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Nonequilibrium optimization of the cooling of a dilute atomic gas

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Outline

1. Nonequilibrium entropy production
   - Microscopic expressions
   - Nonequilibrium statistical distance
2. Minimizing entropy production
   - Optimizing the cooling of a cup of coffee
   - Optimizing the cooling of dilute atomic gas

Thermodynamics: a short reminder

Equilibrium (nonequilibrium) processes:

Entropy: $\Delta S = Q/T + \Sigma$

Work: $W = \Delta F + W_{irr}$ ($F = U - TS$ = free energy)

with $\langle \Sigma \rangle \geq 0$ and $\langle W_{irr} \rangle \geq 0$ (Second law)

➔ thermodynamics does not allow computation of Σ, W_{irr}

Nonequilibrium entropy production:

$$\Sigma = \beta(W - \Delta F) = \beta W_{irr} \qquad \beta = 1/(kT)$$

➔ difference between total work and equilibrium work

Nonequilibrium entropy production

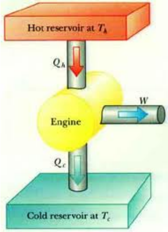
Maximum extractable work:

$$-\langle W \rangle = -\Delta F - kT\langle \Sigma \rangle \leq -\Delta F$$

➔ is reduced by nonequilibrium entropy production

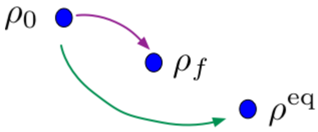
Efficiency of thermodynamic devices:

$$\eta = \left(1 - \frac{T_1}{T_2}\right) - T_1 \frac{\Sigma}{Q}$$



➔ fundamental quantity of nonequilibrium physics

Microscopic expression: single-step process



Case 1: Complete thermalization:

Schlögl Z. Phys. (1966)

$$\langle \Sigma \rangle = S(\rho_0 || \rho^{\mathrm{eq}}) \geq 0 \quad \text{with} \quad \rho^{\mathrm{eq}} = \exp(-\beta H)/Z$$

Relative entropy: $S(\rho_1 || \rho_2) = \mathrm{tr}(\rho_1 \ln \rho_1 - \rho_1 \ln \rho_2)$

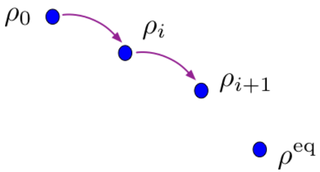
Case 2: Partial thermalization:

Deffner, Lutz PRL (2011)

$$\langle \Sigma \rangle = S(\rho_0 || \rho^{\mathrm{eq}}) - S(\rho_f || \rho^{\mathrm{eq}}) \leq S(\rho_0 || \rho^{\mathrm{eq}})$$

➔ entropy production = "entropic distance"

Microscopic expression: multi-step process



Case 1: Complete thermalization:

Nulton *et al.*, JCP (1985)

$$\langle \Sigma_i \rangle = S(\rho_i || \rho_{i+1}) \geq 0$$

Case 2: Partial thermalization:

$$\langle \Sigma_i \rangle = S(\rho_i || \rho^{\mathrm{eq}}) - S(\rho_{i+1} || \rho^{\mathrm{eq}}) \geq 0$$

Total entropy production: $\langle \Sigma \rangle = \sum_{i=1}^{N} \langle \Sigma_i \rangle$

Statistical distance: $L_i = \sqrt{2 \langle \Sigma_i \rangle}$

Thermodynamic length

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Thermodynamic Length and Dissipated Availability

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Measuring Thermodynamic Length

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Thermodynamic Metrics and Optimal Paths

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➔ statistical distance has not been measured so far

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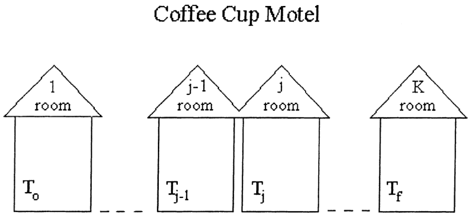
Minimizing entropy production

Cooling a cup of coffee: Salamon *et al.* J. Noneq Therm. (2002), Lima EJP (2015)



Complete thermalization:

