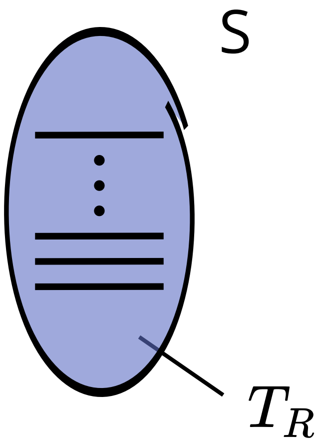
Unifying paradigms of quantum refrigeration: A universal and attainable bound on cooling

Fabien Clivaz, Ralph Silva, Géraldine Haack, Jonatan Bohr Brask, Nicolas Brunner, Marcus Huber

QTD 2019, 23-28 June 2019, Espoo, Finland

Fabien Clivaz et al., arXiv:1903.04970
Fabien Clivaz et al., arXiv:1710.11624 (to appear in PRE)

General Idea



- S arbitrary (finite dim.)
- S initially at T_R
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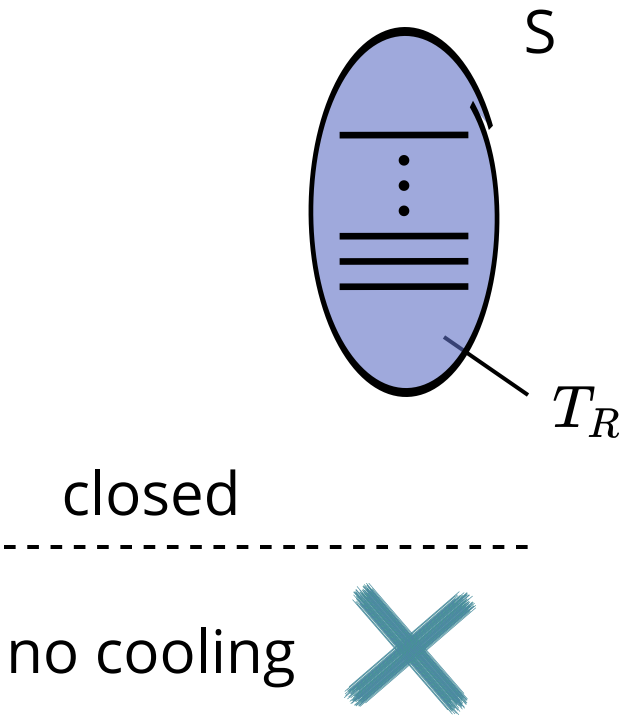

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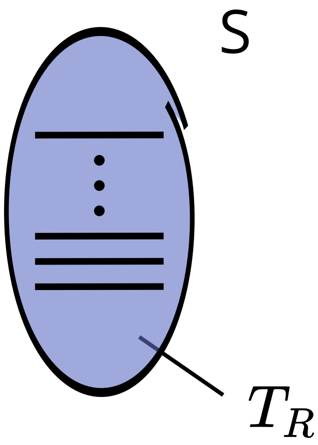

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closed

fully open

no cooling ✗

✗ Loose track of Resource

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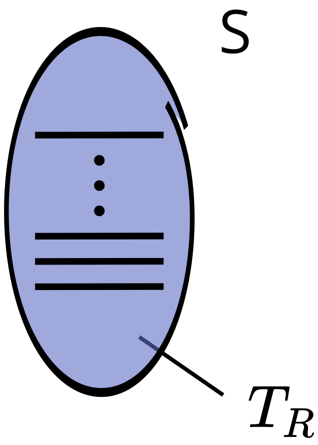

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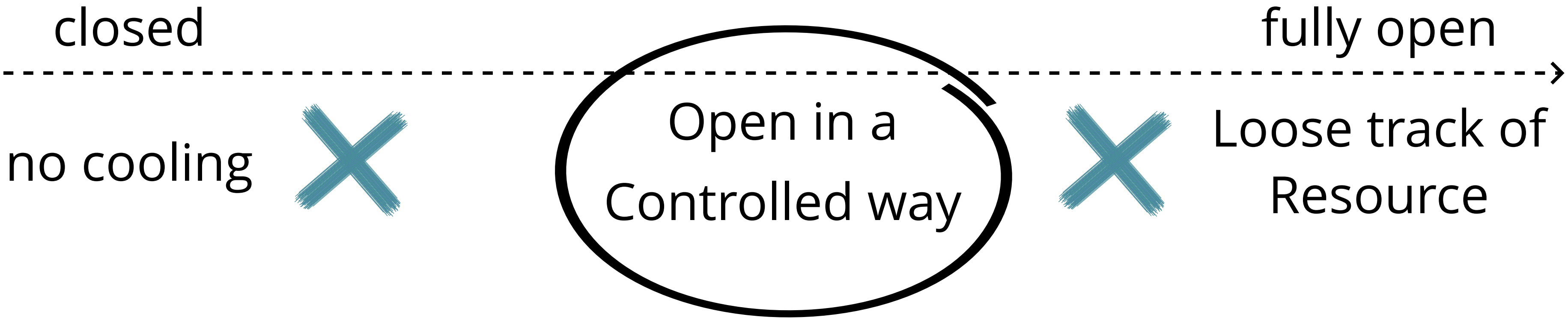

