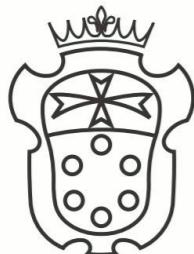


# Quantum Batteries: Extractable works and the role of correlations



Gian Marcello Andolina

**Phys. Rev. Lett. 122, 047702  
(2019)**

## Supervisors:

**M. Polini**



**V. Giovannetti**



## Collaborators:

**M. Keck**



**A. Mari**

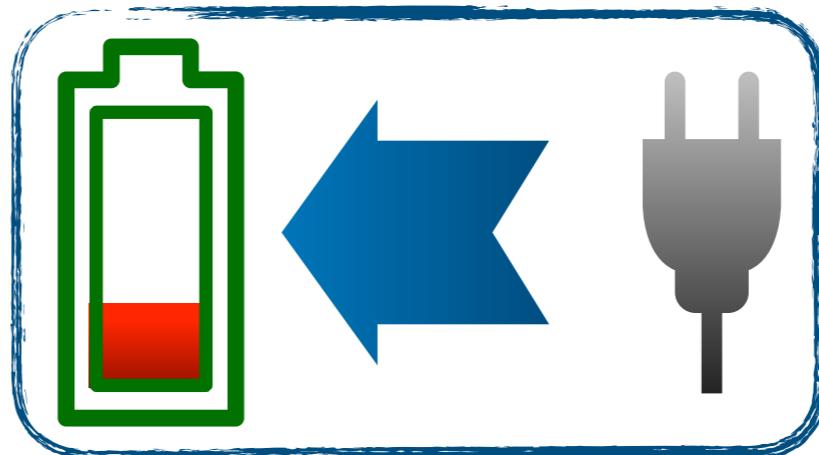


**M. Campisi**

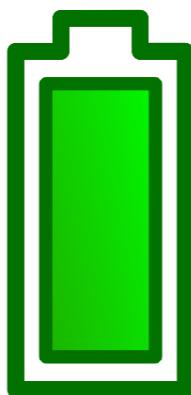


# Outline

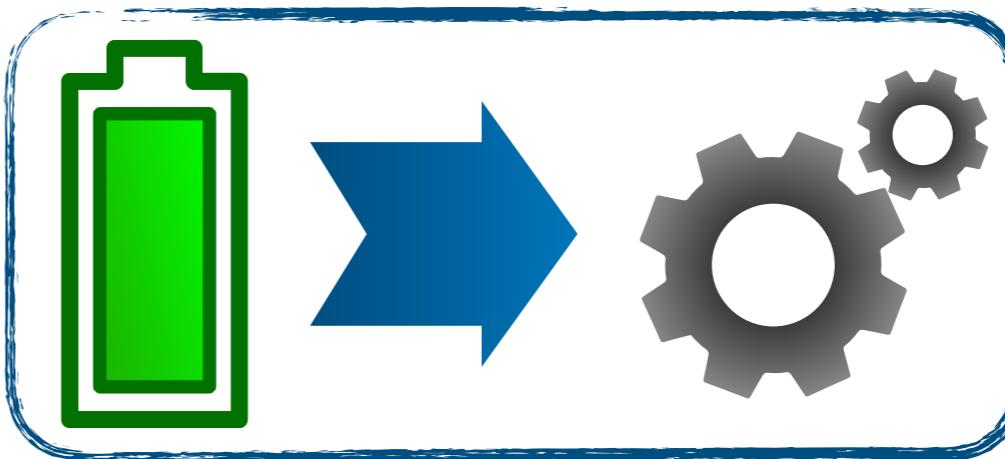
Charging



Storage

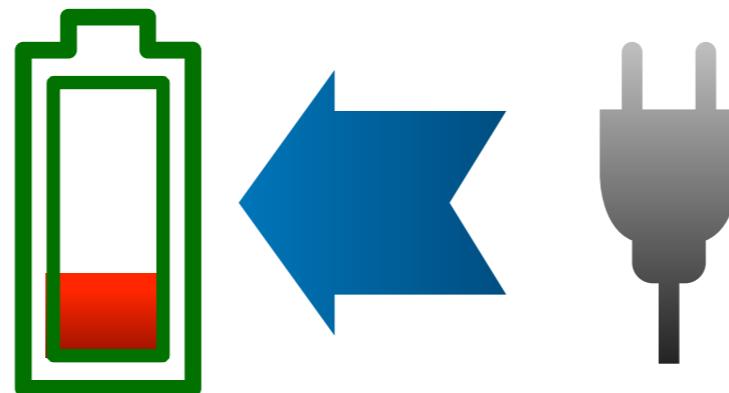


Work Extraction



# Charging

## Collective Speed-Up

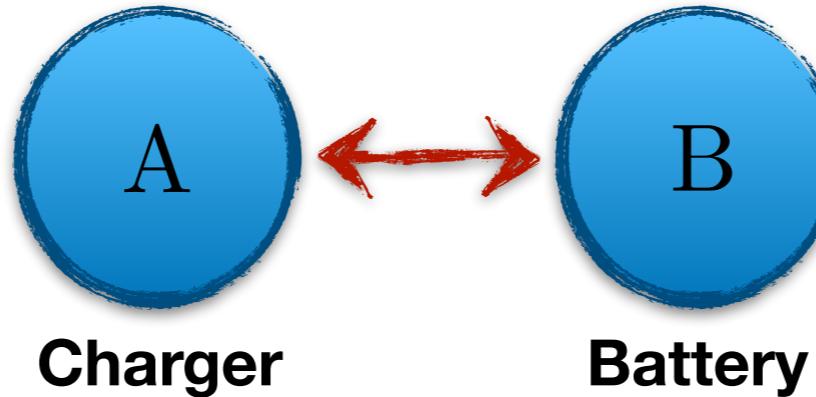


F. C. Binder, S. Vinjanampathy, K. Modi, and J. Goold,  
New J. Phys. **17**, 075015 (2015).

F. Campaioli, F.A. Pollock, F.C. Binder, L.Céleri, J.Goold, S. Vinjanampathy, and K. Modi,  
Phys. Rev. Lett. **118**, 150601 (2017)

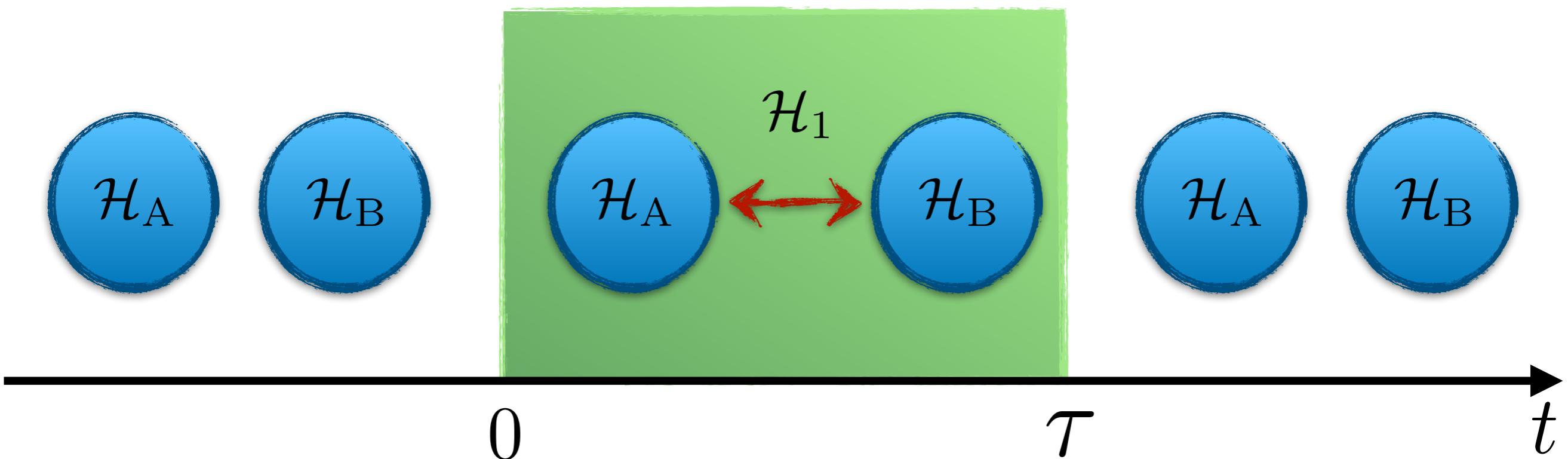
D. Ferraro, M. Campisi, G.M. Andolina, V. Pellegrini, and M. Polini  
Phys. Rev. Lett. **120**, 117702 (2018)