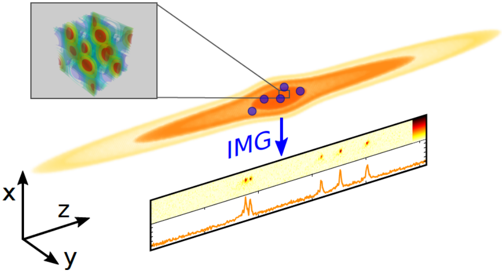
$$\langle \Sigma_i \rangle = S(\rho_i || \rho_{i+1}) \quad \text{with} \quad \delta Q_i = C\, dT_i$$

➔ optimal temperature sequence $\frac{T_{i-1}}{T_i} = \left(\frac{T_f}{T_0}\right)^{-1/K}$ for K steps
(entropy production minimized for equal temperature ratios)

➔ "systems like to stay close to equilibrium"

Minimizing entropy production

Dilute Cesium atoms in a nonharmonic MOT (Widera lab)



Effectively noninteracting atoms

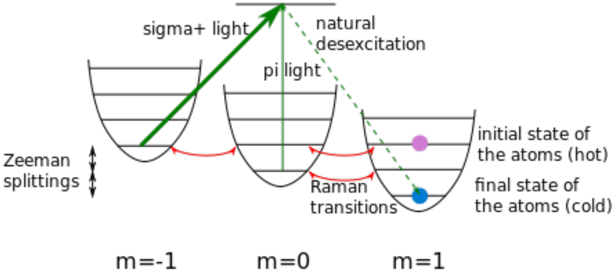
➔ no thermalization (over duration of the experiment)

Questions: how to cool the atomic gas?

Cooling dilute atomic gas

Sideband Raman cooling

Kerman *et al.* PRL (2000)

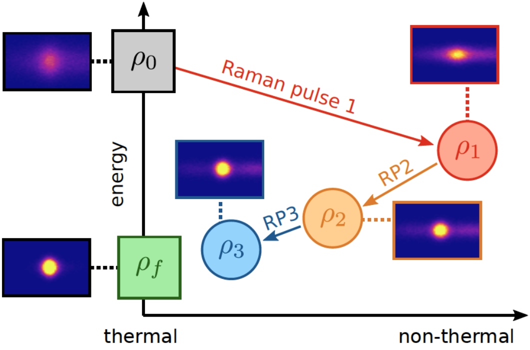


➔ standard subdoppler laser cooling scheme

Sideband Raman cooling

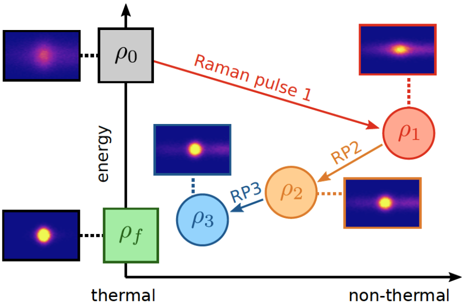
Raman cools momentum (but not position): nonthermal state

➔ apply sequence of pulses (equal spacing)



Question: what is the optimal spacing?

Optimal cooling



Partial thermalization:

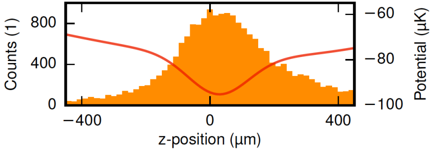
$$\langle \Sigma_i \rangle = S(\rho_i || \rho_f) - S(\rho_{i+1} || \rho_f) \geq 0$$

Minimize entropic distance to final (target) state:

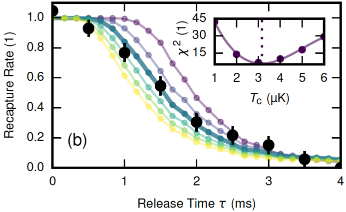
$S(\rho_3 || \rho_f)$ for a sequence of three pulses

What is measured?

1. Axial position distribution $f(z)$ (fluorescence imaging)



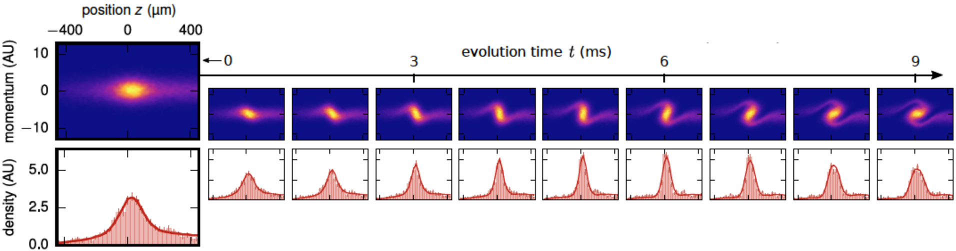
2. Momentum distribution after pulse $\tilde{f}(p_z)$ (release-recapture)