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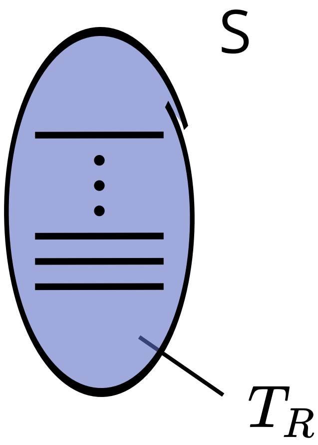

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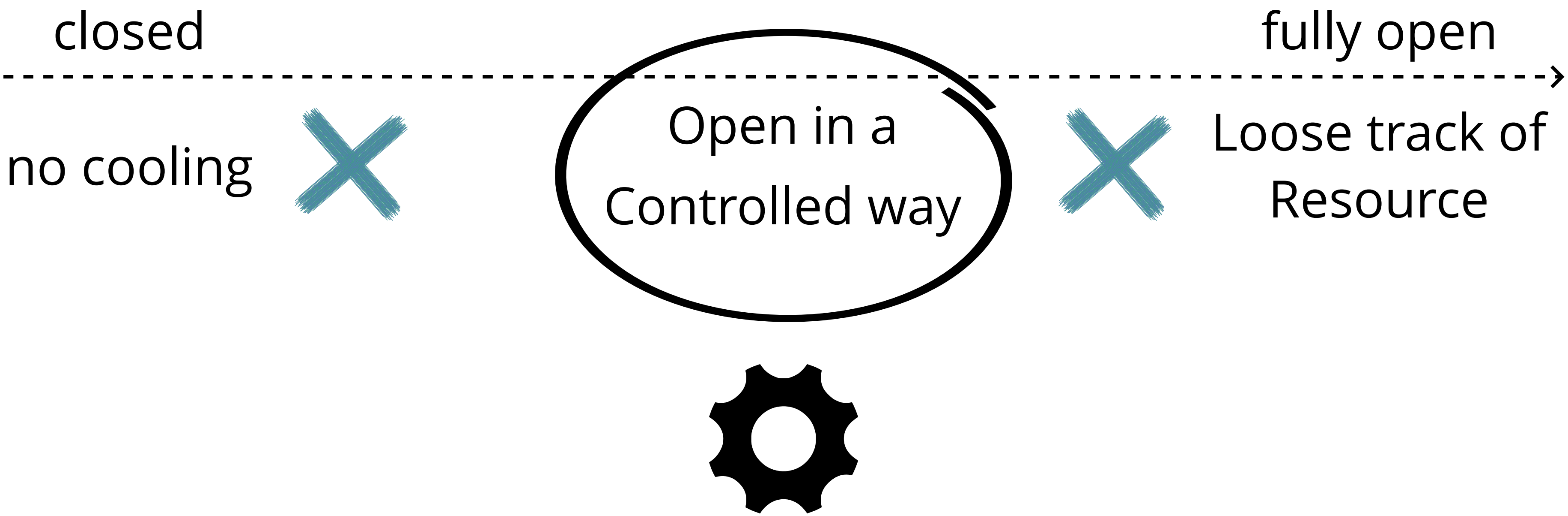

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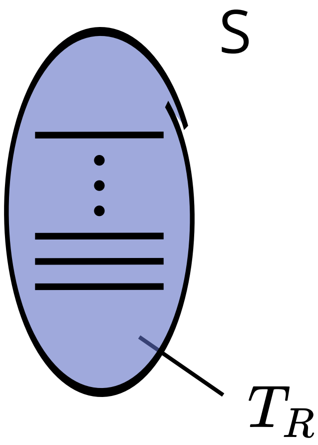

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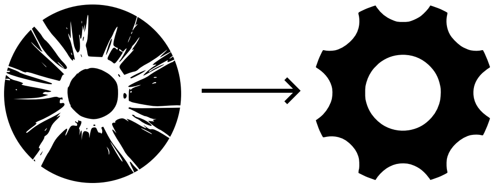

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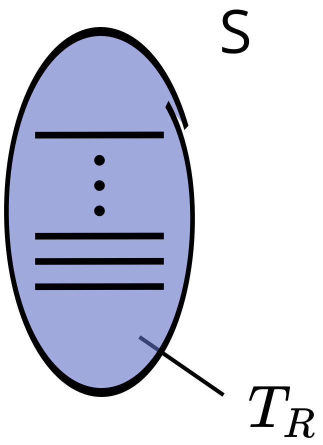

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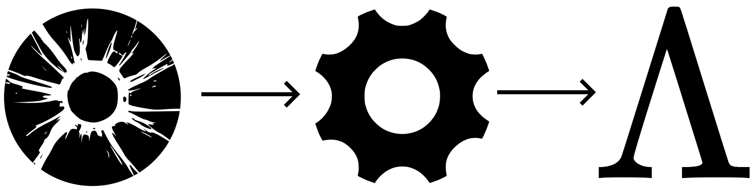