# Charged-mediated energy transfer



$$\mathcal{H}(t) = \mathcal{H}_A + \mathcal{H}_B + \lambda(t)\mathcal{H}_1$$

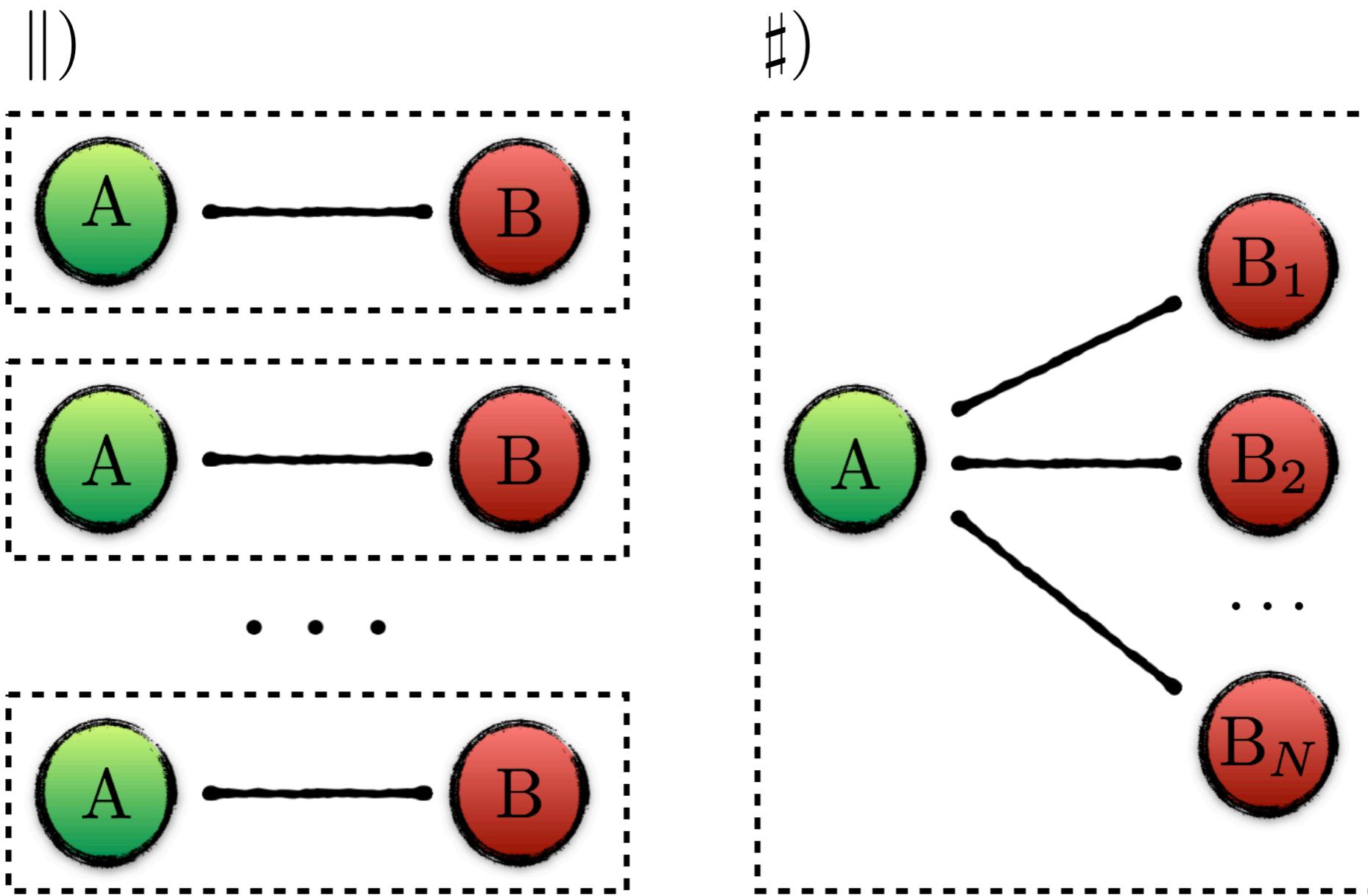
$$\lambda(t) = \mathbb{I}_{[0,\tau]}(t)$$



D. Ferraro, M. Campisi, G.M. Andolina,  
V. Pellegrini, and M. Polini  
Phys. Rev. Lett. **120**, 117702 (2018)

G.M. Andolina, D. Farina, A. Mari, V. Pellegrini,  
V. Giovannetti and M. Polini  
Phys. Rev. B **98**, 205423 (2018)

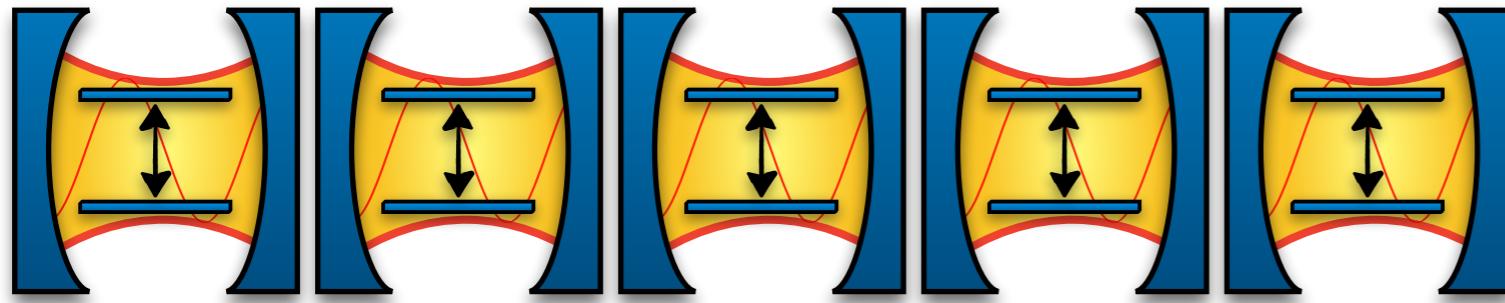
## Parallel vs Collective Charging



$$E_A^{(\#)}(0) = N E_A^{(||)}(0)$$

# Realistic models for cQED

## Parallel charging

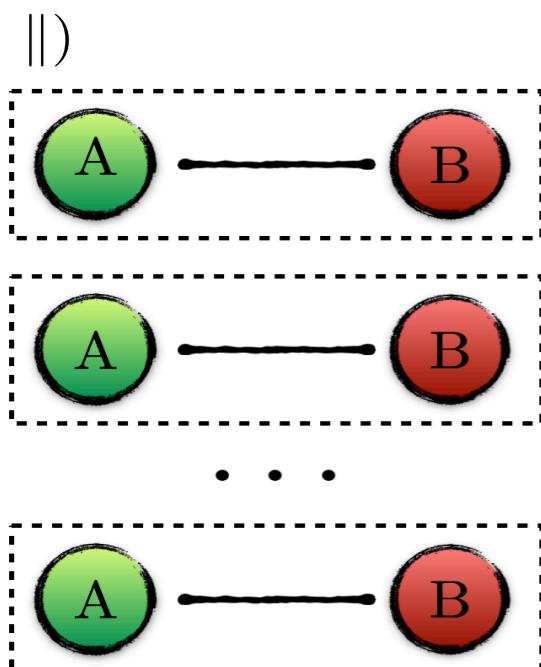


## Rabi Battery

$$\mathcal{H}_A = \omega_0 a^\dagger a$$

$$\mathcal{H}_B = \omega_0 \sigma^+ \sigma^-$$

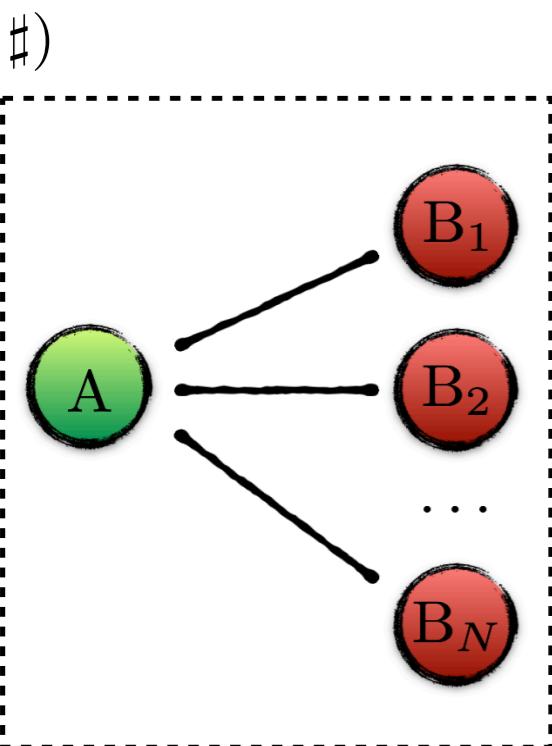
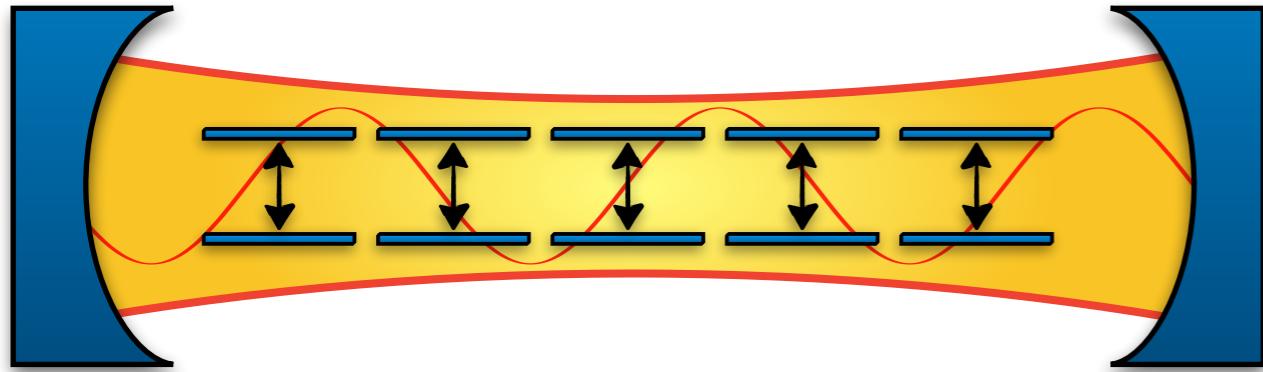
$$\mathcal{H}_1 = g(a + a^\dagger) \sigma^x$$