➔ independent after pulse $\rho(z, p_z) = f(z)\tilde{f}(p_z)$ (phase-space density)

Phase-space dynamics

Comparison theory/experiment:



➔ "whorls" created by nonharmonic potential

K-directed divergence

Relative entropy:

$$S(\rho_1||\rho_2) = \int dz \rho_1 \ln(\rho_1/\rho_2)$$

only defined if ρ_2 different from zero when ρ_1 different from zero

➔ zero bins due to finite statistics (in experiment or numerics) are problematic

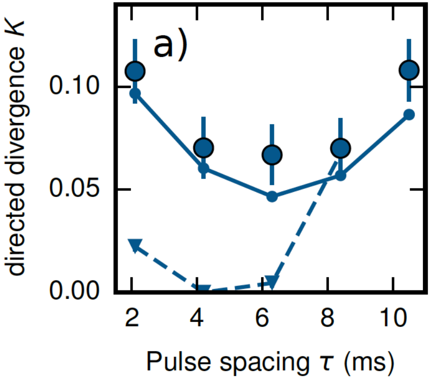
K-directed divergence: Lin , IEEE (1991)

$$K(\rho_1||\rho_2) = S(\rho_1||(\rho_1 + \rho_2)/2)$$

satisfies $K(\rho_1||\rho_2) \geq 0$ and $K(\rho_1||\rho_2) = 0$ iff $\rho_1 = \rho_2$ like $S(\rho_1||\rho_2)$

Results 1

Entropic distance $K(\rho_3||\rho_f)$:

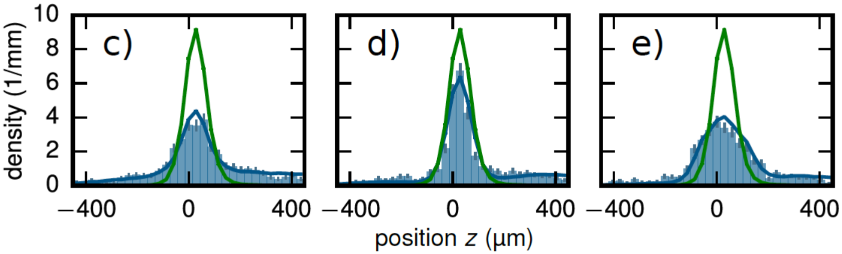


➔ harmonic (simulation) mimimum at 4.2ms (= quarter-period)

➔ nonharmonic (data) mimimum at 6.3ms (= no clear period)

Results 2

Overlap with final state for $\tau = $ 2.1 ms, 6.3 ms and 10.5ms:

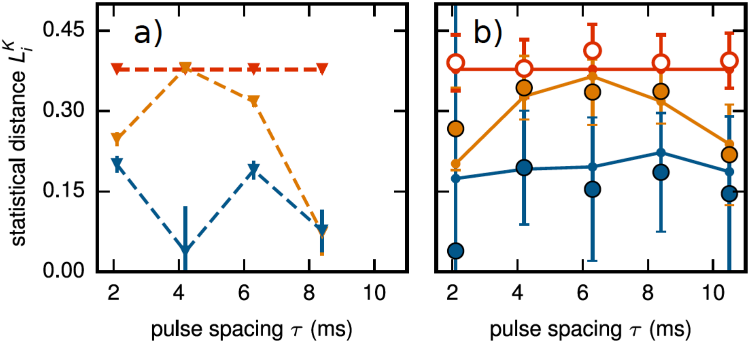


➔ overlap doubled for optimal time (more than 75%)

➔ temperature reduced by a factor 4: from 12.1 to 2.9 μK

Results 3

Statistical distance (harmonic - nonharmonic):



➔ information on the cooling *process*

➔ optimal cooling mainly achieved in first two steps

Summary

- entropy production and statistical distance are fundamental nonequilibrium quantities
- they can be measured in cold-atom experiment
- they can be used to successfully optimize cooling of a dilute atomic gas

Nonequilibrium optimization of the cooling of a dilute atomic gas, D. Mayer, F. Schmidt, S. Haupt, Q. Bouton, D. Adam, T. Lausch, E. Lutz, and A. Widera, arXiv:1901.06188