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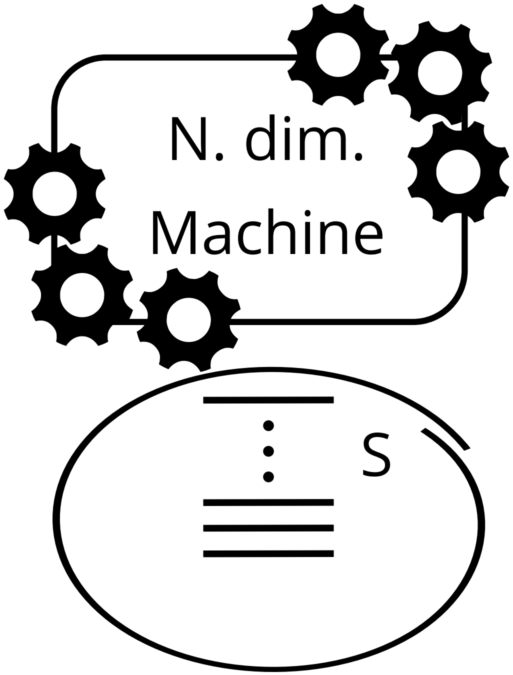

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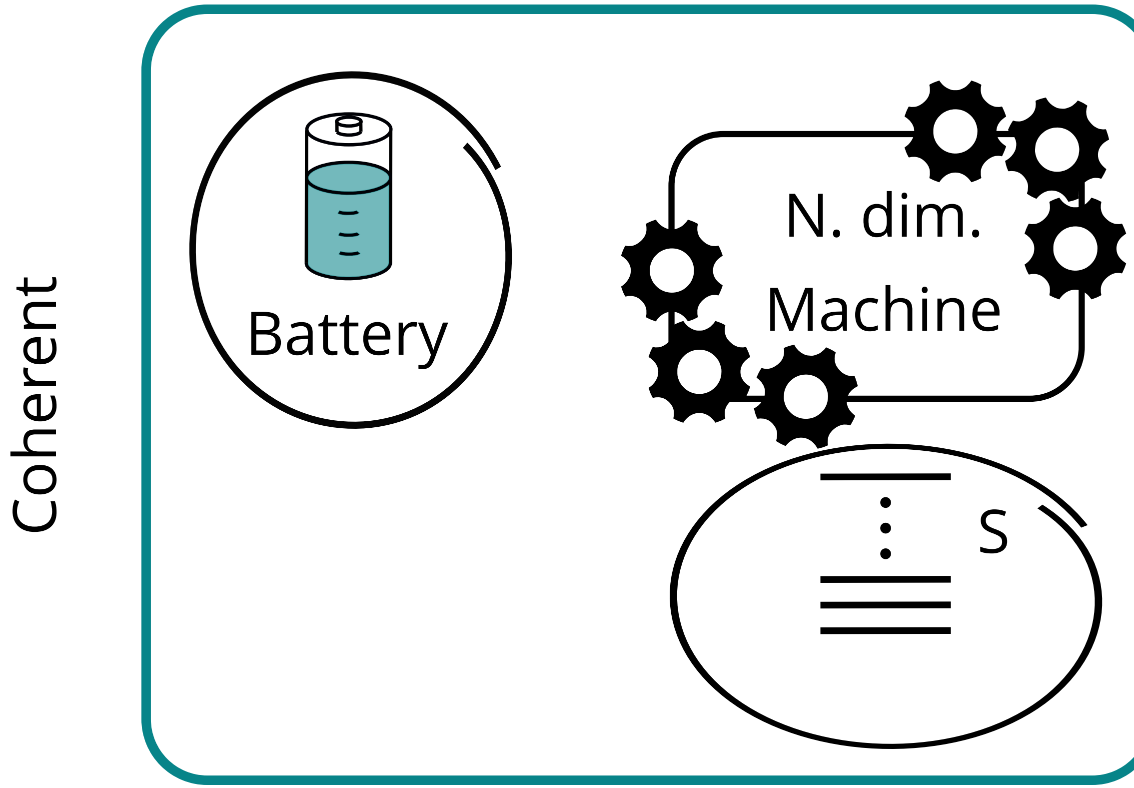