$$|\Psi(0)\rangle = [|\downarrow\rangle |1\rangle]^{\otimes N}$$

# Realistic models for cQED

## Collective charging



## Dicke Battery

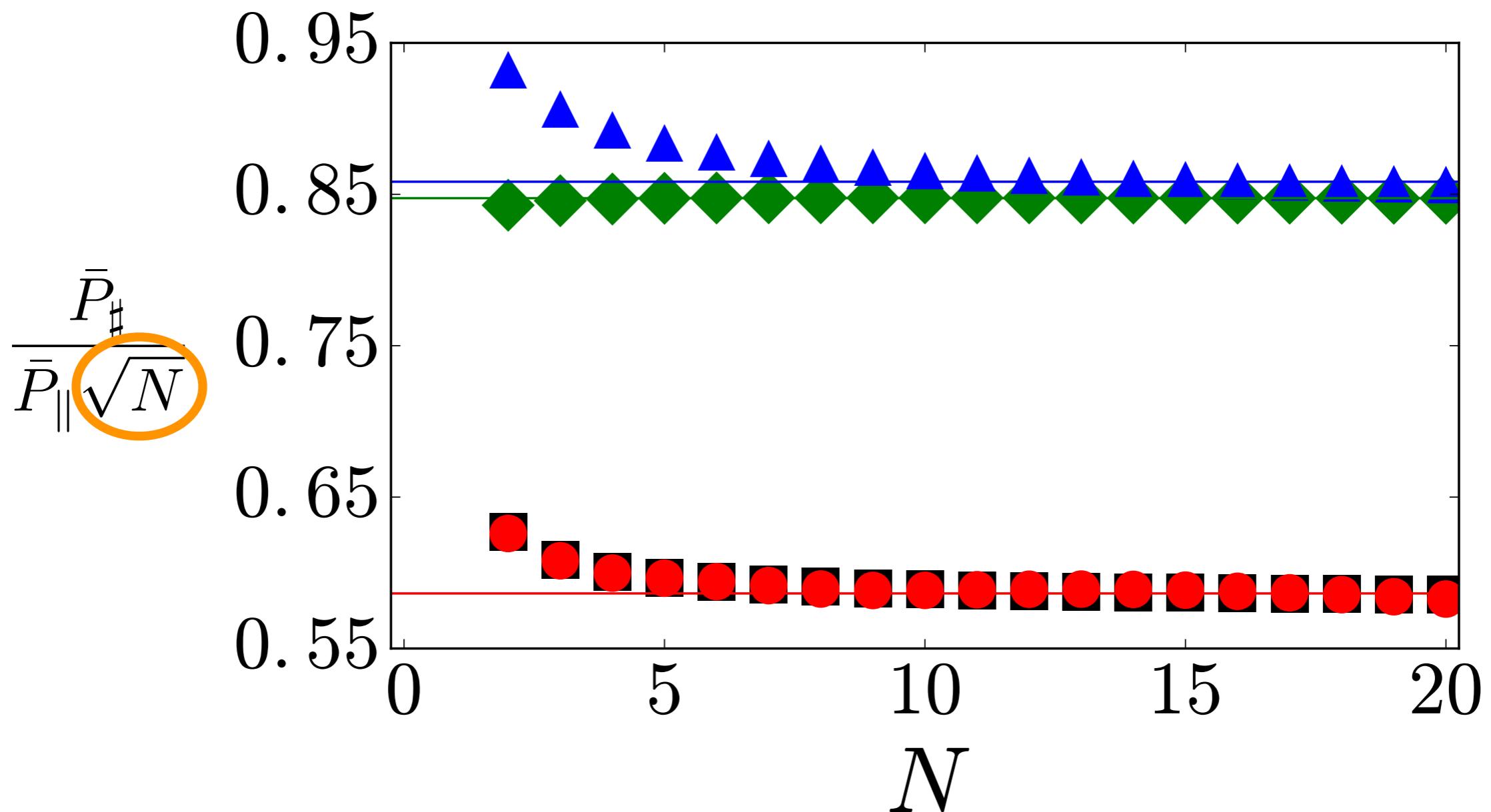
$$\mathcal{H}_A = \omega_0 a^\dagger a$$

$$\mathcal{H}_B = \sum_i \omega_0 \sigma_i^+ \sigma_i^-$$

$$\mathcal{H}_1 = \sum_i g(a + a^\dagger) \sigma_i^x$$

$$|\Psi(0)\rangle = |\downarrow, \downarrow, \dots, \downarrow, \rangle |N\rangle$$

$\bar{\tau}$    **Optimal time**

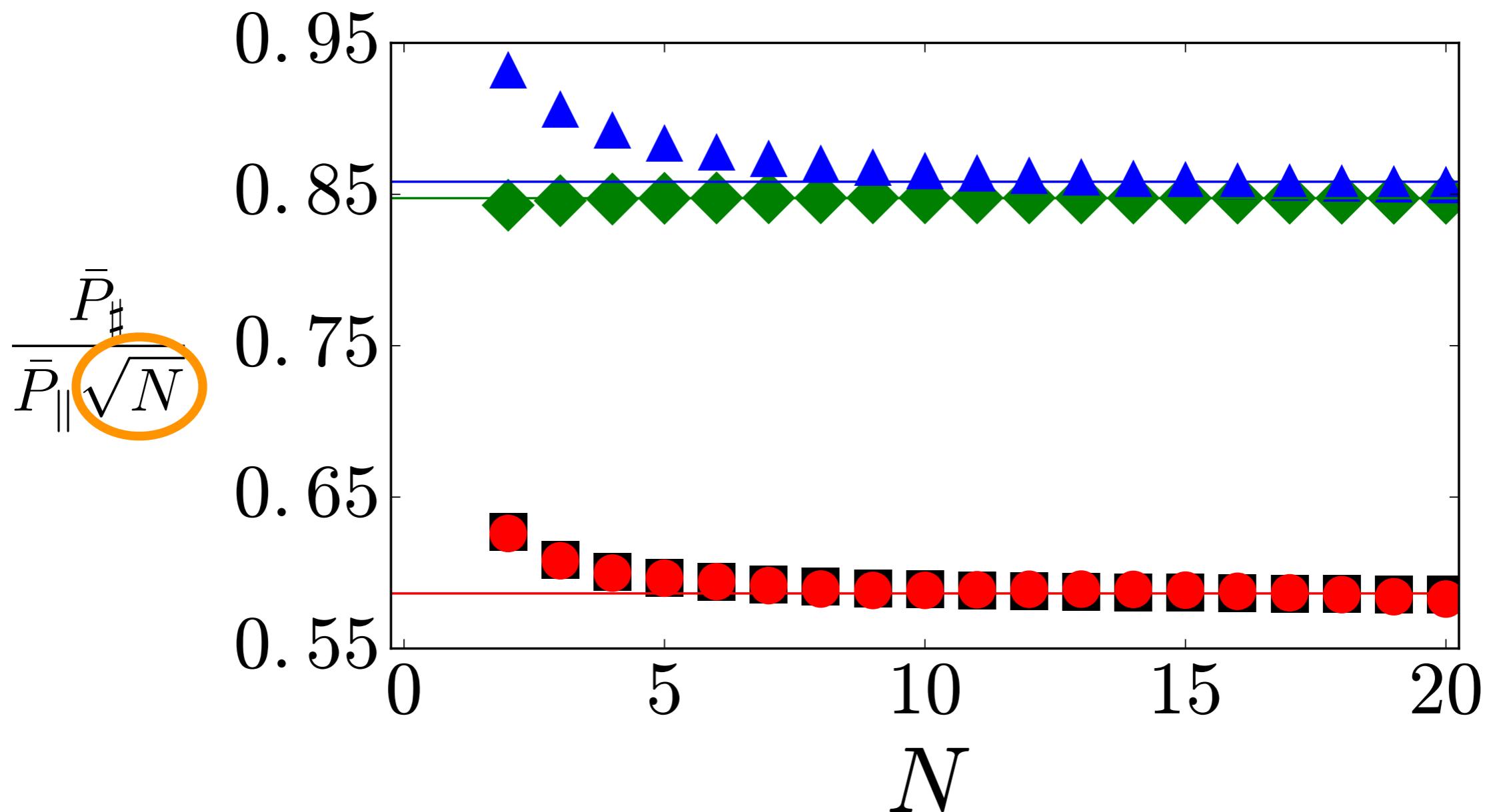


$$P(\tau) = \frac{E(\tau)}{\tau}$$

**Collective advantage**

$$\tau_\# \sim \frac{\tau_\parallel}{\sqrt{N}}$$

$\bar{\tau}$    **Optimal time**

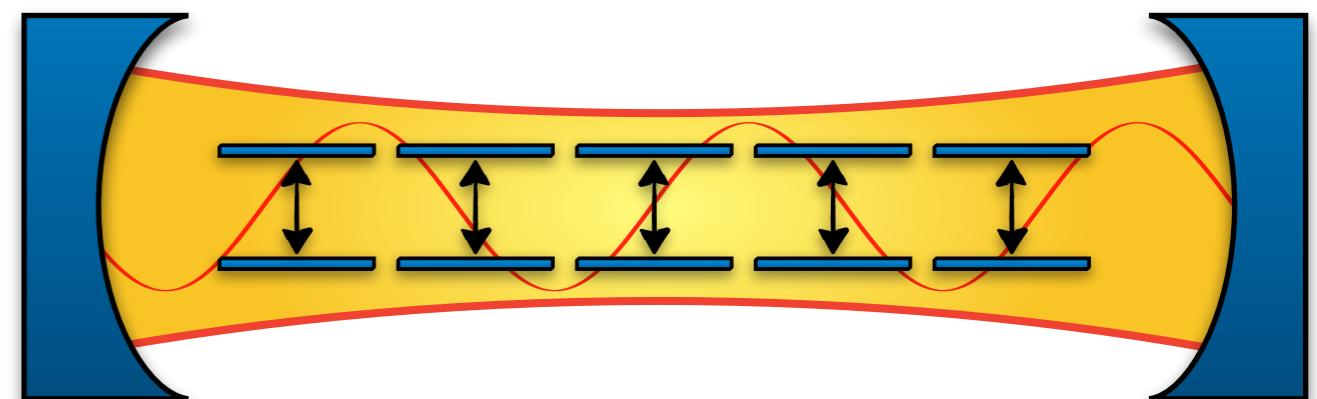
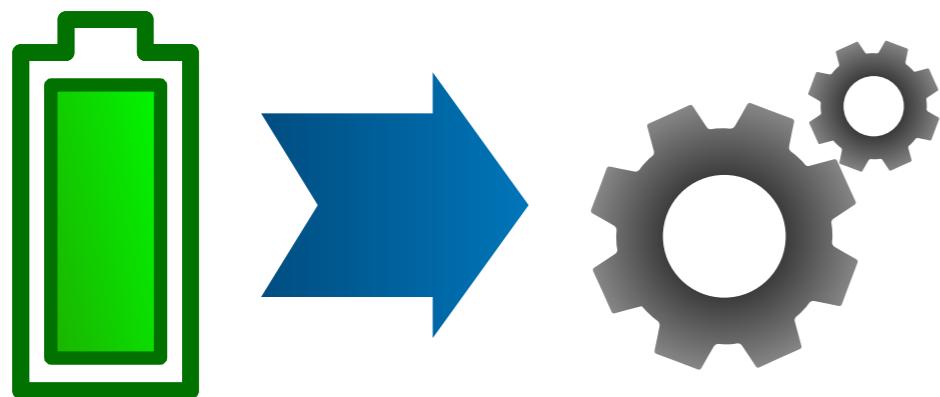


