Low dissipation regime optimisation and applications

Paolo Abiuso

24/06/19

ICFO

OUTLINE

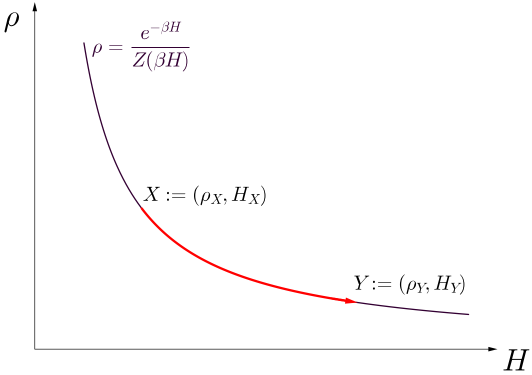
- Low dissipation regime
- Optimisation of thermal machines
- Applications: non-Markovian effects

Quasistatics

Thermal bath $T = 1/\beta$

control on $H(\vec{\lambda}(t))$

operational time $\tau \to \infty$



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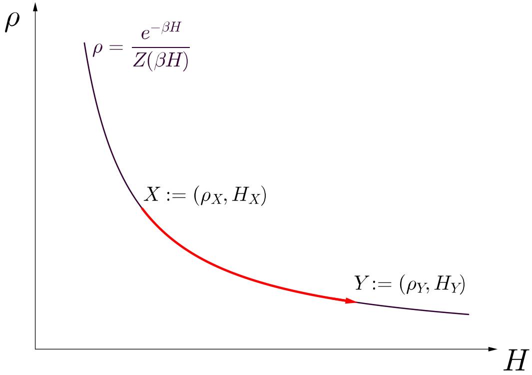

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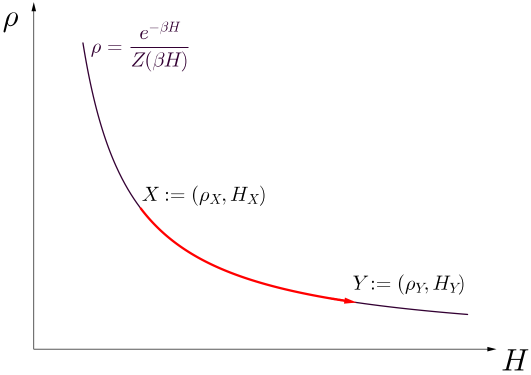

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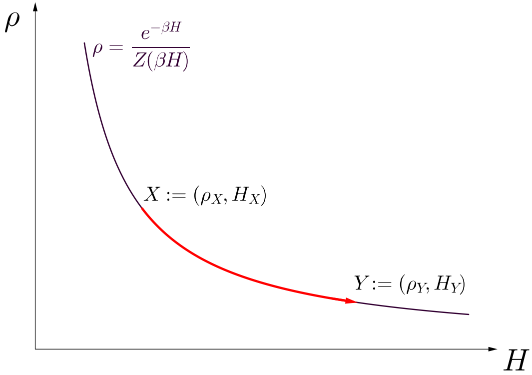

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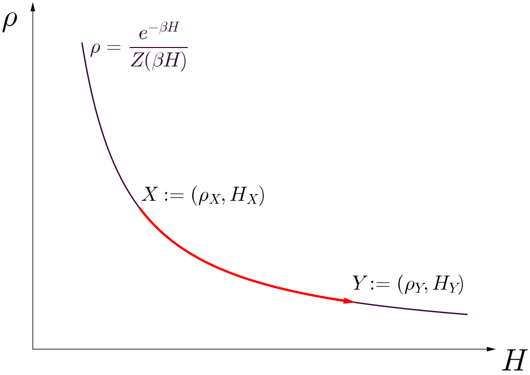

UNIVERSAL...

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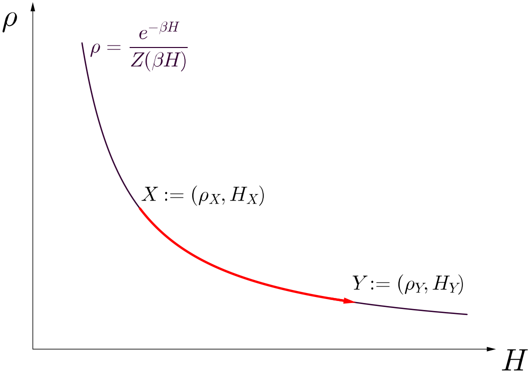

UNIVERSAL...
...but NO POWER!