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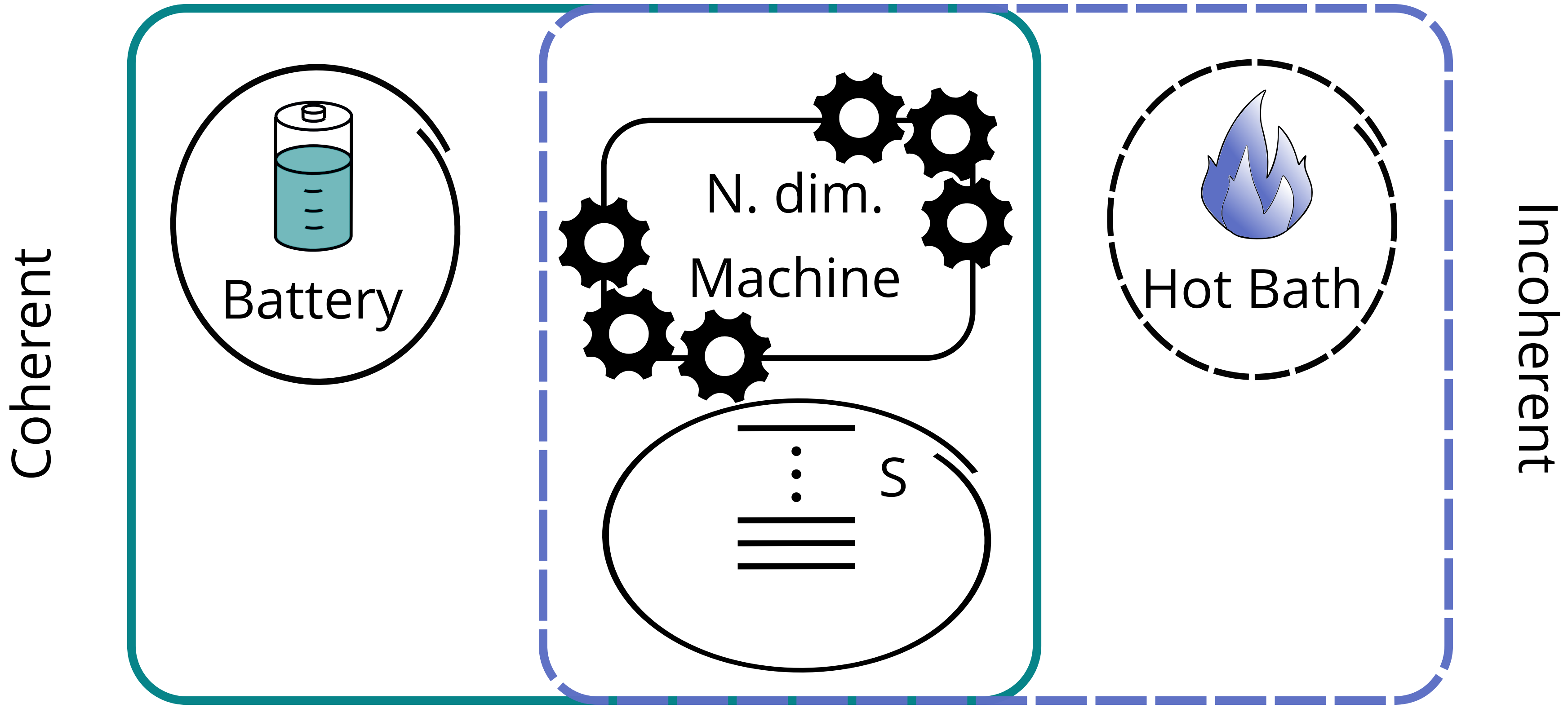

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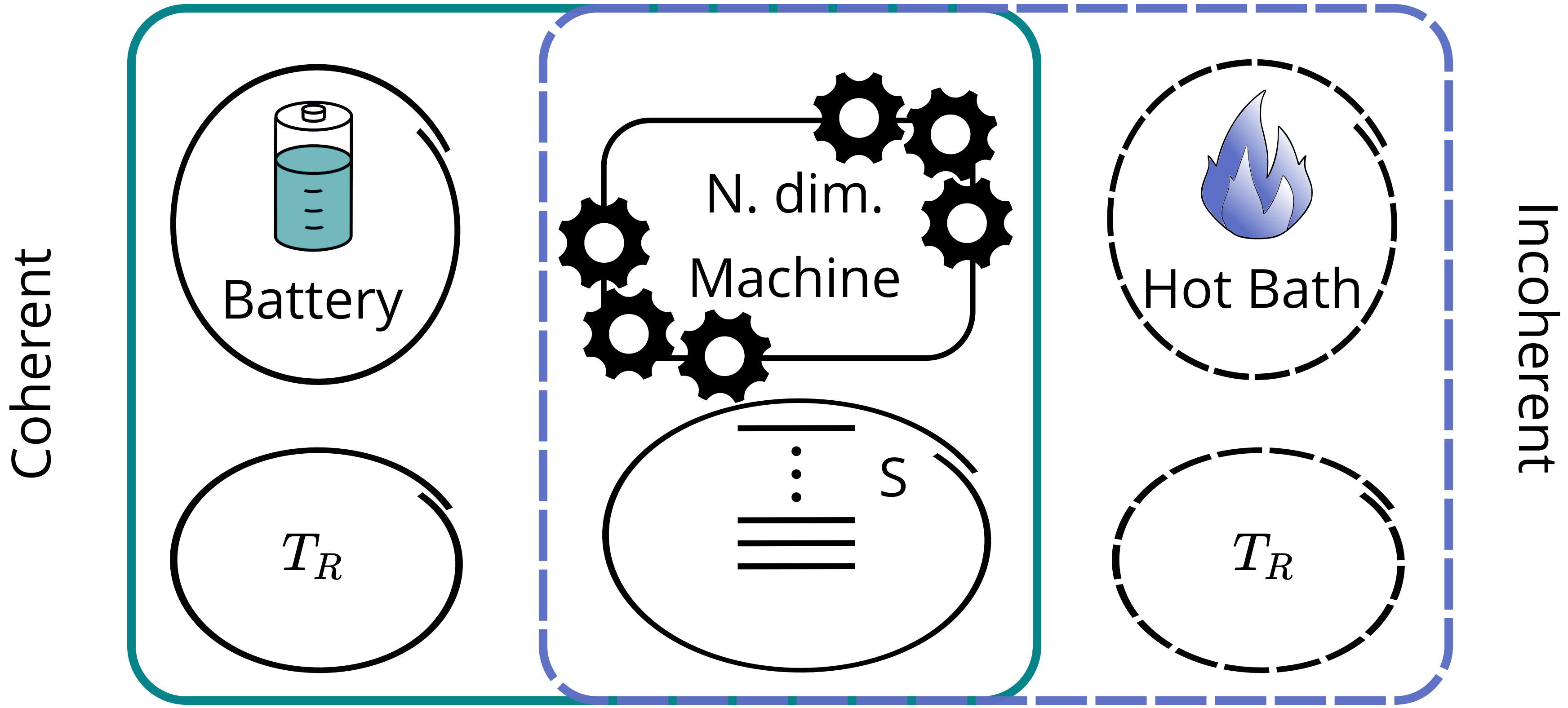