$$P(\tau) = \frac{E(\tau)}{\tau}$$

**Collective advantage**

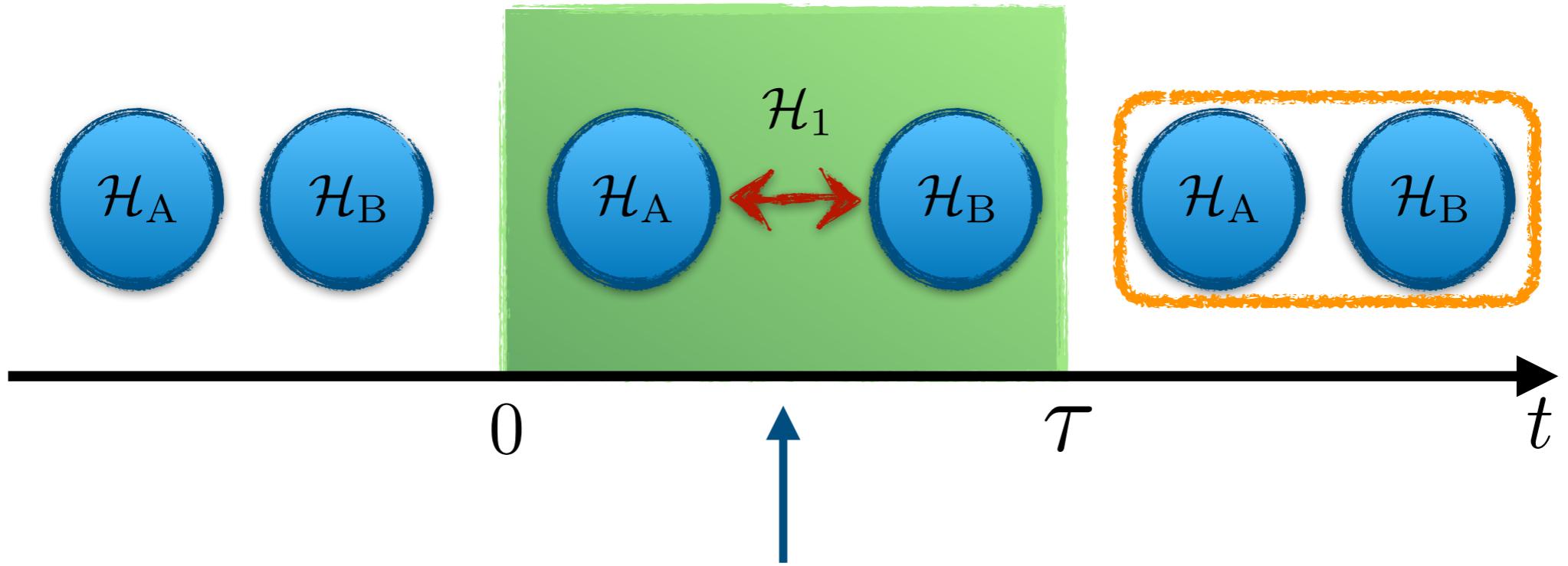
$$\Gamma = \frac{\bar{P}^{(\sharp)}}{\bar{P}^{(\parallel)}} \sim \sqrt{N}$$

# Correlations and work extraction



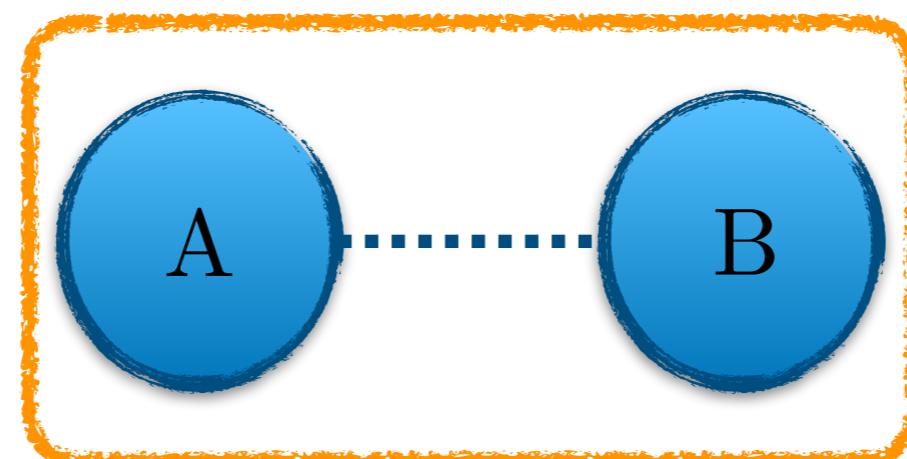
R. Alicki and M. Fannes  
Phys. Rev. E **87**, 042123 (2013)

G.M. Andolina, A. Mari, M. Keck, M. Campisi,  
M. Polini and V. Giovannetti  
Phys. Rev. Lett. **122**, 047702 (2019)

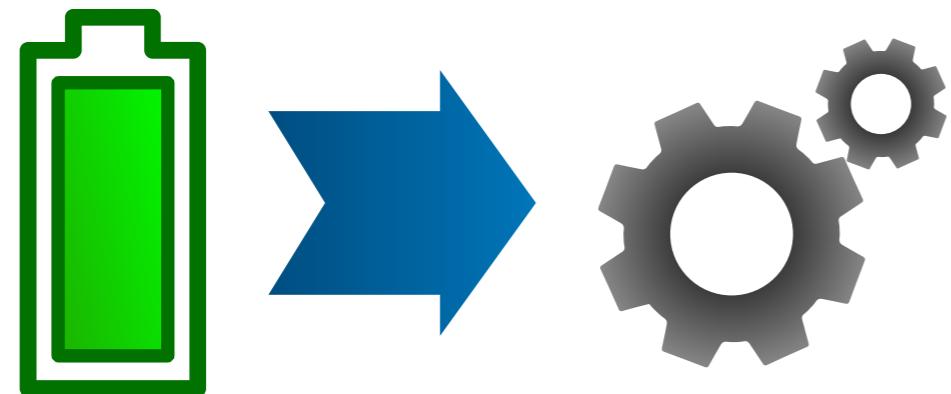


**Previous work focused on the speed of the process**

**During the charging some correlations are created**



# Ergotropy



$$\rho \rightarrow U(t^*)\rho U^\dagger(t^*)$$

$$U(t) = \mathcal{T}\left\{\exp\left[-i\int_0^t dt'(\mathcal{H}_0 + \mathcal{V}(t'))\right]\right\}$$

$$\mathcal{V}(t)$$

$$W = \text{tr}[\rho \mathcal{H}_0] - \text{tr}[U(t^*)\rho U^\dagger(t^*)\mathcal{H}_0]$$

$$\mathcal{E}(\rho) := W_{\max} = \text{tr}[\rho \mathcal{H}_0] - \min_U \text{tr}[U \rho U^\dagger \mathcal{H}_0]$$

$$\mathcal{E}(\rho) \leq \Delta F_\beta(\rho) = E(\rho) - E(\omega_\beta) - \frac{1}{\beta} [S(\rho) - S(\omega_\beta)] \quad \forall \beta$$

R. Alicki and M. Fannes  
Phys. Rev. E **87**, 042123 (2013)

G. Manzano, F. Plastina, R. Zambrini  
Phys. Rev. Lett. **121**, 120602 (2018)

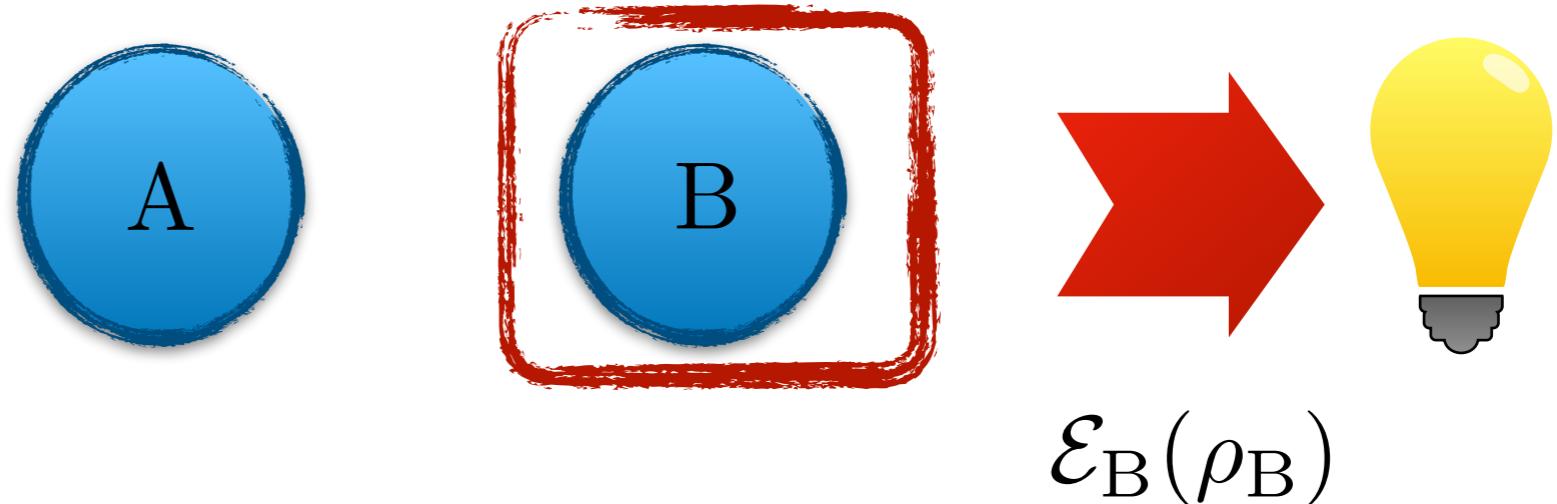
## After the protocol



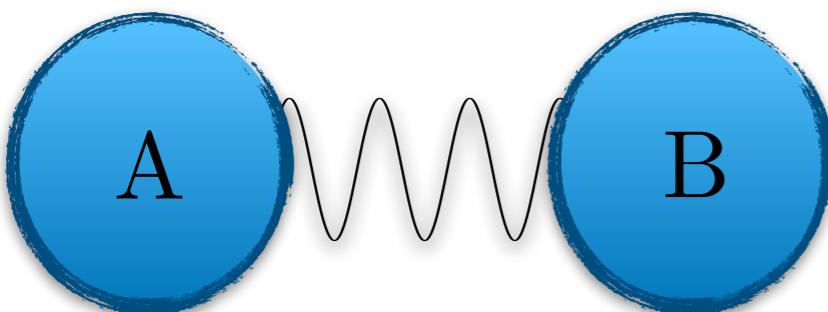
$$\rho_{AB}^f = U^\dagger \rho_{AB}^i U$$

