Work Distributions on Quantum Fields

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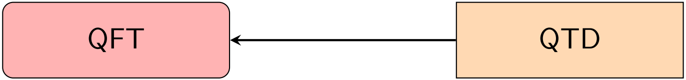
Phys. Rev. Lett. 122, 240604 (2019)

Context and Motivations

QFT → QTD

- Motivation: study the thermodynamics of quantum fields that are undergoing non-equilibrium transformations.
- Why?
- There are plenty of phenomena occurring in quantum fields with interesting thermodynamic properties: **quantum energy teleportation** [Hotta, 2008] (related to **algorithmic cooling** [Rodríguez-Briones et al., 2017]), **entanglement harvesting** [Reznik, 2003, Pozas-Kerstjens and Martín-Martínez, 2015] (generation of **correlations**), the **Unruh** effect (**thermalization**) [Fewster et al., 2016].

Context and Motivations



- Fluctuation theorems, do they hold in QFT?
- If so, this would provide an alternative path to calculate equilibrium properties of thermal states of quantum fields (such as ratios of partition functions), which are notoriously hard to calculate.

Work Fluctuations

- How do we introduce the notion of work fluctuations in QFT?
- There are many candidates to take as a notion for work fluctuation (Two Point Measurement (TPM) scheme, Gaussian measurements...) [Bäumer et al., 2018]
- Since we would like to recover Jarzynski's and Crooks' theorems, we look into the TPM scheme.

TPM Scheme

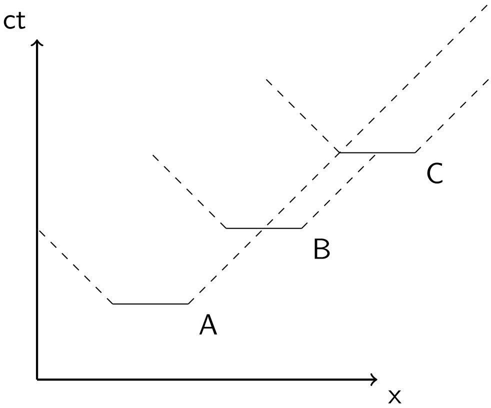
1. A projective measurement of $\hat{H}(0)$ is done on the initial state $\hat{\rho}$. This yields the energy measured as E_i and the post-measurement state $|E_i\rangle\langle E_i|$.
2. Unitary evolution of the post-measurement state according to the unitary associated to the process $\hat{U}(T,0)$.
3. A projective measurement of $\hat{H}(T)$ is done on $\hat{U}(T,0)\,|E_i\rangle\langle E_i|\,\hat{U}^\dagger(T,0)$, returning the value E_j'.

The possible values of the work $w^{(ij)}$ are defined as $w^{(ij)} = E_j' - E_i$. The work probability distribution is

$$P(W) = \sum_{(ij)} \delta\left(W - w^{(ij)}\right) \langle E_i|\,\rho\,|E_i\rangle\, |\langle E_j'|\,\hat{U}(T,0)\,|E_i\rangle\,|^2, \quad (1)$$

Problems with the TPM Scheme

- We cannot readily apply the TPM scheme in QFT, as doing (even local) projective measurements in quantum fields imply the possibility of superluminal signaling [Sorkin, 1993].

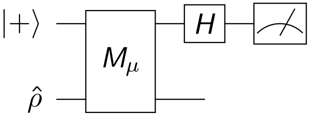


Measuring Quantum Fields

- The correct way to measure quantum fields is by coupling a non-relativistic quantum system to them and measuring it [Christopher J. Fewster, 2018].
- This procedure has been used to study a wide variety of phenomena in QFT [Simidzija and Martín-Martínez, 2018, Henderson et al., 2019].
- A possible operational definition could involve coupling an ancillary system to the quantum field and measuring it $\Rightarrow$ Ramsey scheme.

Ramsey Scheme and Definition

- The Ramsey scheme was first introduced in Quantum Thermodynamics as a way to experimentally measure the work fluctuations given by the TPM scheme [Dorner et al., 2013, Mazzola et al., 2014].



$$\hat{M}_\mu = \hat{U}_S e^{-\mu \hat{H}(0)} \otimes |0\rangle\langle 0| + e^{-\mu \hat{H}(T)} \hat{U}_S \otimes |1\rangle\langle 1|$$

- We use it instead as a way to operationally introduce work distributions in QFT.

Application

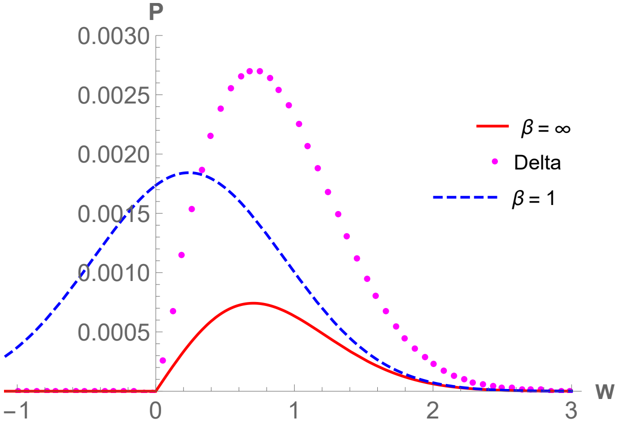
- We study processes generated by local Hamiltonians of the form $\hat{H}_\phi(t) = \hat{H}_0 + \lambda\chi(t)\int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x} F(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x}) = \hat{H}_0 + \hat{H}_I(t)$, where $\hat{\phi}(t,\boldsymbol{x})$ is a free scalar field, and $\hat{H}_I(0) = \hat{H}_I(\tau) = 0$. The initial state of the field is a KMS state [Kubo, 1957, Martin and Schwinger, 1959]

KMS States

KMS states generalize Gibbs' notion of thermality to cases where, due to the dimensionality of the Hilbert space, Gibbs thermal states are not well-defined.

They are characterized by properties of their two-point correlator.

P(W)



- The results are analytical.
- $\frac{\sigma_W^2}{\langle W \rangle} \to \infty$ as the unitaries become more localized in spacetime.

Fluctuation Theorems

- Jarzynski's equality: $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$
- Crooks' theorem: $\frac{P(W)}{P_{\text{rev}}(-W)} = e^{\beta W} \frac{Z_2}{Z_1}$
- Jarzynski's and Crooks' theorem hold (to second order) for the previous family of local unitaries $\hat{H}_\phi(t) = \hat{H}_0 + \lambda\chi(t) \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{x} F(\boldsymbol{x}) \hat{\phi}(t, \boldsymbol{x})$.
- The fluctuation theorems have been used to experimentally obtain differences in free energies between states where doing the calculations analytically was out of reach. [Liphardt et al., 2002, Park et al., 2003, Collin et al., 2005]
- Ratios of partition functions are very hard to calculate in QFT. If these theorems were to hold for quantum fields, a potentially easier way to obtain ratios of partition functions in QFT would be opened.

Further work

- Extend Jarzynski's and Crooks' theorems to more general classes of unitaries. A full proof would probably involve using tools from AQFT [Christopher J. Fewster, 2018, Manuceau and Verbeure, 1968], due to the need to work with KMS states.
- Possible experimental implementations in superconducting circuits [Sabín et al., 2012, L. García-Álvarez and Sabín, 2017, García-Álvarez et al., 2015, Forn-Díaz et al., 2017].
- Studying work fluctuations could shed some new light into the thermodynamics of local processes in QFT (i.e, entanglement harvesting, quantum energy teleportation, Unruh effect, among others).

Thank you for your attention.

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