of detailed balance is responsible for the external work

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#### **EXAMPLE: 2 HARMONIC OSCILLATORS**

$$H_S = H_1 + H_2 + H_I$$
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Steady state:

$$\dot{Q}_1 = \frac{2\gamma\epsilon^2}{\Delta^2}\omega_1(n_1 - n_2),$$
  
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Considerations:  

$$\dot{Q}_1$$
 and  $\dot{Q}_2$  do not depend on  
 $T_1 - T_2$  but rather on  $n_1 - n_2$   
 $\omega_1 = \omega_2 \Rightarrow \dot{Q}_1 = -\dot{Q}_2 \Rightarrow \dot{W}_{ext} = 0$ 

Oven (or accelerator)





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 $\frac{T_1}{T_2} < \frac{\omega_1}{\omega_2} < 1$ 

Engine



Oven (or accelerator)





Engine



 $\frac{T_1}{T_2} < \frac{\omega_1}{\omega_2} < 1$ 

Refrigerator



Engine



$\frac{T_1}{1} < \frac{\omega_1}{1} < 1$	$n = \frac{ \dot{W}_{ext} }{ \dot{W}_{ext} } = 1$	$\omega_1$
$T_2 \omega_2$	$\eta = \frac{1}{\dot{Q}_2} = 1$	$\omega_2$

Engine





Refrigerator



Engine





Refrigerator

Engine







Refrigerator

 $\frac{\omega_1}{\omega_2} < \frac{T_1}{T_2} \qquad \text{COP} = \frac{Q_1}{\dot{W}_{\text{ext}}} = \frac{\omega_1}{\omega_2 - \omega_1}$ 

Carnot's limit: zero work/zero cooling

$$\frac{\omega_1}{\omega_2} = \frac{T_1}{T_2}$$

 $\eta = 1 - \frac{T_1}{T_2}$ 

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Although our network is linear, it uses time-dependent Hamiltonians so the network is essentially driven.

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- The collisional model is an example of generalised environments defined by Strasberg et al., PRX 2017, providing a source of both heat and work.
- Our analysis is general and works for quantum systems of any dimension.
- See our paper for a chain of many oscillators coupled by general interactions, e.g. counter-rotating terms.

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## SPECIAL ISSUE



- "Fluctuation Relations and Nonequilibrium Thermodynamics in Classical and Quantum Systems"
- Journal: MDPI Entropy (IF 2.3)
- Topics:
  - Fluctuation relations in classical stochastic thermodynamics
  - Definitions of work, heat, and entropy and related fluctuation theorems in quantum systems
  - Quantum engines and refrigerators
  - Resource theory of quantum thermodynamics
  - Role of quantum correlations and coherence in quantum thermodynamics
- Deadline: 31/12/2019

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- Daniel Alonso, Tenerife
  J. Onam González, Tenerife

Postdoc position on Quantum Thermodynamics to start in Spring 2020!





Engineering and Physical Sciences Research Council