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ΔS_{irr} ΔW_{dis}

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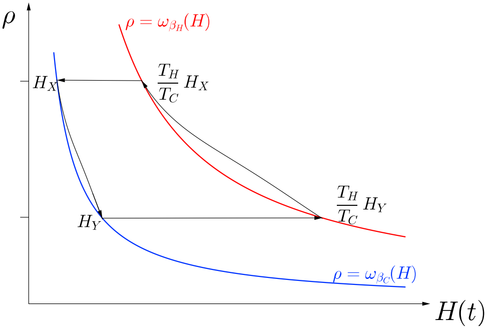

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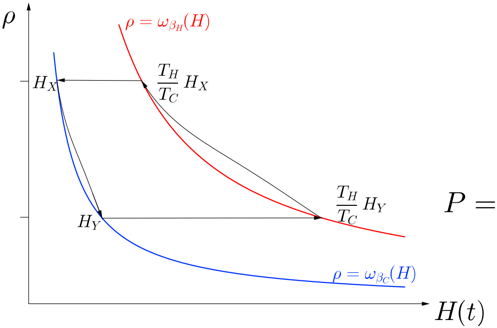
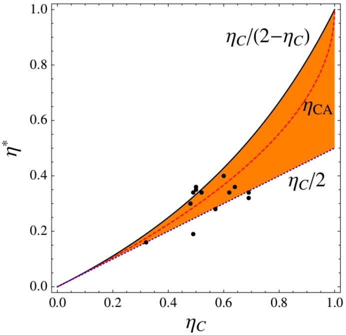


$$P = \frac{(T_H - T_C)\Delta S - T_H \frac{\Sigma_H}{\tau_H} - T_C \frac{\Sigma_C}{\tau_C}}{\tau_C + \tau_H}$$

Speed optimization (τ_H, τ_C)



M. Esposito et al.
PRL 105, 150603 (2010)

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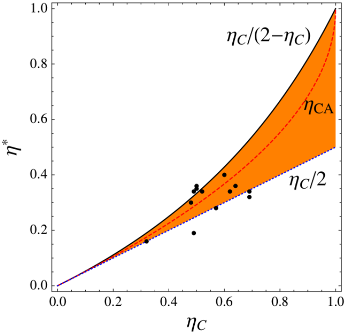

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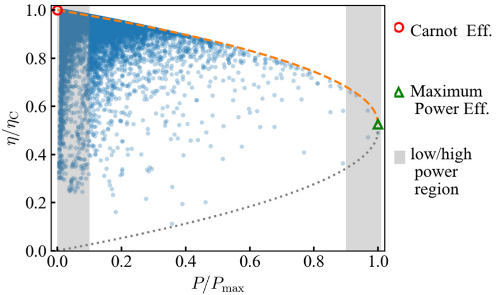

Y.-H. Ma et al.
PRE 98, 042112 (2018)

Symmetric case $\Sigma_H = \Sigma_C := \Sigma$

$$\eta = \gamma \eta_{\text{Carnot}} \qquad \gamma < 1$$

$$P_\gamma^{(\max)} = \frac{(\Delta S)^2}{4\Sigma} \frac{(T_H - T_C)^2 \gamma (1-\gamma)}{\gamma T_H + (1-\gamma) T_H}$$

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$$\omega_\beta(\vec{\lambda}) \Rightarrow \Delta S = \int_0^1 \vec{s}(\lambda) \cdot \dot{\vec{\lambda}}\, \mathrm{d}t$$

scalar product

Recipe for Σ?[1]

$\Sigma \equiv$ thermodynamic length $(X \xrightarrow{\lambda(\vec{t})} Y)$
$(0 \leq t \leq 1)$

- slow driving
- linear response
- discrete processes

$$\Sigma = \int_0^1 \dot{\vec{\lambda}}^T m(\lambda) \dot{\vec{\lambda}} \, \mathrm{d}t$$

quadratic form $(m > 0)$

[1] Scandi, Perarnau arXiv 1810.05583

Infinitesimal cycles are optimal

P.Abiuso, M.Perarnau arXiv 1907.xxxxx

$$\frac{\Delta S^2}{\Sigma} = \frac{\left(\int_0^1 \vec{s}(\lambda) \cdot \dot{\vec{\lambda}} \, \mathrm{d}t \right)^2}{\int_0^1 \dot{\vec{\lambda}}^T m(\lambda) \dot{\vec{\lambda}} \, \mathrm{d}t} \leq \int_0^1 \vec{s}^T m^{-1}(\lambda) \vec{s} \, \mathrm{d}t \leq \max_{\lambda} \left[\vec{s}^T m^{-1}(\lambda) \vec{s} \right]$$

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fullfilled for $\quad \vec{\lambda}(t) = \lim_{\varepsilon \to 0} \left[\vec{\lambda}^* + \varepsilon t \, m^{-1} \vec{s}(\lambda^*) \right]$

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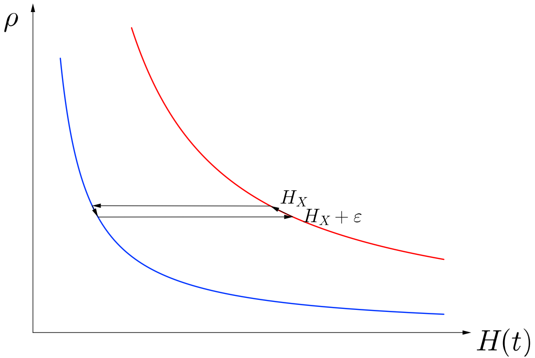

Applications

optimal control for the performance of:

- independent N-Qubits array, Ising chain, XY model
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Qubit Carnot Cycle ($H = \frac{E(t)}{2}\sigma_z$)

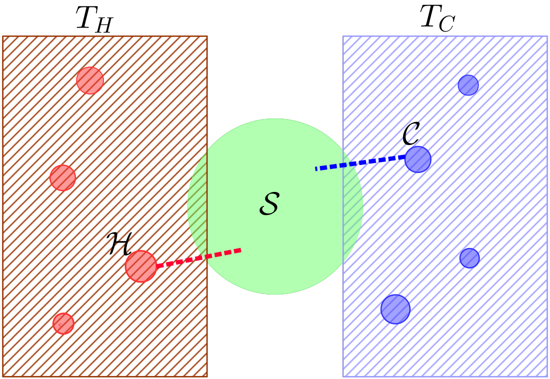
Using slow-driving[a] technique

$$\boxed{P_{max} \sim 0.11 \frac{(\sqrt{T_H} - \sqrt{T_C})^2}{A}}$$

$$A(\mathbf{model}) \simeq \tau_{\mathrm{rel}}$$

[a] Cavina, Mari, Giovannetti - Phys.Rev.Lett. **119**, 050601 (2017)

non-Markovian setup



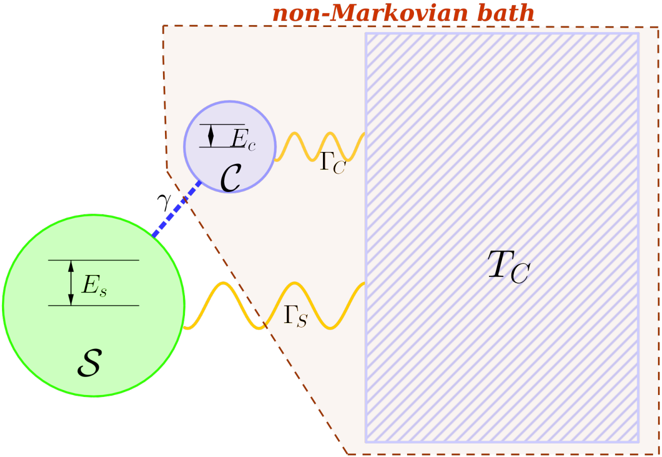
ρ_s

$\downarrow$

ρ_{hsc}

n-M model

$$\dot{\rho}_{sc} = -i[H_s + H_c + H_I, \rho_{sc}] + \mathcal{D}^s[\rho_{sc}] + \mathcal{D}^c[\rho_{sc}]$$



$y := \frac{\gamma}{\Gamma_S} \neq 0 \rightarrow$ non-Markovian dynamics on $\mathcal{S}$

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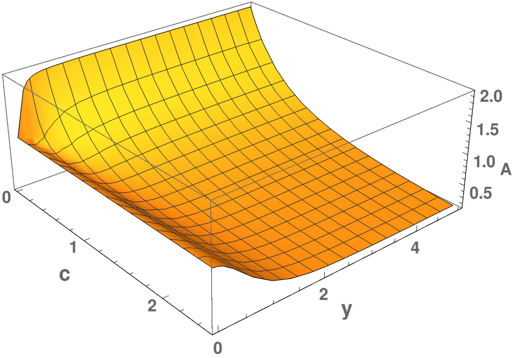

$$c = \frac{\Gamma_C}{\Gamma_S}$$

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Why faster thermalisation?

$$F_{sc} = F_s + F_c + T\mathcal{I}(\mathcal{S}:\mathcal{C})$$

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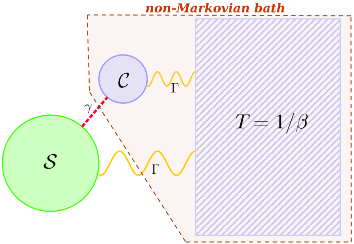

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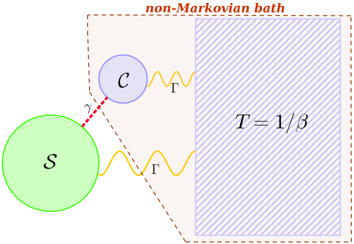

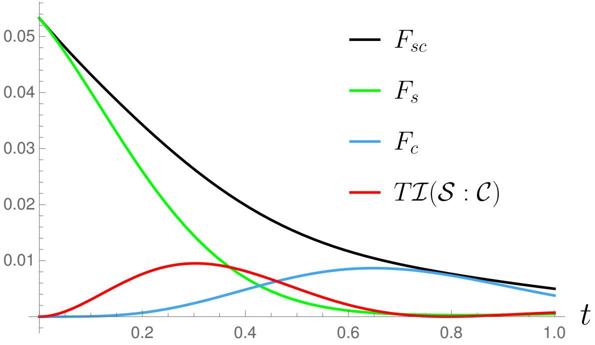

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- Non-Markov mechanism for accelerated thermalisation

THANK YOU!

P.Abiuso, M.Perarnau arXiv 1907.xxxxx
P.Abiuso, V.Giovannetti PRA **99**, 052106 (2019)