$\rho_{AB}$  **remains pure**

## After the protocol



$\rho_B$  is no longer pure  
Due to correlations established with A



$$\mathcal{E}_B(\rho_B) \neq E_B(\rho_B)$$

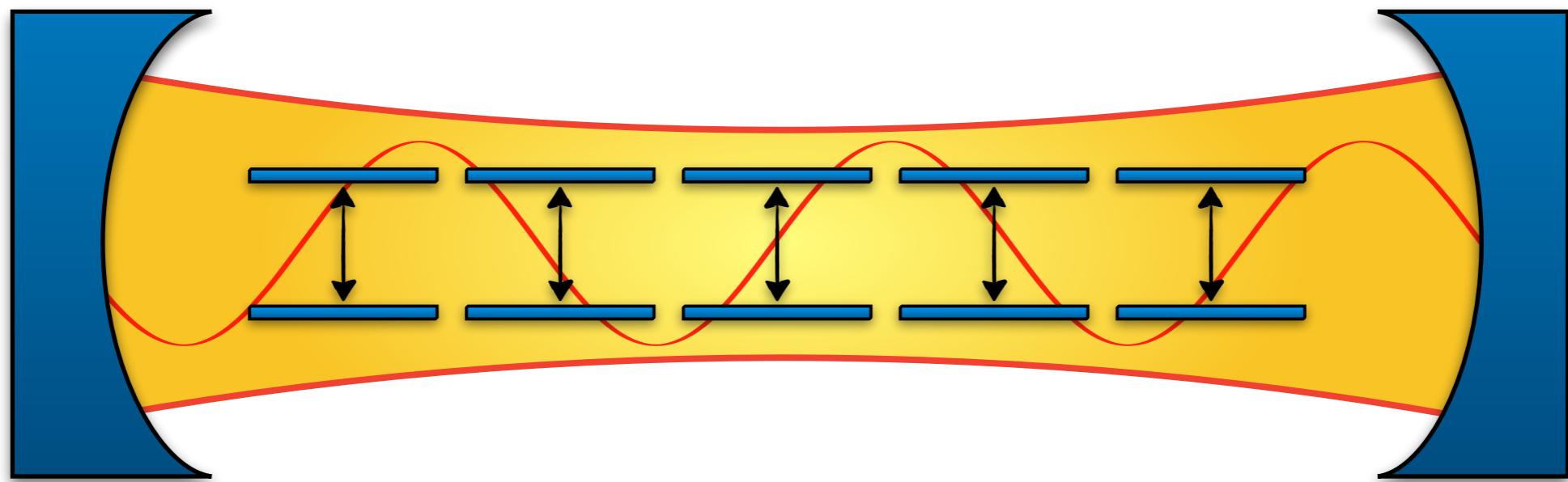
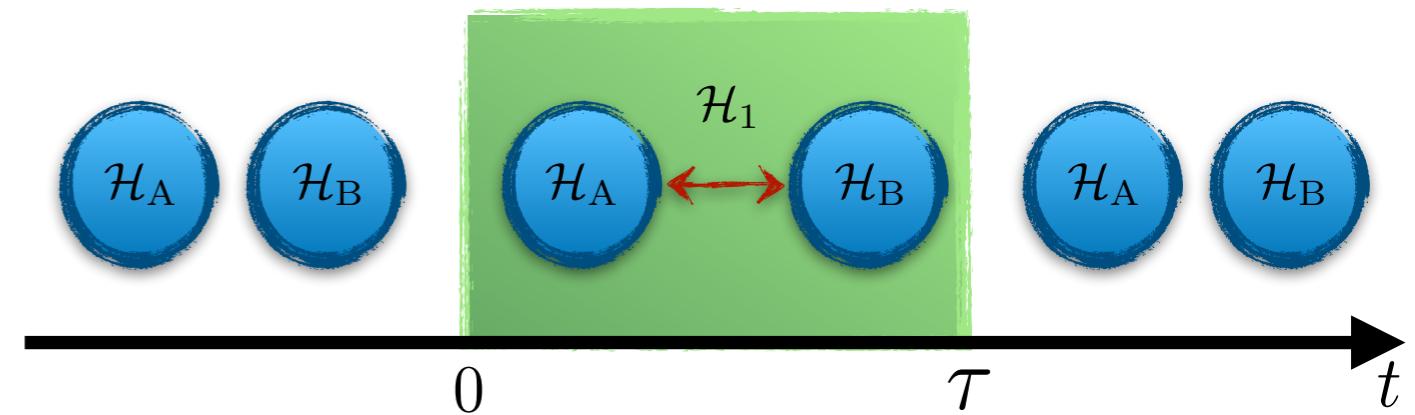
There is energy locked in correlations

## Tavis-Cummings battery

$$\mathcal{H}_A = \omega_0 a^\dagger a$$

$$\mathcal{H}_B = \sum_i \omega_0 \sigma_i^+ \sigma_i^-$$

$$\mathcal{H}_1 = \sum_i g(a\sigma_i^+ + a^\dagger\sigma_i^-)$$



## Fock

$$p_n^{(N)} = \delta_{n,N}$$

## Coherent

$$p_n^{(N)} = e^{-N} N^n / n!$$

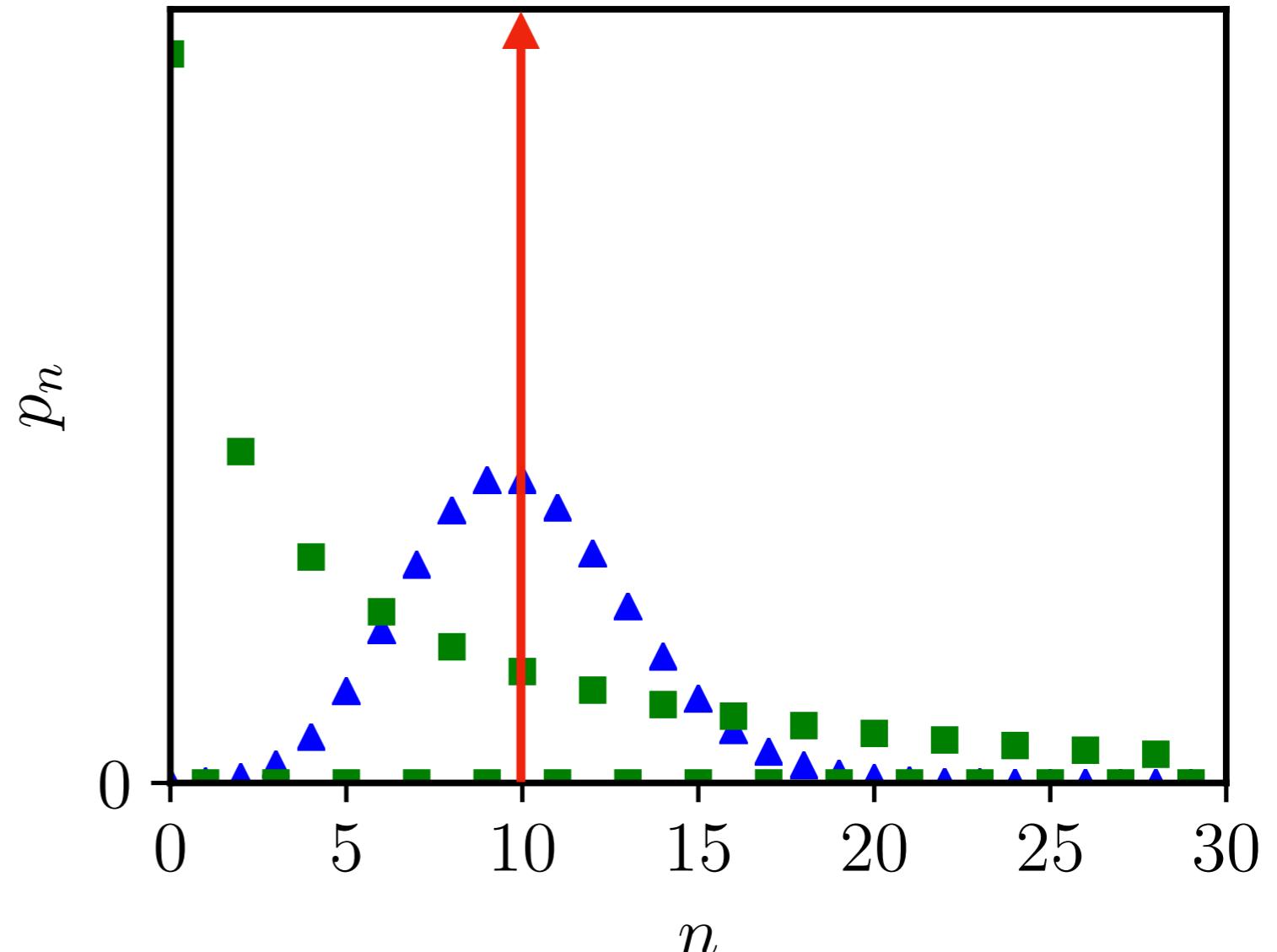
## Squeezed vacuum

$$p_{2n}^{(N)} = \frac{1}{\cosh(r_N)} \left[ \frac{\tanh(r_N)}{2} \right]^{2n} \frac{2n!}{n!^2}$$