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- allow repetitions: $\Lambda^n(\rho_S) := \Lambda(\cdots\Lambda(\Lambda(\rho_S))) \rightarrow \Lambda^\infty(\rho_S)$

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Related Paradigms

Finite machine
Coherent Λ_{coh}
Incoherent Λ_{inc}

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Finite machine	**Infinite machine**
Coherent Λ_{coh}	CPTP map
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[1] Brandao, Horodecki, Oppenheim, Renes, Spekkens, PRL **111** (2013)

[2] Gour, Müller, Narasimhachar, Spekkens, Yunger Halpern, Phys. Rep. **583** (2015)

[3] Gallego, Eisert, Wilming, NJP.**18** (2016)

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Coherent
- Heat bath algorithmic cooling [4,5,6]
- Quantum Otto engines [7]

Incoherent
- Autonomous cooling [8]

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[4] Schulman, Vazirani, Proc. 31’st ACMSTOC, 322-329 (1999)
[5] Rodriguez-Briones, Martin-Martinez, Kempf, Laflamme, PRL.**119** (2017)
[6] Alhambra, Lostaglio, Perry, arXiv:1807.07974 (2018)
[7] Niedenzu, Gelbwaser-Klimovsky, Kof-man, Kurizki, NJP. **18** (2016).
[8] Skrzypczyk, Brunner, Linden, Popescu, JPA. **44** (2011)

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Universal Bound

Definition: ρ_1 colder than ρ_2 iff $\rho_1 \succ \rho_2$

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Theorem: the ground state population of $\Lambda^\infty(\rho_S)$ is upper bounded by:

$$p_0^* = \frac{1}{\sum_{n=0}^{d_S-1} (e^{-\frac{1}{T_R}\mathcal{E}_{\max}})^n}$$

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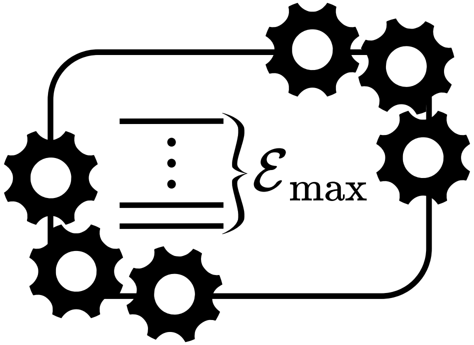

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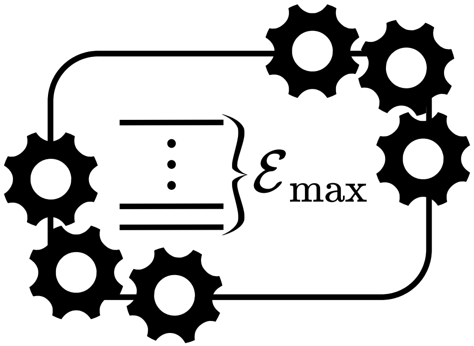

Proof (sketch):

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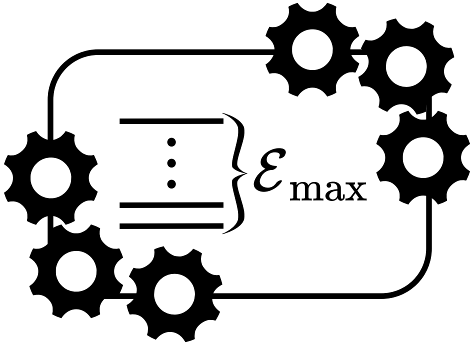

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- Hot bath not more powerful than a battery
  $\Rightarrow \Lambda_{\mathrm{inc}}^{\infty}(\rho_S) \prec \Lambda_{\mathrm{coh}}^{\infty}(\rho_S)$ $\square$

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Theorem: the bound is reachable within the coherent paradigm

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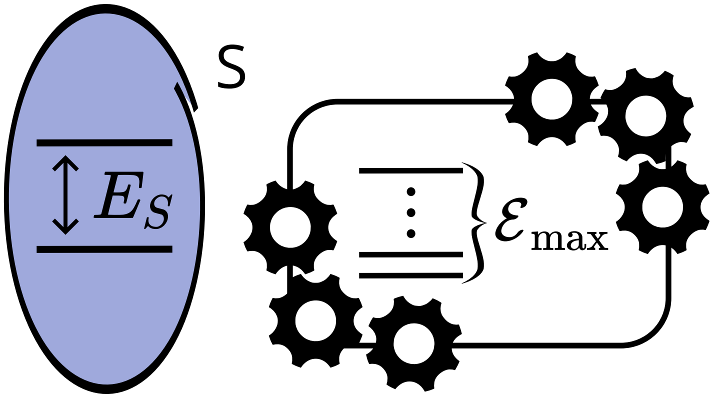