$$p_{2n+1}^{(N)} = 0$$

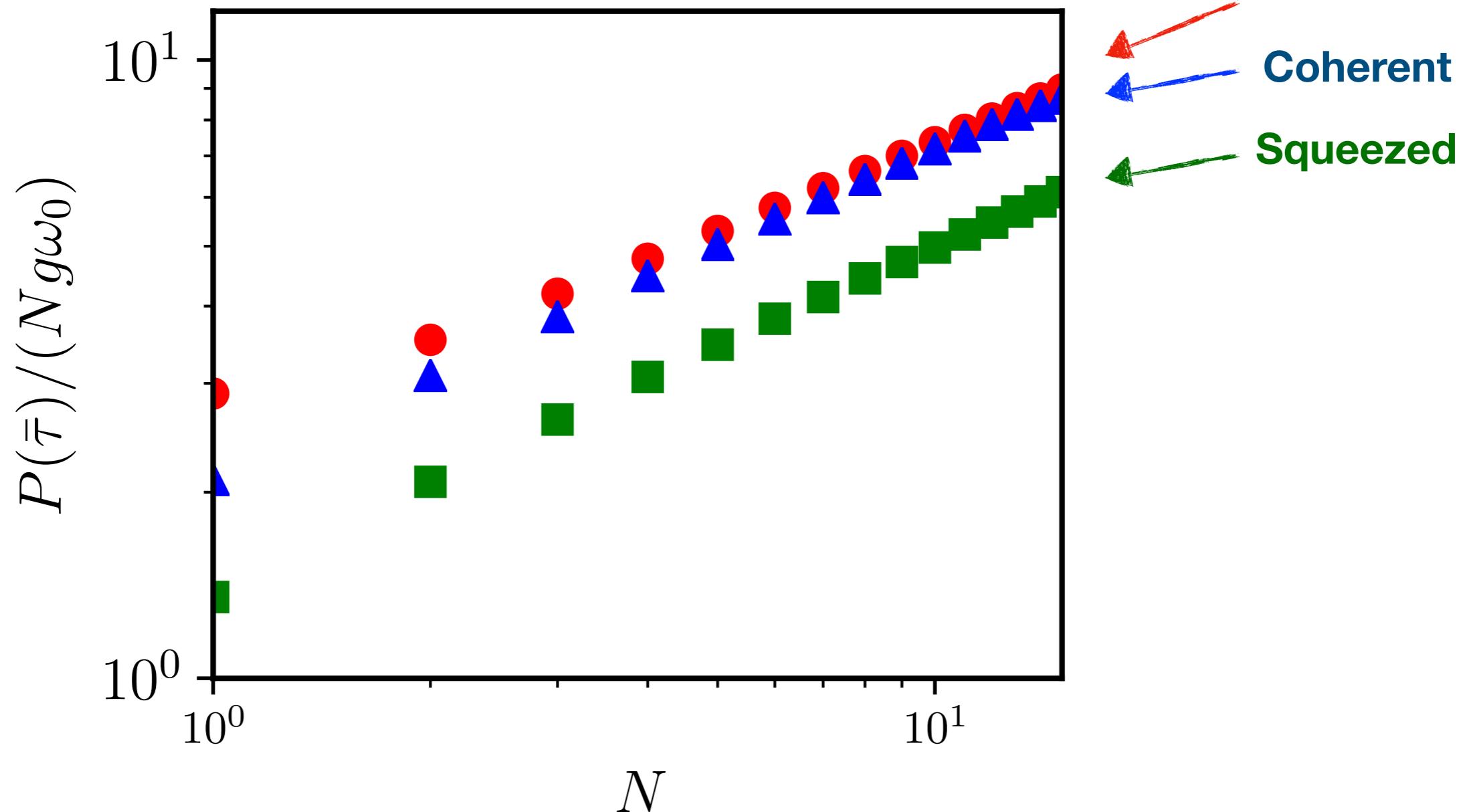
$$r_N = \text{arcsinh}(\sqrt{N})$$

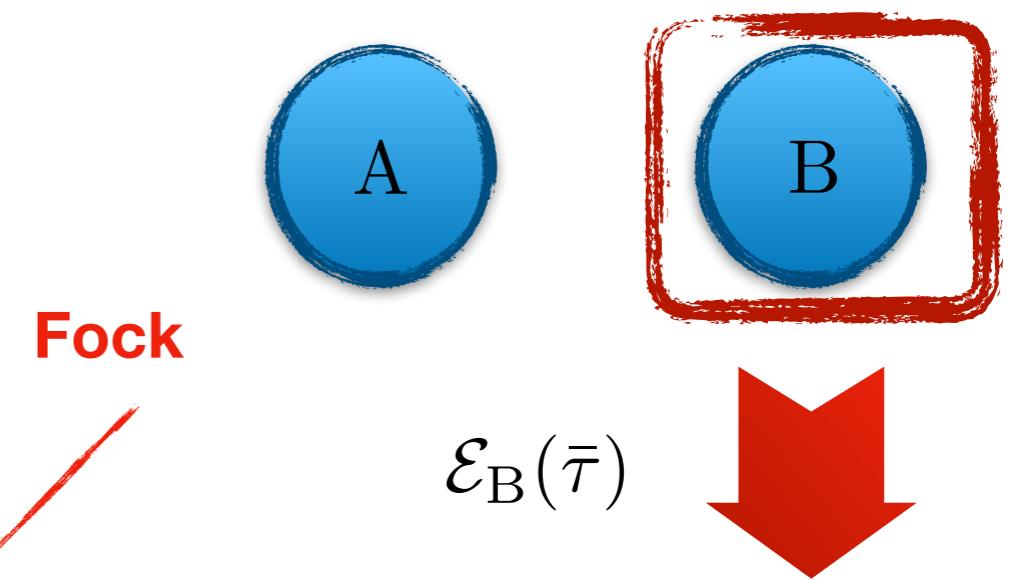
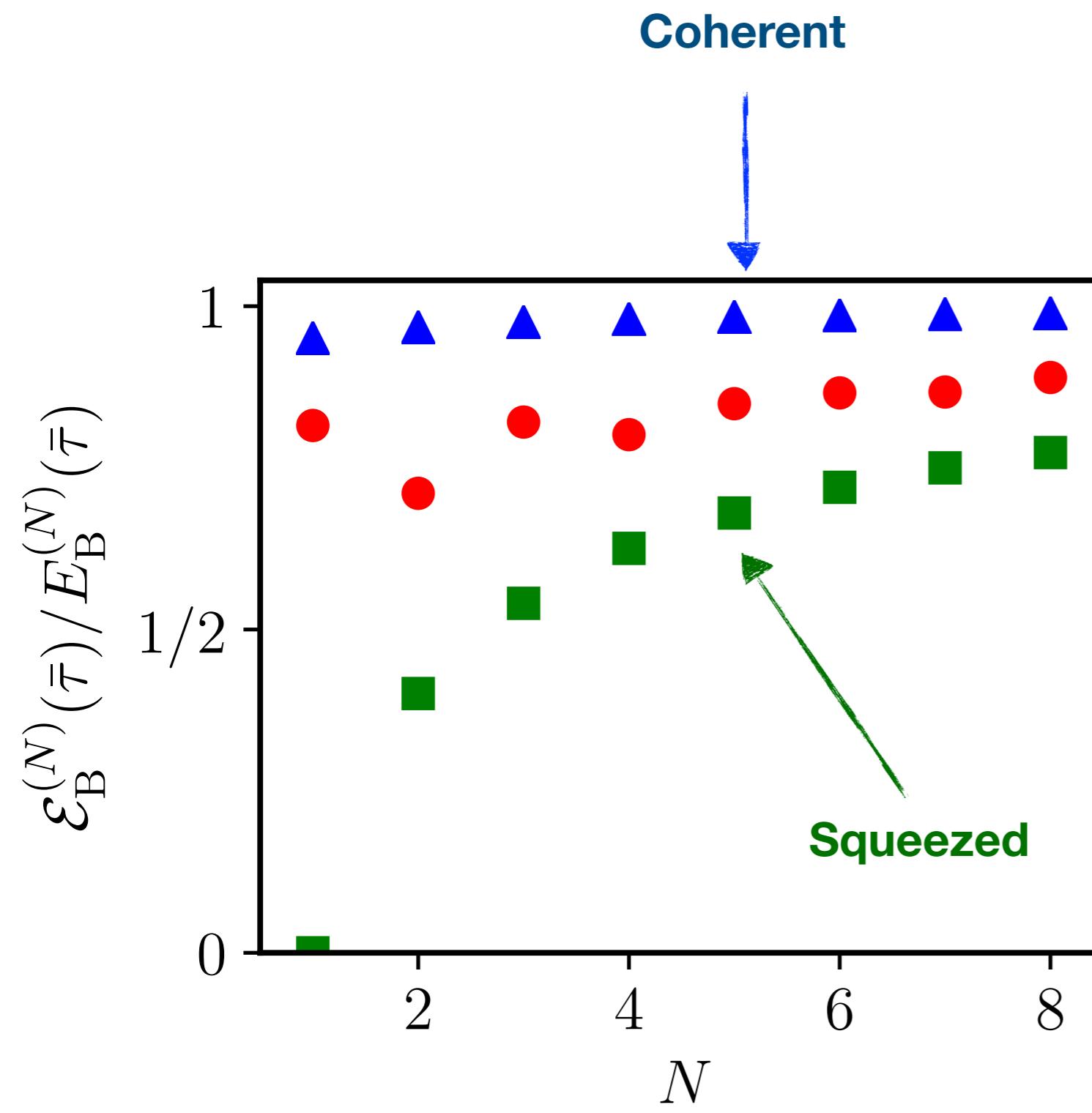
$$|\Psi(0)\rangle = |\downarrow, \downarrow, \dots, \downarrow\rangle |\phi_{\text{ph}}\rangle$$



$$\langle \mathcal{H}_A \rangle = N\omega_0$$

All states share the same collective advantage





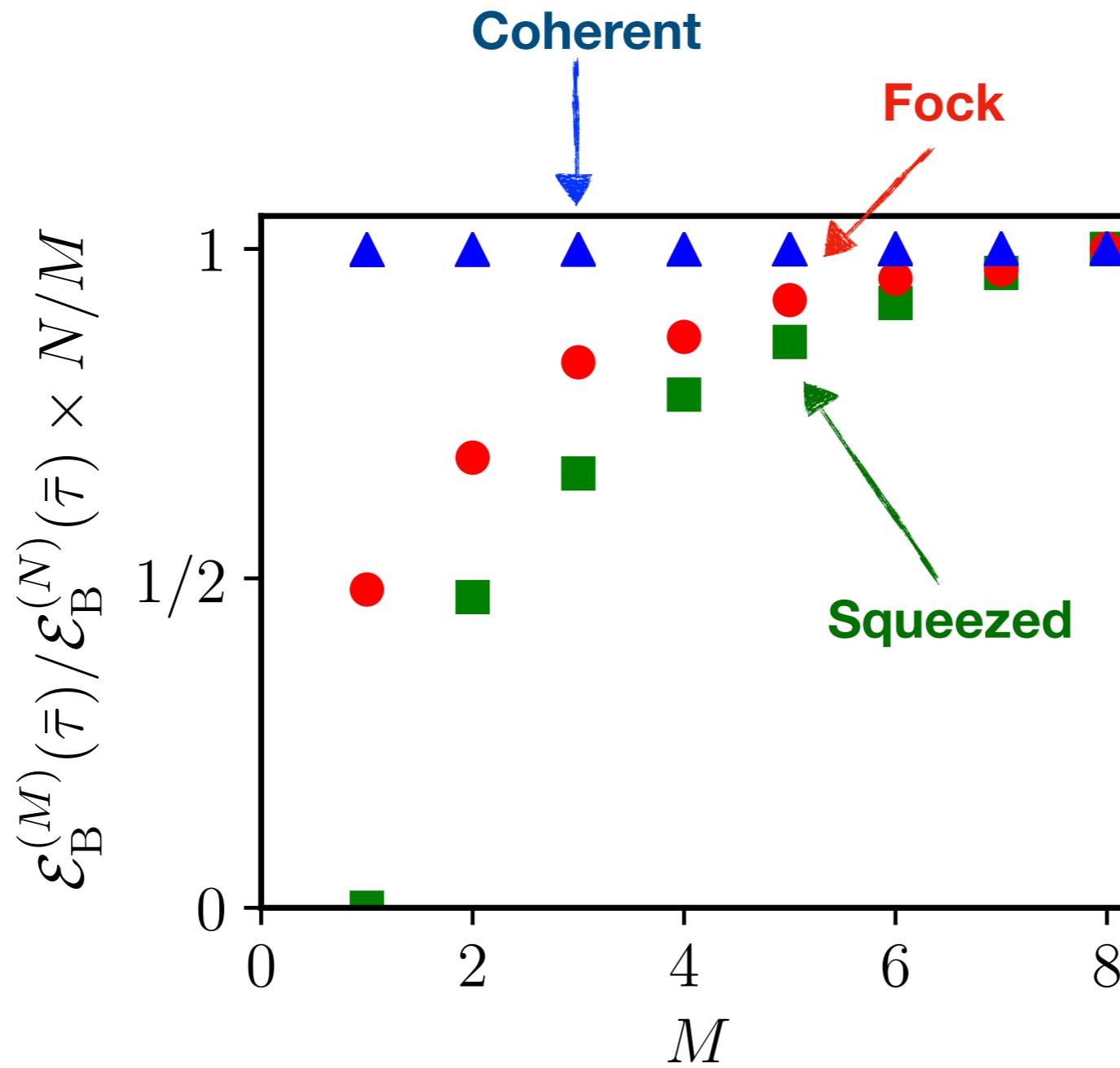
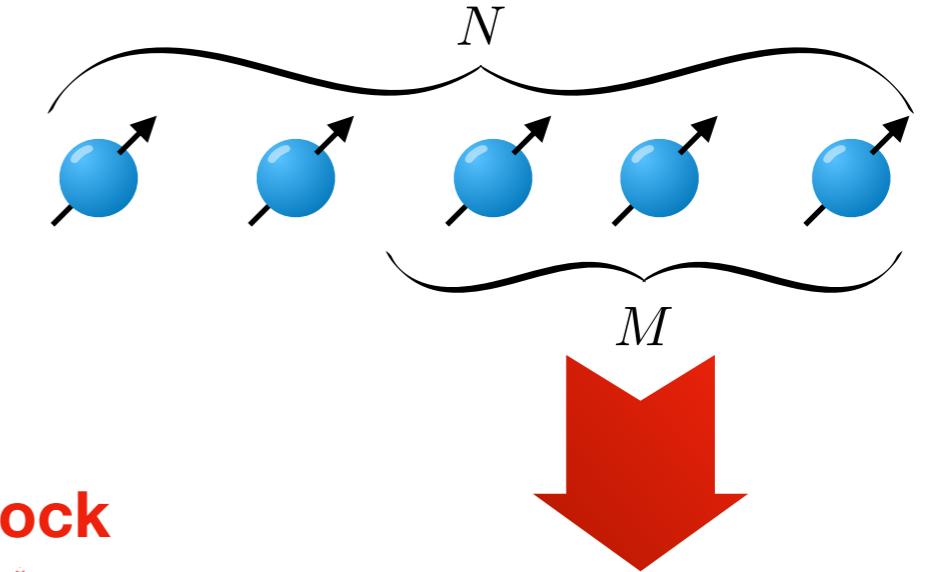
In integrable models:

$$N \rightarrow \infty \quad \frac{\mathcal{E}_B^{(N)}}{E_B^{(N)}} \rightarrow 1$$

Can be proven analytically

We extract energy from  $M$  over the total  $N$  qubits

$$N = 8$$



Coherent states are better!

# Energy charging via quench



$$\mathcal{H}(t) = [1 - \lambda(t)]\mathcal{H}_0 + \lambda(t)\mathcal{H}_1 \quad \lambda(t) = \mathbb{I}_{[0,\tau]}(t)$$

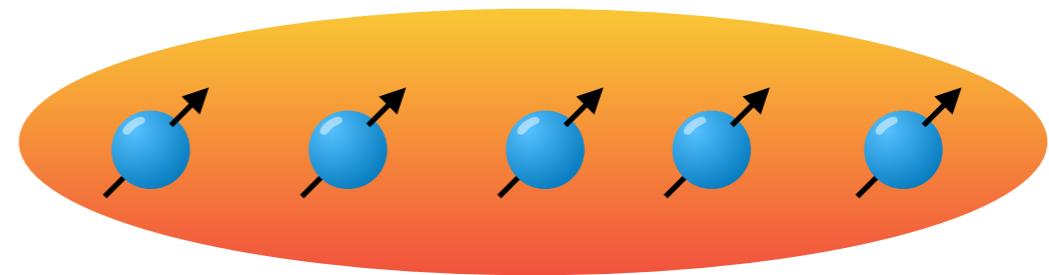


# Many-body localized Quantum Battery

## Disordered Quantum Ising Chain

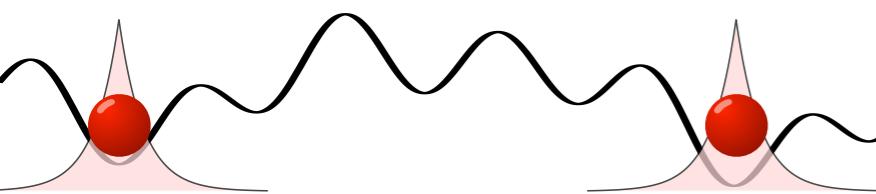
Jonas A. Kjäll, Jens H. Bardarson,  
and Frank Pollmann

Phys. Rev. Lett. **113**, 107204 (2014)



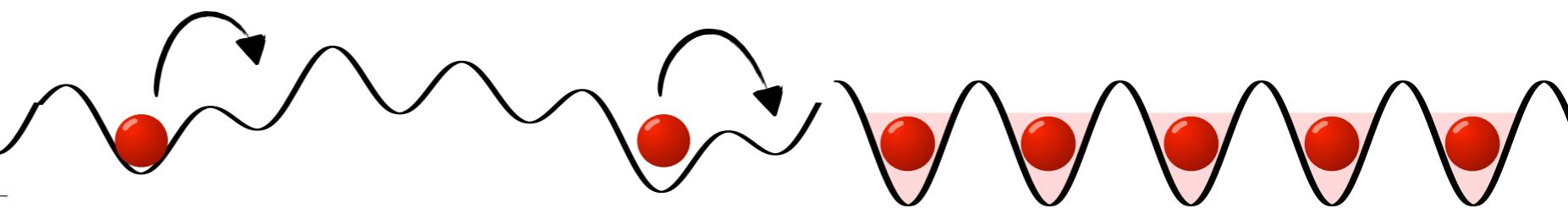
**Anderson localized**

Area law



**Many-body localized**

Area law (with log-corrections)