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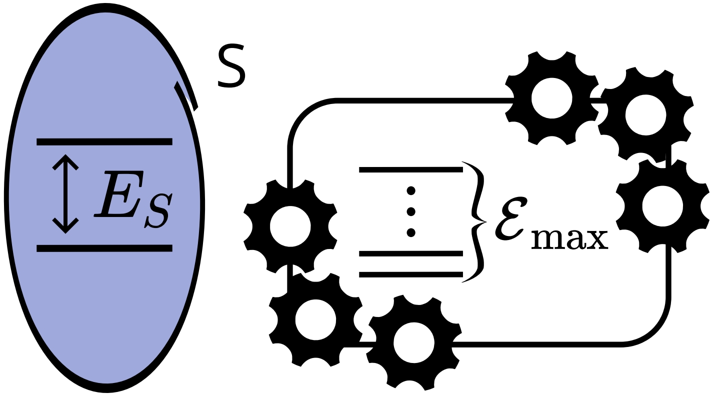

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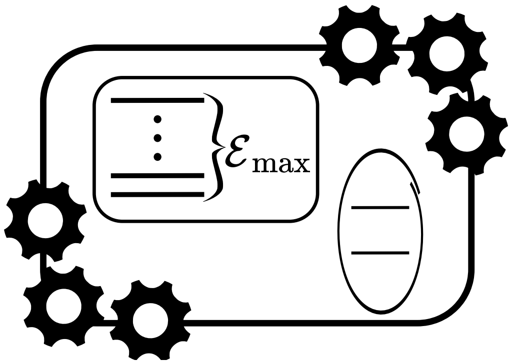
Theorem: For S qubit, can incoherently cool to T^* if add one qubit (of gap $\mathcal{E}_{\max} - E_S$) to machine



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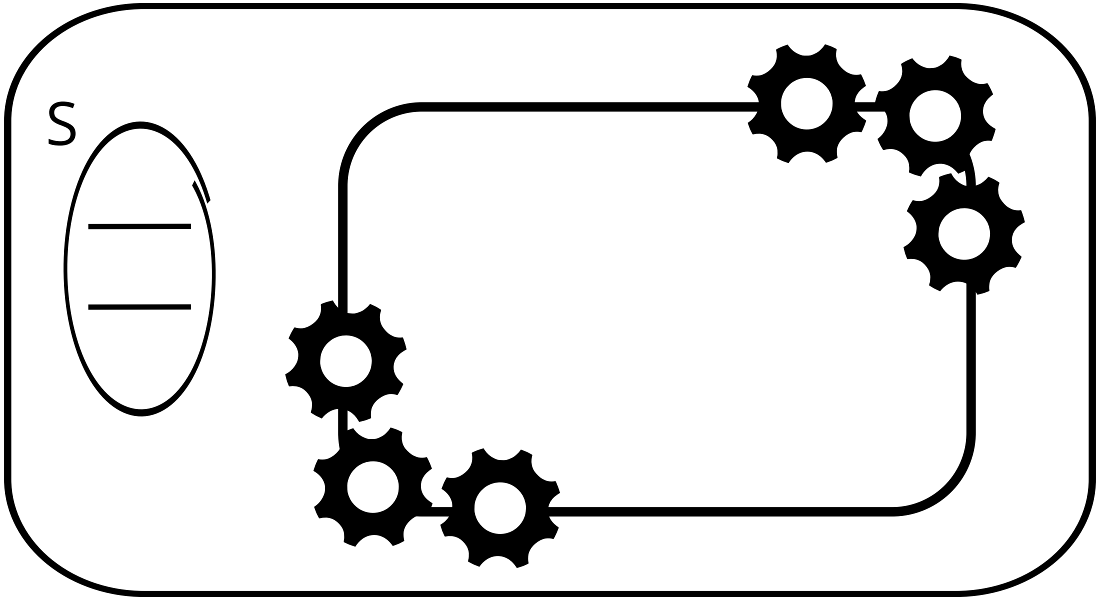