**Ergodic**

Volume law

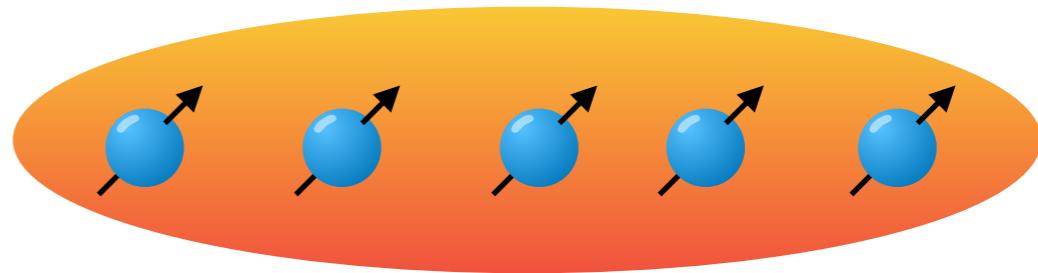
**Not a ground state property!**

# Many-body localized Quantum Battery

## Disordered Quantum Ising Chain

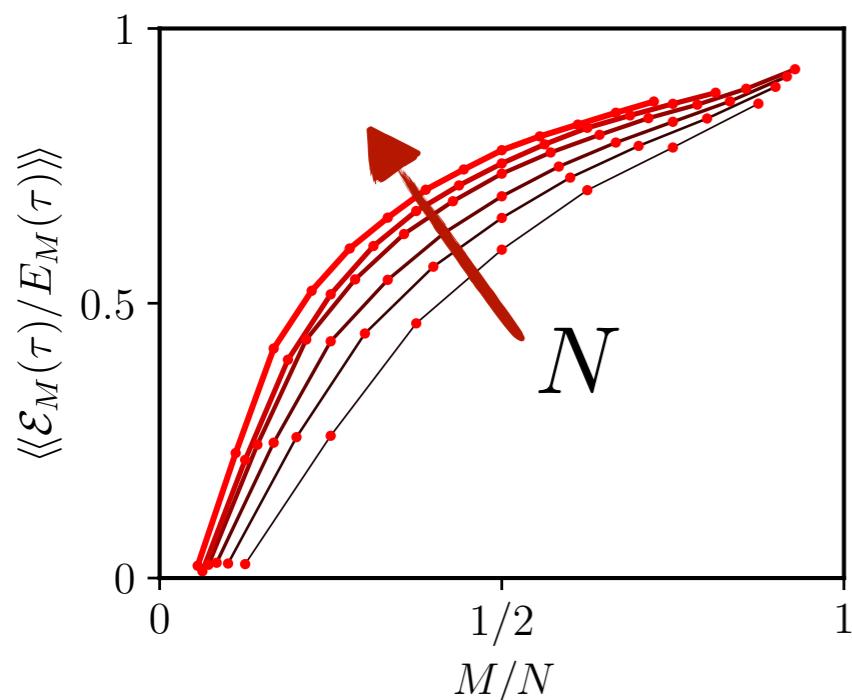
Jonas A. Kjäll, Jens H. Bardarson,  
and Frank Pollmann

Phys. Rev. Lett. **113**, 107204 (2014)



### Anderson localized

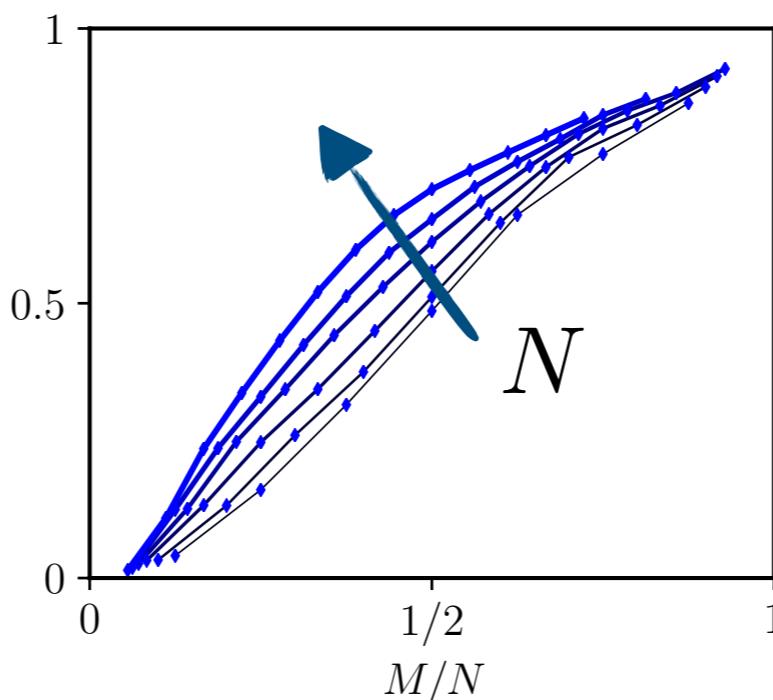
Area law (integrable)



$$\left\langle\left\langle \frac{\mathcal{E}_M(\tau)}{E_M(\tau)} \right\rangle\right\rangle \rightarrow 1$$
$$N \rightarrow \infty$$

### Many-body localized

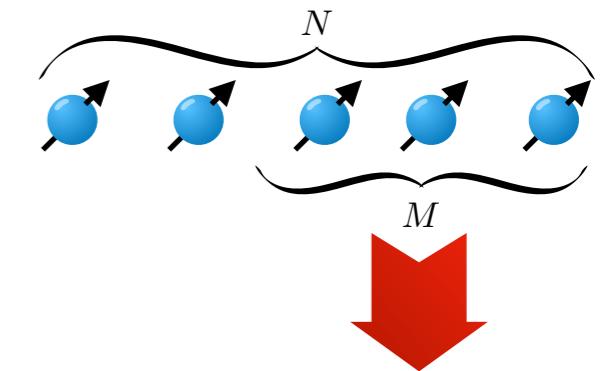
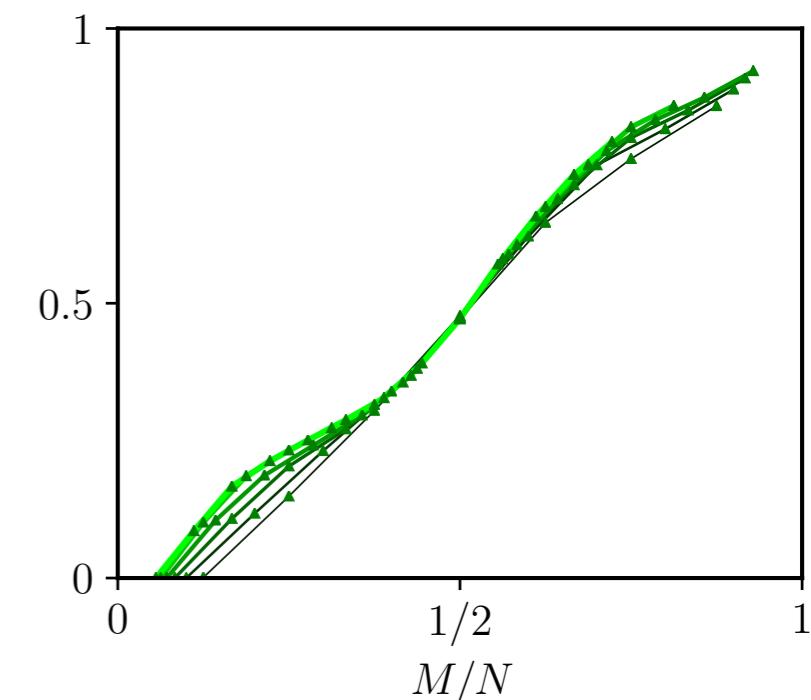
Area law (with log-corrections)



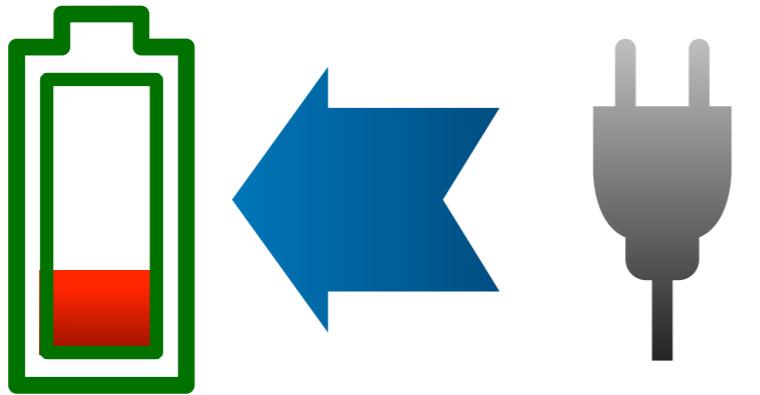
D. Rossini, G.M. Andolina and M. Polini  
arXiv:1906.00644

### Ergodic

Volume law

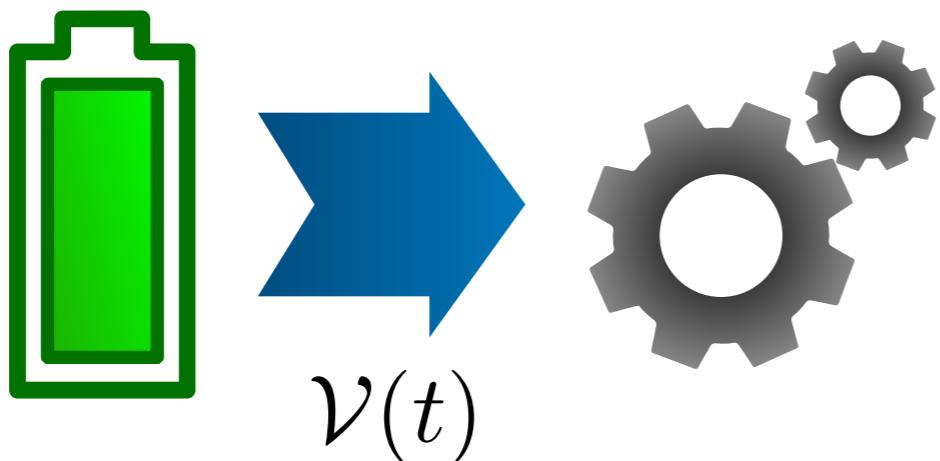


# Conclusions



**Collective evolution gives  
an advantage in the power**

$$\Gamma \sim \sqrt{N}$$



**Coherent states are “optimal” for  
work extraction**

**Integrable systems are  
convenient in the limit**  $N \rightarrow \infty$

# Thanks for your attention!