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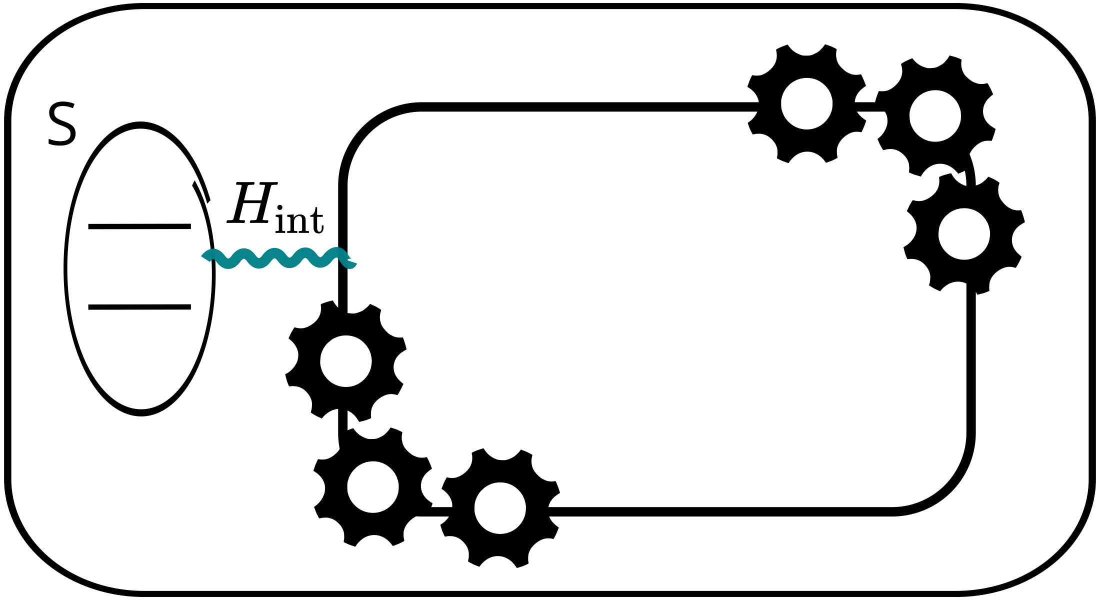

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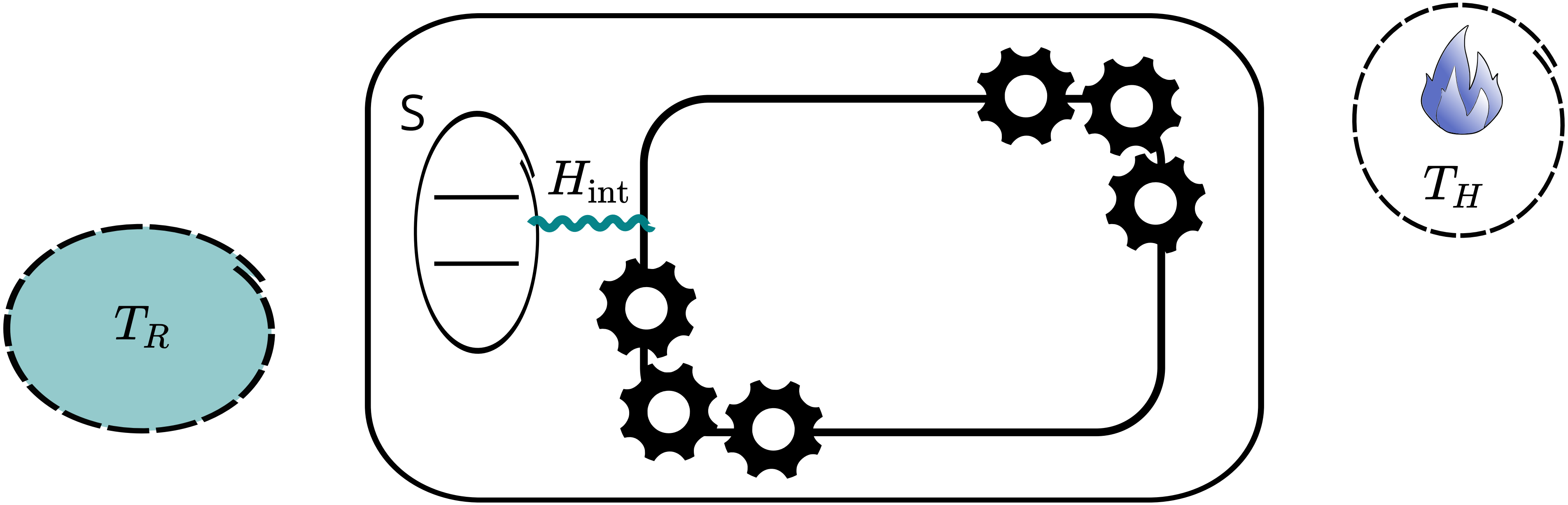

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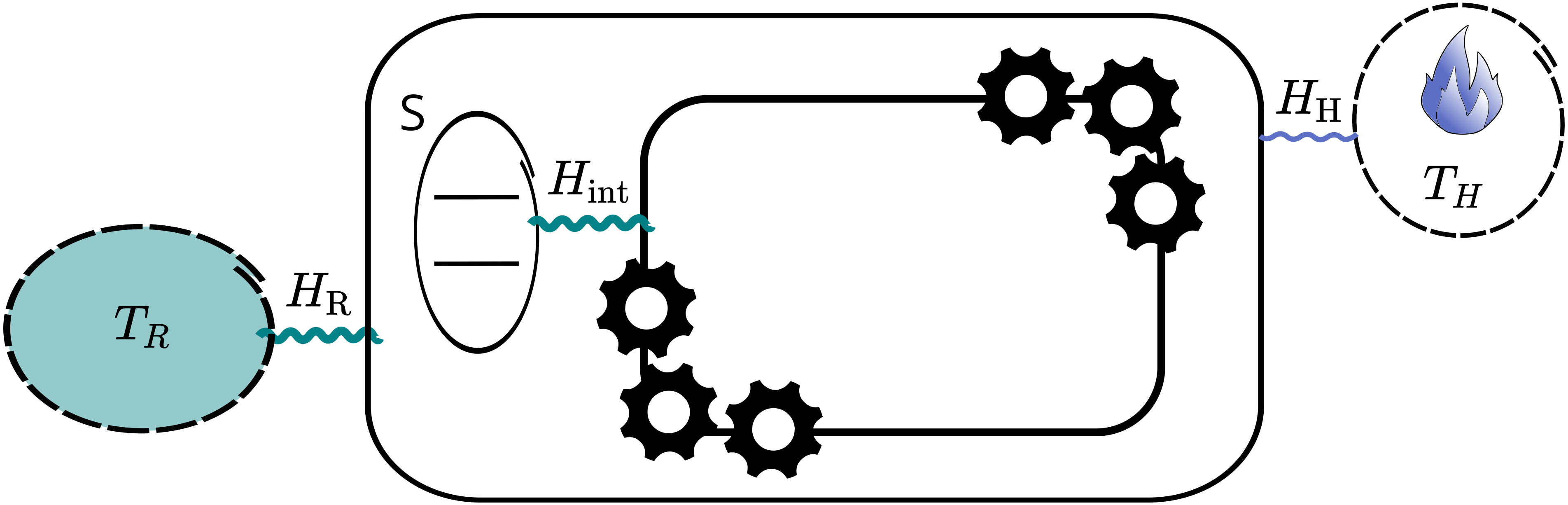

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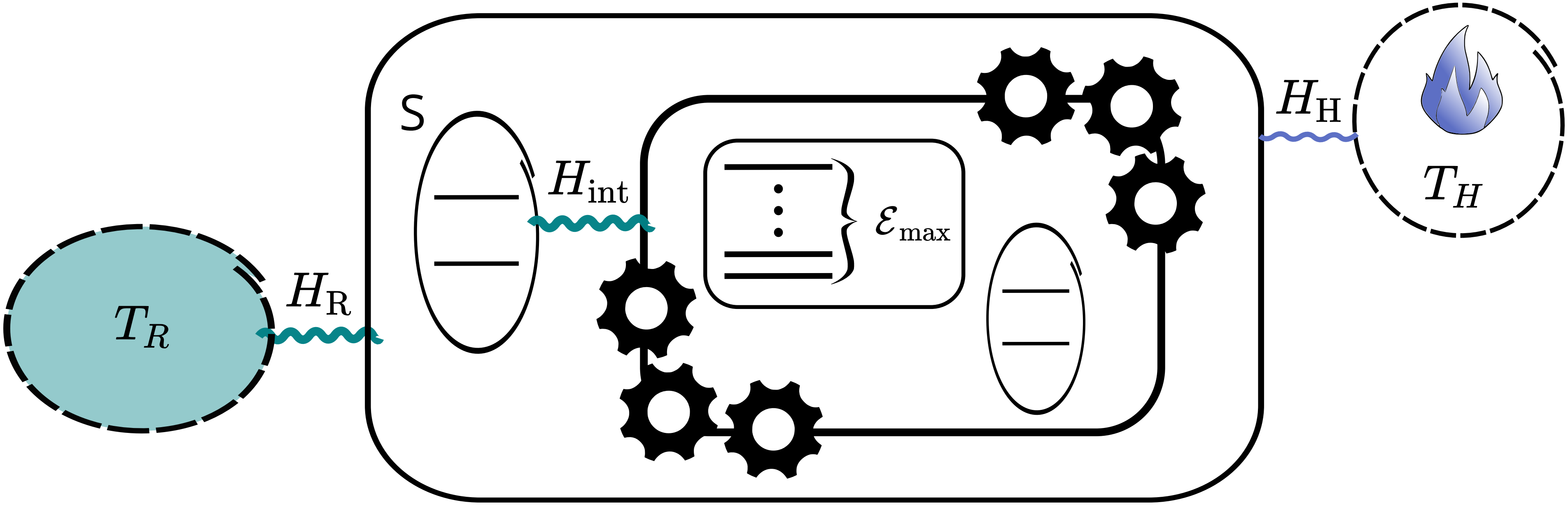

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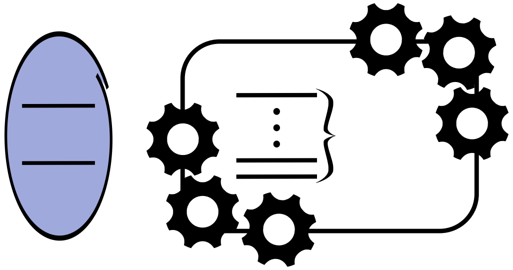
Summary



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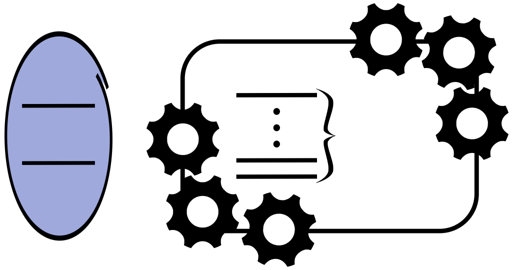

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